FLUID MECHANICS & FLUID MACHINES

Course Code: CE402

Module I: Basic concepts and Definitions- Distinction between a fluid and a solid Density, Specific weight, Specific gravity, Kinematic and dynamic viscosity, variation of viscosity with temperature, Newton law of viscosity; vapor pressure, boiling point, cavitations; surface tension, capillarity, Bulk modulus of elasticity, compressibility (4 hrs)

Module II: Fluid Statics- Fluid Pressure: Pressure at a point, Pascal's law, pressure variation with temperature, density and altitude. Piezometer, U-Tube Manometer, Single Column Manometer, U-Tube Differential Manometer, Micromanometers, pressure gauges, Hydrostatic pressure and force: horizontal, vertical and Inclined surfaces. Buoyancy and stability of floating bodies (6 hrs)

Module III: Fluid Kinematics- Classification of fluid flow: steady and unsteady flow; uniform and non- uniform flow; laminar and turbulent flow; rotational and irrotational flow; compressible and incompressible flow; ideal and real fluid flow; one, two and three dimensional flows; Stream line, path line, streak line and stream tube; stream function. velocity potential function. One, two and three dimensional continuity equations in Cartesian coordinates (6 hrs)

Module IV: Fluid Dynamics – Surface and body forces: Equations of motion- Euler's equation; Bernoulli's equations- derivation; Energy Principle; Practical applications of Bernoulli's equation: venturimeter, orifice meter and pitot tube; Momentum principle; Forces exerted by fluid flow on pipe bend; Vortex Flow – Free and Forced (8 hrs)

Module V: Boundary layer theory, laminar and turbulent flow and flow through pipes (6 hrs)

Module VI: Dimensional Analysis and Dynamics Similitude- Definitions of Reynolds Number, Froude Number, Mach Number, Weber Number and Euler Number; Buckingham's π- Theorem . (4 hrs)

Module VII: Fluid machines; Impact of Jets; Introduction to Turbines and Pumps (8 hrs)

Text/Reference Books:

1. Fluid Mechanics and Machinery, C.S.P. Ojha, R. Berndtsson and P.N. Chandramouli, Oxford University Press 2010
2. Hydraulics and Fluid Mechanics, P.M. Modi and S.M. Seth, Standard Book House.
3. Theory and Applications of Fluid Mechanics, K. Subramanya, Tata McGraw Hill
4. Fluid Mechanics with Engineering Applications, R.L. Daugherty, J.B. Franzini and E.J. Finnemore, International Student Edition, Mc Graw Hill.
5. Elementary fluid mechanics, Dr. R.J. Garde.
6. Fluid Mechanics, R.K. Bansal.

1. Density (ρ) = $\frac{M}{V}$ $\left(\frac{Kg}{m^3}\right)$

Dimensional formula = $[ML^{-3}]$

ρ_{H_2O} at 4°C = 1000 kg/m^2

Ques:- Why density of water is maximum at 4°C?

For the gas
ideal gas equation

$PV = mRT$

$P = \left(\frac{m}{V}\right) RT$.

$\boxed{P = \rho RT}$

P ↑	ρ ↑
T ↑	ρ ↓

→ For the liquids no change of density w.r.t pressure and temperature.

2. Specific weight (weight density) = $\frac{\text{weight}}{\text{volume}}$ (KN/m^3)

denoted by γ (gamma)

$$\gamma = \frac{\text{weight}}{\text{Volume}} = \frac{mg}{V} = \rho g$$

$\boxed{\therefore \gamma = \rho g}$, $\gamma_{H_2O} = 9810\ N/m^3$

Dimensional formula = $[ML^{-2}T^{-2}]$

3. Specific gravity (s)

$$S_{fluid} = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}$$

$$= \frac{\text{wt. density of fluid}}{\text{wt. density of standard fluid}}$$

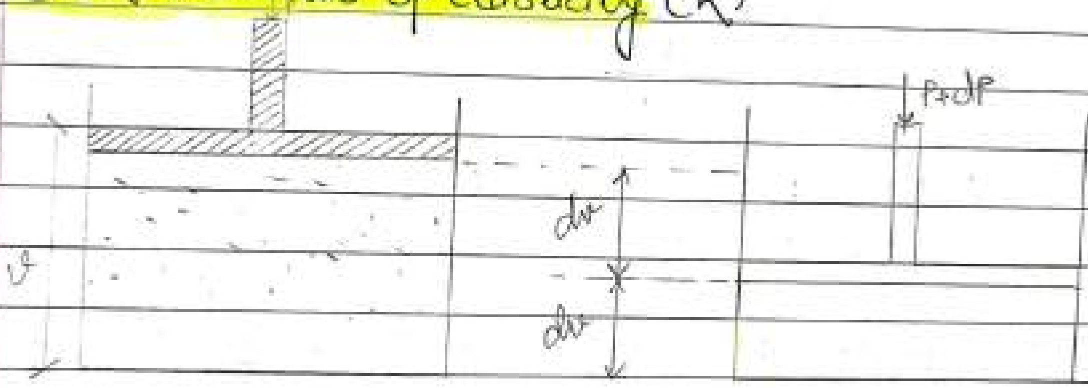
Standard fluid for liquid → water

Standard fluid for gas → air

$S_{H_2O} = 1$, $S_{Hg} = 13.6$

NOTE :- The term Relative density is also sometimes used when the comparison is not made w.r.t to the standard fluid.

5. Bulk modulus of Elasticity (K)



$V_i = V$ $\quad\quad$ $V_f = V - dV$

$P_i = P$ $\quad\quad$ $P_f = P + dP$

$$K = \frac{\text{Change in pressure}}{\text{Volumetric strain}}$$

$$= \frac{\text{Change in pressure}}{\frac{\text{Change in volume}}{\text{Original volume}}}$$

$$= \frac{P_f - P_i}{\frac{V_f - V_i}{V_i}} = \frac{(P+dP)-(P)}{\frac{(V-dV)-(V)}{V}} = \frac{dP}{\frac{-dV}{V}}$$

$$\therefore \boxed{K = -V \cdot \frac{dp}{dV}}$$

∴ The -ve sign appears in the above equation as with increase of pressure, volume decreases.

6. Compressibility (β)

It is defined as the property of virtual of which fluid undergo a change of volume under the action of external pressure forces.

$$\therefore \beta = \frac{-dV/V}{dP} = \frac{1}{K}$$

$$\rho = m/V$$

$$\therefore m = \rho V$$

Differentiating both sides.

$\Rightarrow dm = d(\rho V)$

$\Rightarrow 0 = \rho dV + V \cdot d(\rho)$ (∵ mass is constant)

$\Rightarrow -\rho dV = V \cdot d\rho$

$$\therefore \boxed{\frac{-dV}{V} = \frac{d\rho}{\rho}}$$

$$\boxed{K = -V \cdot \frac{dp}{dV} = \frac{dp}{\frac{d\rho}{\rho}} = \frac{\rho \cdot dp}{d\rho}}$$

For constant pressure process

$$dP = 0$$

$$\therefore K = 0$$

For constant volume process

$$dv = 0$$

$$\therefore K = \infty$$

For, Isothermal Bulk modulus of elasticity

$$PV = \text{constant}$$

differentiating on both sides.

$$Pdv + VdP = 0$$

$$-\frac{dv}{V} = \frac{dP}{P}$$

$$\therefore K = \frac{dP}{\frac{-dv}{V}} = \frac{dp}{\frac{dp}{p}} = P$$

$$\boxed{K_{iso} = P}$$

Adiabatic bulk modulus of elasticity.

$$pv^{\gamma} = C \quad \text{(Adiabatic process)}$$

$$\Rightarrow p \cdot \gamma \cdot v^{\gamma-1} dv + v^{\gamma} \cdot dp = 0$$

$$\Rightarrow \frac{p \cdot \gamma \cdot v^{\gamma}}{v} \cdot dv + v^{\gamma} \cdot dp = 0$$

$$\Rightarrow \frac{p \cdot \gamma \cdot v^{\gamma}}{v} \cdot dv = -v^{\gamma} \cdot dp$$

$$\therefore p\gamma \cdot \frac{dv}{v} = -dp$$

$$\therefore -\frac{dv}{v} = +\frac{dp}{p} \times \frac{1}{\gamma}$$

$$\therefore K = \frac{dp}{\frac{-dv}{v}} = \frac{dp}{\frac{dp}{p} \times \frac{1}{\gamma}} = p \times \gamma$$

$$\boxed{K_{adibatic} = P\gamma}$$

Here,

$\gamma \rightarrow$ Adiabatic index

$\gamma = C_P / C_V$

C_p — Specific heat of constant pressure.

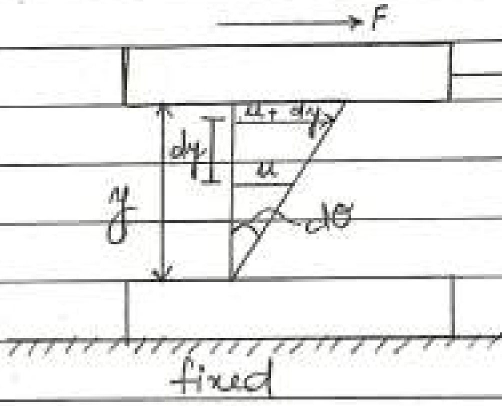
C_v — Specific heat of constant volume.

7. Viscosity

It is the internal resistance offered by the one fluid layer for the movement of adjacent fluid layer. It is due to the intermolecular exchange and the cohesive force between the molecules.

$$V = d/t$$

$$\therefore d = V \times t$$



→ For the fluid particles which is on the top plate, travelled a distance $= v \times dt$

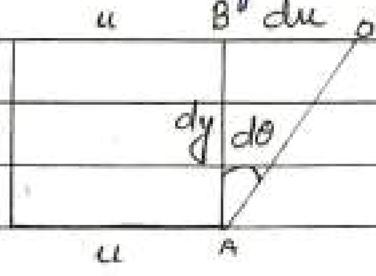
$$\tan d\theta = \frac{v \times dt}{y}$$

For small angles $\tan d\theta \approx d\theta$

$$d\theta = \frac{v \times dt}{y}$$

$$\boxed{\therefore \frac{d\theta}{dt} = \frac{v}{y}}$$

$\therefore$ The term $\frac{d\theta}{dt}$ is called 'Rate of angular deformation'.



From ΔABO

$$\tan d\theta = \frac{du \cdot dt}{dy}$$

For small angle $\tan d\theta \approx d\theta$

$$\therefore d\theta = \frac{du \cdot dt}{dy}$$

$$\boxed{\therefore \frac{d\theta}{dt} = \frac{du}{dy}}$$

$\frac{d\theta}{dt} \rightarrow$ Rate of shear strain or, Rate of angular deformation as, shear force, $F \uparrow$, Rate of angular deformation $\frac{d\theta}{dt} \uparrow$

$$F \propto \frac{d\theta}{dt}$$

$$\tau = F/A$$

$\tau \propto F$ ($\because A \rightarrow$ area of plate is constant)

$$\tau \propto \frac{d\theta}{dt}$$

$$\boxed{\tau = \mu \frac{d\theta}{dt}}$$

where, μ is the proportionality constant and is known as coefficient of dynamic viscosity.

$$\mu = \frac{\tau}{\frac{d\theta}{dt}}$$

If $\frac{d\theta}{dt} = 1$, $\mu = \tau$

$\therefore$ Coefficient of dynamic viscosity is defined as the amount of shear stress required to produced unit

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rate of angular deformation or, rate of shear strain.

$$\tau = \frac{\mu \cdot v}{y} = \frac{\mu \cdot dv}{dy}$$

$$\boxed{\tau = \mu \cdot \frac{du}{dy}} \rightarrow \text{Newton's law of viscosity}$$

The term $\frac{du}{dy}$ unit of $\frac{m/s}{m} = s^{-1}$

∴ The term $\frac{du}{dy}$ is called 'velocity gradient'.

Unit of μ :

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\therefore \mu = \frac{\tau}{\frac{du}{dy}} = \frac{N/m^2}{s^{-1}} = \frac{N \cdot s}{m^2} = Pa\text{-}s$$

$$1\,Pa\text{-}s = \frac{N}{m^2} \times s = \frac{kg \cdot m s^{-2} \times s}{m^2} = \frac{kg}{m \times s} \text{ (S.I unit)}$$

$$\frac{1kg}{m \cdot s} = \frac{1000g}{100cm \times s} = \frac{10g}{cm \times s}$$

angular velocity

$$\frac{1kg}{m \cdot s} = 10 \text{ poise}$$

$v = \omega \times r$

$\omega = \frac{2\pi N}{60}$ $\frac{rad}{sec}$

$$\boxed{\therefore 1 \text{ poise} = 0.1 \frac{kg}{m \times s} = 0.1 \frac{N \cdot s}{m^2}}$$

$\omega = \frac{1}{sec}$

* If $\mu = 0$, then $\tau = 0$, the fluid is called an 'Ideal fluid'. The fluid which satisfies Newton's law of

viscosity is called 'Newtonian Fluid'.

For Newtonian fluid $\tau = \mu \cdot \frac{du}{dy}$

e.g. Petrol, diesel, water etc.

The fluid which does not obey Newton's law of viscosity is called Non-newtonian fluid.

The study of Non-Newtonian fluid is known as "Rheology".

E.g: Blood, milk, Clay etc.

The Relationship between shear stress and velocity gradient for Non-newtonian fluid.

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

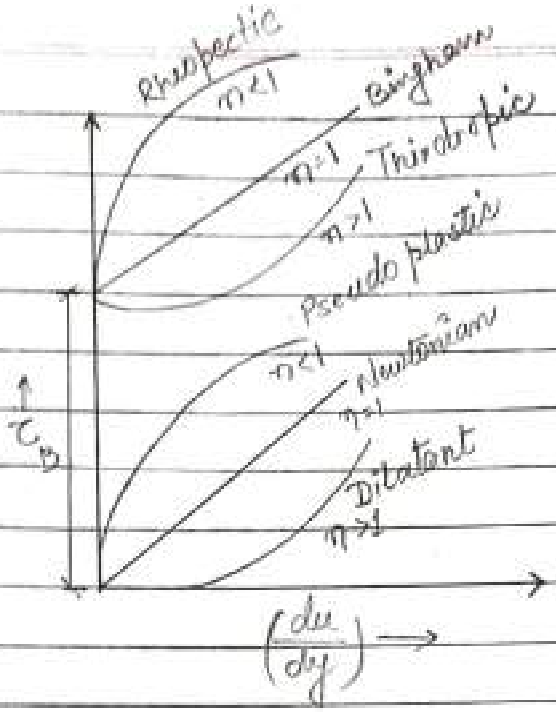
If $n=1$, $A=\mu$, $B=0$, then the modal is identified as Newtonian modal

Fluids

Newtonian fluid — Non-newtonian fluid

Time Independent — Time dependent

Dilatant (milk) — Pseudo (Blood) — Bingham (Toothpaste) — Thixotropic (Lipstick) — Rheopectic (Gypsum soln in water)



Kinematic viscosity (ν)

$$\nu = \frac{\text{dynamic viscosity}}{\text{density}}$$

Unit of kinematic viscosity

$$\nu = \frac{\mu}{\rho} = \frac{\frac{kg}{m \cdot s}}{\frac{kg}{m^3}} = \frac{m^2}{s} \text{ (S.I)}$$ 1 stroke = 10^{-4} m²/s

$$= (100)^2 \frac{cm^2}{s}$$

$$= 10^4 \text{ stroke}$$

$$\boxed{\therefore 1 m^2/s = 10^4 \text{ stroke}}$$

Vapour pressure

Consider a closed container in which a liquid is filled by leaving an empty space above the liquid surface of the temperature is increased the vapour bubble will starting forming and leaves from the liquid surface and get collected in empty space. The stage will reach where the Rate of collapsing of vapour bubble into the liquid surface. Under the condition vapour phase is said to be in phase equilibrium with the liquid.

The pressure exerted by the vapour upon the liquid surface is called vapour pressure.

If the partial pressure of water vapour is less than the vapour pressure, then no water is present in the lake.

If the partial pressure of water vapour is equal to the vapour pressure then water is present in the lake.

~~If the pressure falls below~~

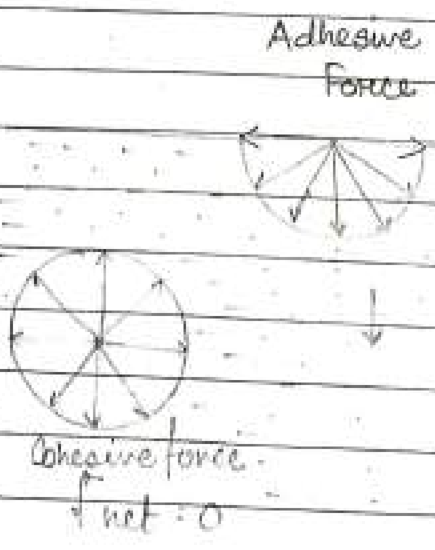
If any fluid flow problems, at any section the pressure should not fall less than or equal to vapour pressure, to avoid the cavitation.

If the pressure falls below the vapour pressure then vapour bubble will form and these will be carried along with flowing fluid into the high pressure region where they will burst and it may damage the surfaces and the action is called the pitting action.

Surface Tension

Tension exerted by fluid at the surface is called Surface tension.

Consider a molecule which is completely inside the liquid and



SI unit (N/m) of surface tension

Force per unit length = Surface tension

it is surrounded by equal no. of molecules.
∴ The net force acting upon such molecules is zero.

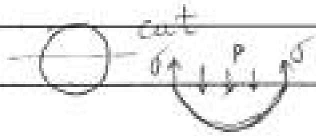
If we consider another molecule which is at the surface, it is under of the influence of attractive force by its own molecules and the gas molecules attractive force.

The cohesive force are greater than the adhesive force and it is acted by a net downward vertical force as well as the surface molecules are also subjected to cohesive force by the molecules which lies to the left and right of the surface of the liquid is under tension. The tension per unit length is called surface tension.

Units - N/m or J/m^2

Eg. - (a) A small needle can float
(b) Collection of dust particles on the liquid surface
(c) Small birds can sit in the ponds for drinking water.
(d) formation of liquid droplets.

Case (1) Liquid droplets



d = diameter of droplet

σ = surface tension = surface tension × length

p = pressure in excess of atmosphere under equilibrium.

Pressure force = surface tension

$$p \times \frac{\pi}{4} \times d^2 = \sigma \times (\pi \times d)$$

$$\therefore \boxed{p = \frac{4\sigma}{d}}$$

Patm = 10.3 m
Patm = Fgh
Hg = 13600 kg/m^3
g = 9.81

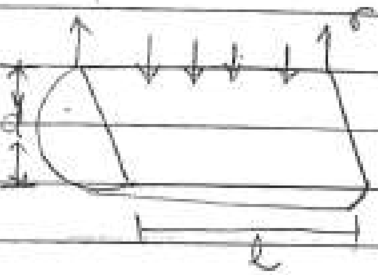
Case (2) soap bubble

Pressure force = Surface tension

$$p \times \frac{\pi}{4} \times d^2 = \sigma \times (2 \times \pi \times d)$$

$$\therefore \boxed{p = \frac{8\sigma}{d}}$$

Case (3) Liquid jet



Pressure force = surface tension

$$p \times l \times d = \sigma \times 2l$$

$$\boxed{\therefore p = \frac{2\sigma}{d}}$$

Surface tension is due to cohesive forces, surface tension decrease with increase in temperature.

$P_{atm} = \rho_{water}\, g\, h$

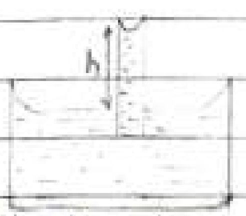
$1 = 1000 \times 9.81$

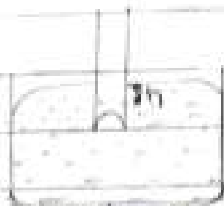
$\frac{1}{1000 \times 9.81} = h$

Effect of pressure in viscosity

For liquid, viscosity is practically independent of pressure. except at extremely high pressure for gases. dynamic viscosity is generally independent of pressure particularly (at low to moderate pressure) but kinamatic pressure viscosity decreases as density is proportional to pressure.

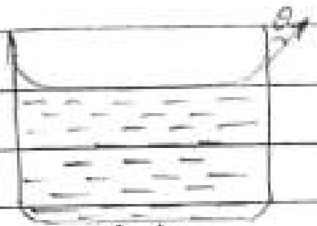
Capillarity:



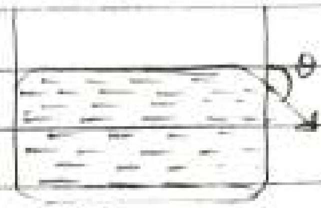


→ Capillarity rise
→ Adhesive > Cohesive

→ Capillarity fall
→ Cohesive > Adhesive



Water
Concave up
$\theta < 90^\circ$
wetted case
Adhesive > Cohesive



Mercury
Concave down
$\theta > 90^\circ$
not wetted case
Cohesive > Adhesive

The Rise and Fall of the liquid level in a narrow vertical tube when its inserted in a liquid from the level of the liquid in a container under equilibrium.

Surface tension force = Weight of liquid in the tube

$$\pi d \times \sigma \cos\theta = \rho g \times \frac{\pi}{4} \times d^2 \times h$$

$$\therefore h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\gamma d}}$$

If $\theta > 90^\circ$, $\cos\theta = -ve$

$\therefore$ h becomes -ve (capillarity fall)

A d↑, h↓

$\therefore$ For the tubes of monometer to avoid the correction for capillarity, the d > 6mm is preferred.

For water,
$\theta = 0°$

$$\boxed{h = \frac{4\sigma}{\rho g d}}$$

NOTE : Capillarity is due to both Cohesion and Adhesion

Numericals :-

1. A plate with surface area of $0.4 m^2$ and weight of 500N slides down on an inclined plane at 30° to the horizontal at a constant speed of 4 m/s. If the inclined plane is lubricated with an oil of dynamic viscosity 2 poise, find the thickness of lubricant film.

Sol:- Surface area of plate 'A' = $0.4 m^2$
Weight of plate 'W' = 500 N
Velocity of sliding plate 'V' = 4 m/sec
Dynamic Viscosity 'μ' = 2 poise
= 0.2 poise

Let,

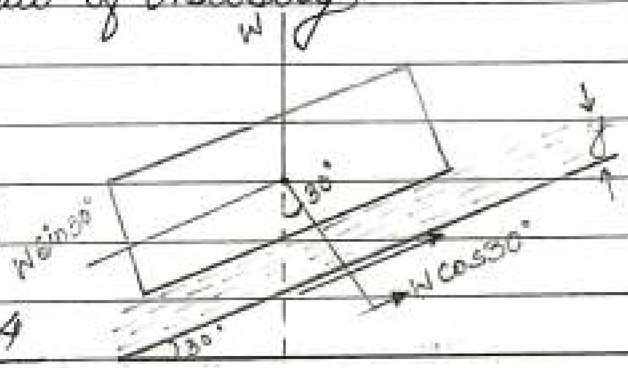
The thickness of lubricant film - 'y'

As the plate slides down the shear stress 'τ' generates in a upward direction.

$\therefore$ For equilibrium at terminal velocity component of weight of plate in the Direction of motion = Share friction in opposite direction

$\therefore W \sin 30^\circ = \tau \cdot A$

According to Newton's Law of Viscosity



$$\tau = \mu \frac{du}{dy}$$

$$\therefore W \sin 30^\circ = \frac{\mu \cdot V}{y} \cdot A$$

$$500 \times \frac{1}{2} \times 0.2 \times 4 \times 0.4 \over y$$

Solving, $y = 1.20 \times 10^{-3}$ m $= 1.20$ mm

$\therefore$ The thickness of lubricant film $= 1.20$ mm.

2. A thin square plate 1m x 1m is placed horizontally in a gap of height 2cm. Filled with oil of viscosity 10 poise and pulled at a constant velocity of 0.10 m/sec. Find the force on the plate.

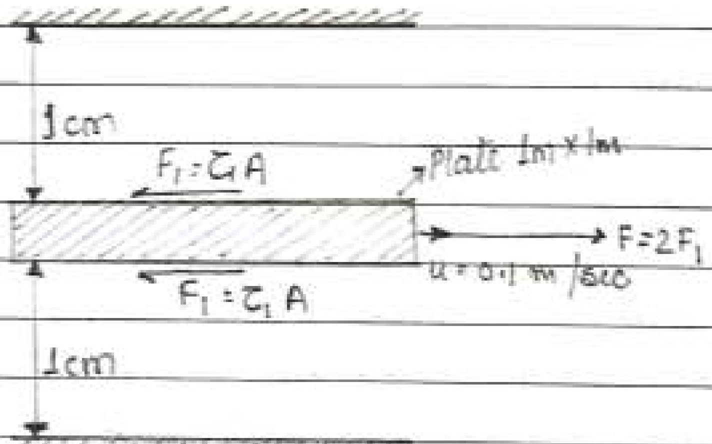
The gap is now filled with another oil, when the plate is placed at a distance of 0.5 cm, from one of the surfaces of the gap and pulled with the same velocity, the force on the plate remains same as before. Find the viscosity of new oil.

Sol:- Data given,

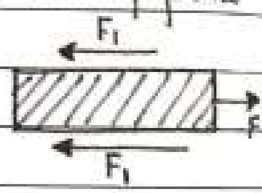
$$\mu = 10\,Poise = 1.0\,\frac{N\cdot s}{m^2}$$

$$u = 0.1\ m/sec.$$

Case 1:-



F·B·D of plate



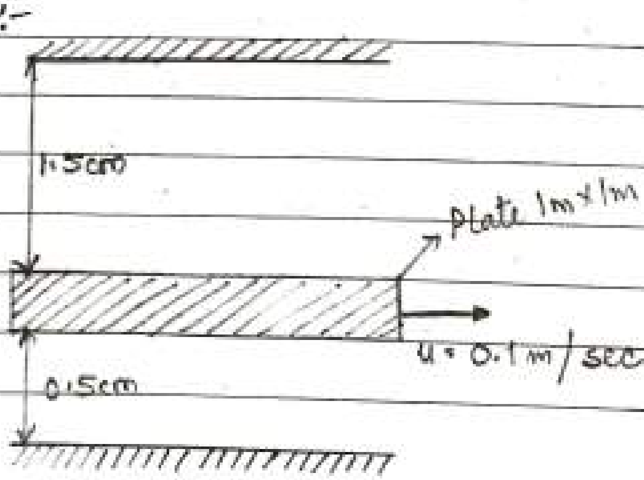
Force on one side of plate.

$$\Sigma F = m\vec{a} \quad (\vec{a} = 0)$$

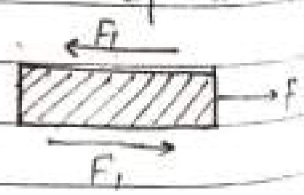
$$F_1 = \mu\left(\frac{du}{dy}\right) \times A = 1 \times \frac{0.1}{(1\times10^{-2})} \times 1 = 10\,N$$

$$F = 2F_1 = 2\times10 = 20\,N$$

Case 2:-



FBD of Plate



Force on top of plate $= \mu \left(\frac{du}{dy}\right)_{top} \times Area$

$$= \mu \cdot \frac{0.1}{1.5 \times 10^{-2}} \times 1 = \frac{20}{3}\mu$$

Force on bottom of plate

$$= \mu \left(\frac{du}{dy}\right)_{bottom} \times Area$$

$$= \mu \frac{0.1}{(0.5 \times 10^{-2})} \times 1 = 20\mu$$

Total force in Case (2) $= \left(\frac{20}{3}\mu + 20\mu\right) = \left(\frac{80}{3}\mu\right)$

From (1) and (3)

$$\frac{80}{3}\mu = 20$$

$$\mu = \left(\frac{3}{4}\right) = 0.75 \left(\frac{Ns}{m^2}\right) \quad \text{Ans:-}$$

Q. Velocity $= 5\sin(2\pi y)$

$\mu = 5$ poise

τ (stress) at $y = 0.05m = ?$

Sol:- $\tau = \mu \frac{du}{dy}$

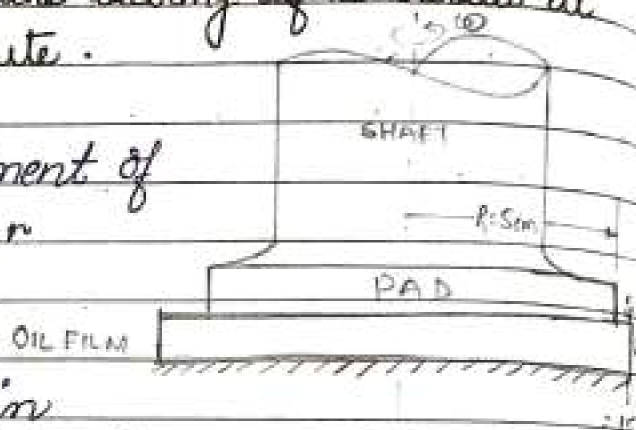
$$\tau = \mu \cdot \frac{5\sin 2\pi y}{dy}$$

$$\tau = \mu \times 5 \times \cos 2\pi y \times 2\pi$$

$$\tau = 0.5 \times 5 \times (\cos 2\pi \times 0.05) \times 2\pi$$

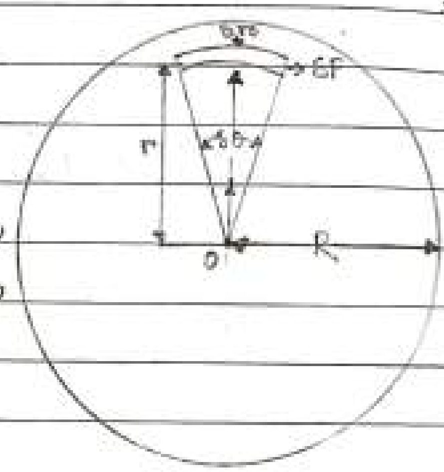
$$\tau = 15.70 \ N/m^2 \quad \text{Ans:-}$$

Q. A thrust bearing consists of a 10cm diameter pad rotating on another pad separating by an oil film $\mu = 80\,cP$ by 1.50mm. Compare the power dissipated in the bearing if it rotates at 100 revolutions per minute.



Sol:- Consider a small element of width δr at a radius r subtending a small angle $\delta\theta$ at the centre of the pad as shown in the plan view. The area of the element is $\delta r \times r\delta\theta$

It experiences a shear force δF due to shearing action of the fluid contained between the pads.



The rotational speed of the pad is given by

$$\omega = 2\pi \times \frac{100}{60} = 10.47 \text{ rad/s}$$

The linear velocity u at a radial distance r must be

$$u = r\omega = 10.47r \text{ m/s}$$

Since the lower pad is at rest, the velocity gradient

$$\frac{du}{dy} = \frac{10.47r - 0}{0.0015} = 6980r \text{ rad/s}$$

From the data,

$$\mu = 80cP = 0.8P = 0.08\ Ns/m^2$$

The shearing stress at the location of the element must be

$$\tau = \mu \frac{du}{dy} = 0.08 \times 6980r = 558.4\,r\ N/m^2$$

and the shearing force δF is given by

$$\delta F = \tau \cdot \delta r \cdot r\delta\theta = 558.4 r^2 \delta r \delta\theta\ N$$

The torque δT required to provide the shearing force over the element is

$$\delta T = r\delta F = 558.4 r^3 \delta r \delta\theta\ Nm$$

and total torque on the shaft must be

$$T = \int_0^R \int_0^{2\pi} 558.4\, r^3 dr\, d\theta = 2\pi \times 558.4 \int_0^R r^3 dr$$

$$= 2\pi \times 558.4 \times \frac{R^4}{4} = 2\pi \times 558.4 \times \frac{0.05^4}{4}$$

$$= 0.00548\ Nm$$

The power dissipated i.e. the rate of energy lost in the bearing is given by

$$\text{Power} = T \cdot \omega = 0.00548 \times 10.47$$
$$= 0.0574\,W. \quad \text{Ans:-}$$

Q. Calculate the gauge pressure and the absolute pressure within

a) a droplet of water 0.4cm in dia and

(b) a jet of water 0.4 cm in dia.

Sol:- σ water $= 0.073\ N/m$

a) $P = P_{gauge} + 1\ atm$

$$P = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.073}{0.004} = 73\ N/m^2$$

b) $P_{abs} = P_{atm} + P_{gauge}$

$= 101300 + 73$

$= 101373\ N/m^2$

$= 101.373\ KN/m^2$

for the jet

$$P_{gauge} = \frac{2\sigma}{d} = \frac{2 \times 0.073}{0.004}$$

$= 36.5\ N/m^2$

$P_{atm} = 101.300 + 36.5$

$= 101336.5\ N/m^2$

$= 101.34\ KN/m^2$ Ans:-

PRESSURE MEASUREMENT

Intensity of Pressure

The pressure is defined as the normal force exerted by static fluid per unit area.

$$\text{Intensity of pressure } (P) = \frac{\text{Normal force}}{\text{Area}}$$

→ Pressure is compressive in nature.

Unit :- (i) N/m^2 (Pa)
(ii) KPa, MPa, GPa
(iii) mm of column of liquid
(iv) 1 bar $= 10^5$ Pa.

Types of pressure

1. Atmospheric Pressure (Patm)

It is defined as the pressure exerted by the atmosphere air upon the surface.

$$P_{atm} = 760 \text{ mm of Hg}$$
$$= 10.33 \text{ m of } H_2O$$

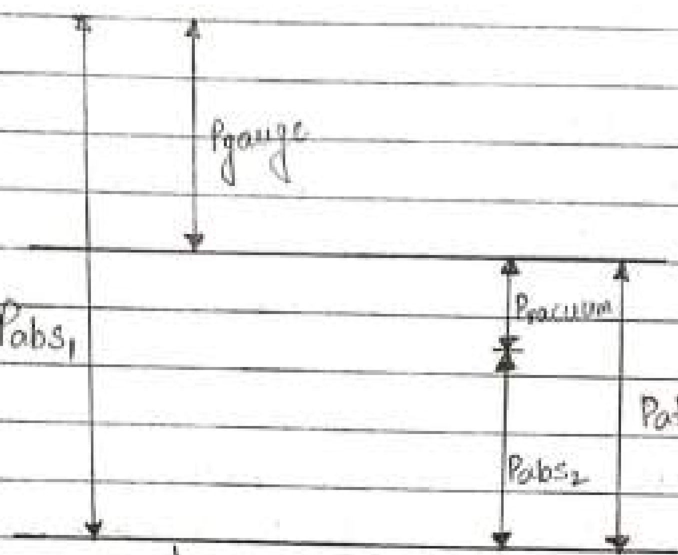
$$P_{atm} = \rho_{Hg} \times g \times h$$
$$= 13.6 \times 1000 \times 9.81 \times 760 \times 10^{-3} \text{ Pa}$$
$$= 101.396 \text{ KPa} = 1.01396 \text{ bar} \simeq 1 \text{ bar}$$

2. Gauge Pressure (P_{gauge})

It is defined as the pressure measured by the gauge (instruments) in which atmosphere pressure is taken as datum. If the measure pressure is lower than atmospheric pressure, then it is called negative gauge pressure or vacuum pressure.

3. Absolute zero pressure -

It is the pressure exerted by absolute vacuum.



$P_{abs_1} = P_{atm} + P_{gauge}$
$P_{abs_2} = P_{atm} - P_{vacuum}$

Hydrostatic Law

It states that rate of increase of pressure in the vertically downward direction in a static fluid as equal to the specific wt. of the fluid.

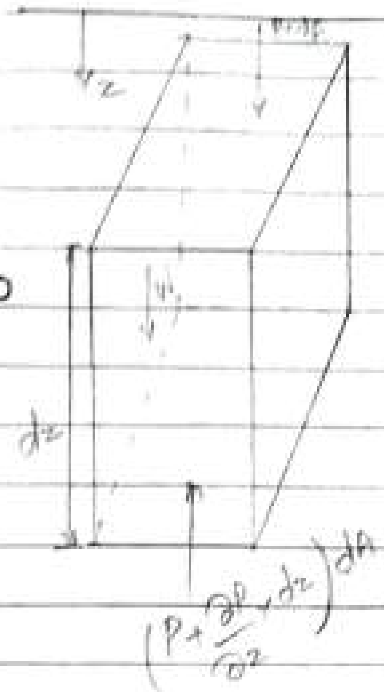
$$\boxed{\frac{\partial P}{\partial z} = \gamma}$$

under equilibrium condition

$$\Sigma f_y = 0$$

$$-P \times dA + \left(P + \frac{\partial P}{\partial z} \cdot dz\right) \times dA - W = 0$$

$$\frac{\partial P}{\partial z} \cdot \partial z \cdot dA = \gamma \times dA \times dz$$



$$\boxed{\therefore \frac{\partial P}{\partial z} = \gamma = \rho g}$$

$$\partial P = \rho g \cdot dz$$

both side integration.

$$P = \rho g z + C$$

At, $z = 0$, $P = P_{atm}$

$$P_{atm} = \rho g \times 0 + C$$

$$\therefore C = P_{atm}$$

$$\therefore P = \rho g z + P_{atm}$$

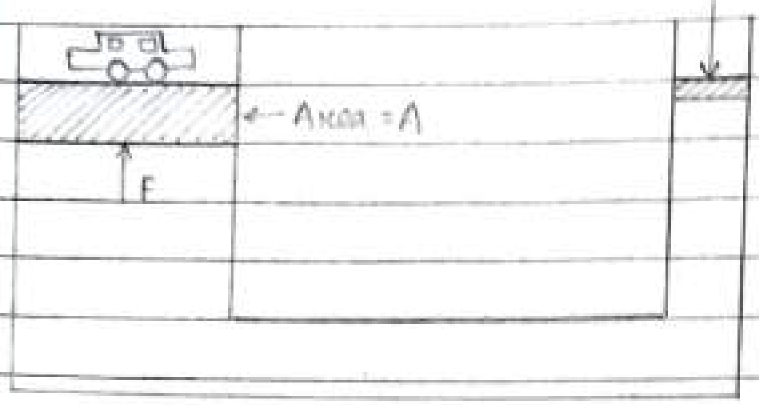
$$P - P_{atm} = \rho g z$$

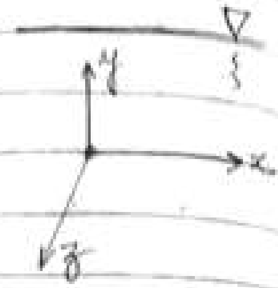
$$\boxed{P_{gauge} = \rho g z}$$

At distance z from free surface, $P = \rho g z$

Pascal's law:

It states that in a static fluid, the pressure acts equally in all the directions.



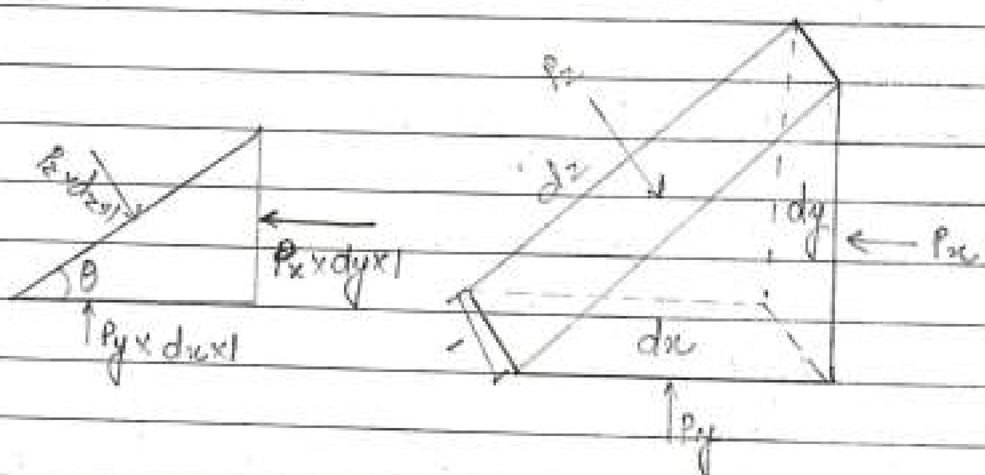


$P_x = P_y = P_z$

$\frac{W}{a} = \frac{F}{A}$; application of Pascal's law (Hydraulic lift)

$$\boxed{\therefore\ F = \left(\frac{A}{a}\right) W}$$

Consider a wedge shape fluid elements
Consider unit elements,
Resolve the force along x-direction



$\Rightarrow P_x \times dy \times 1 - P_z \times dz \times 1 \times \sin\theta$

$$P_x\, dy = P_z\, dz \sin\theta \quad \text{——— (1)}$$

Resolve forces along y-direction

$$P_y dx = P_z dz\cos\theta \quad \text{--- (2)}$$

$$\sin\theta = \frac{dy}{dz}$$

$$\therefore dy = dz\sin\theta$$

$$(1) \Rightarrow P_x \cdot dy = dy P_z$$

$$\therefore P_x = P_z \quad \text{--- (3)}$$

$$\cos\theta = \frac{dx}{dz}$$

$$\therefore dx = dz\cos\theta$$

$$(2) \Rightarrow P_y dx = P_z dx$$

$$\therefore P_y = P_z \quad \text{--- (4)}$$

From eqn (3) & (4)

$$P_x = P_y = P_z$$

Pressure measurement device

Manometer

Simple — Differential

1. Piezometer
2. U-tube manometer
3. Single column manometer
   i) Verticle column
   ii) Inclined column

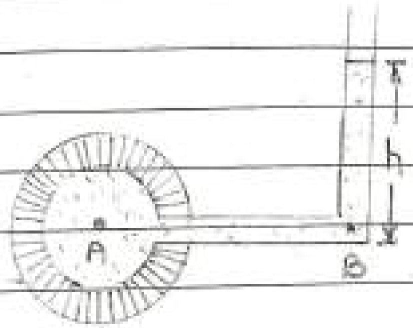
Manometer :

These are the devices which are used to measure the pressure at any point in the pipe by balancing the column of fluid by the different kind of the fluid.

a) Simple manometer

These are the device which contains a narrow tube whose one end is connected to the point where the pressure is to be measured and the other end is open to the atmosphere.

a) Piezometer



$$\boxed{P_A = P_B = \rho g H + P_{atm}}$$

$\therefore P_A = \rho g H$ (gauge pressure)

Piezometer is used for measurement of moderate pressure.

Piezometer is not useful for measurement of gas pressure since gases never form a surface with the atmosphere.

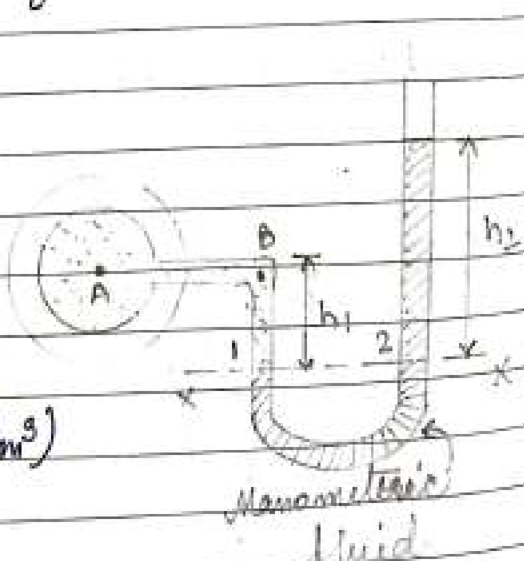
b) U-tube manometer

Manometer has manometric fluid.

→ Generally Hg as it has

i) Low vapour pressure.

ii) Highest density ($13600\ kg/m^3$)



At datum line $x - x$

$P_1 = P_2$

$\Rightarrow P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}$

$P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}$

$\therefore P_A = \rho_2 g h_2 - \rho_1 g h_1 + P_{atm}$

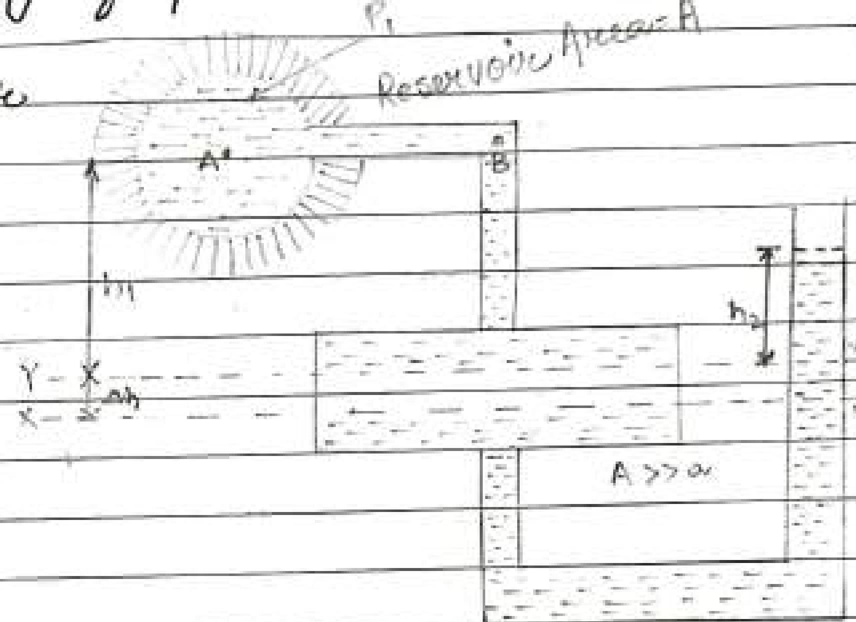
$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1} \longrightarrow$ gauge pressure

(c) Single column manometer

At datum line

$P_A + \rho_1 g (h_1 + \Delta h) = P_{atm} + \rho_2 g (h_2 + \Delta h)$

$\therefore P_A = (\rho_2 g h_2 - \rho_1 g h_1) + (\rho_2 - \rho_1) g \Delta h$



For volume equal

$A' \Delta h = a \times h_2$

$\therefore \Delta h = \frac{a h_2}{A}$

Since $a << A$, then Δh can be neglected

$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1}$ (gauge pressure)

(d) Single column Inclined manometer

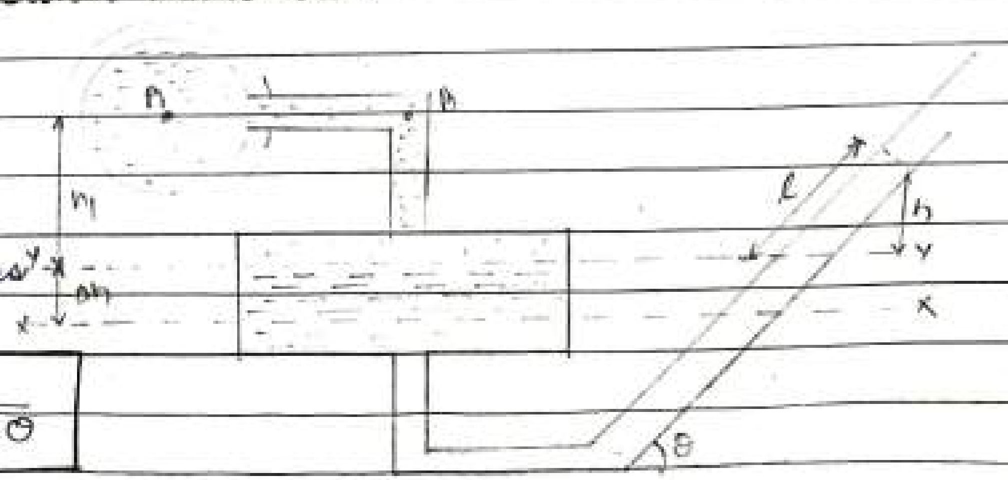
$P = \rho g h$

$h_2 = l \sin\theta$

As θ increases

Sensitivity decreases

$\boxed{\therefore \text{Sensitivity} = \frac{1}{\sin\theta}}$

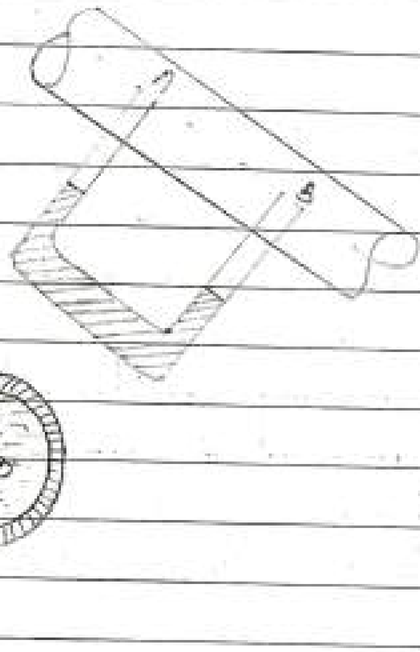


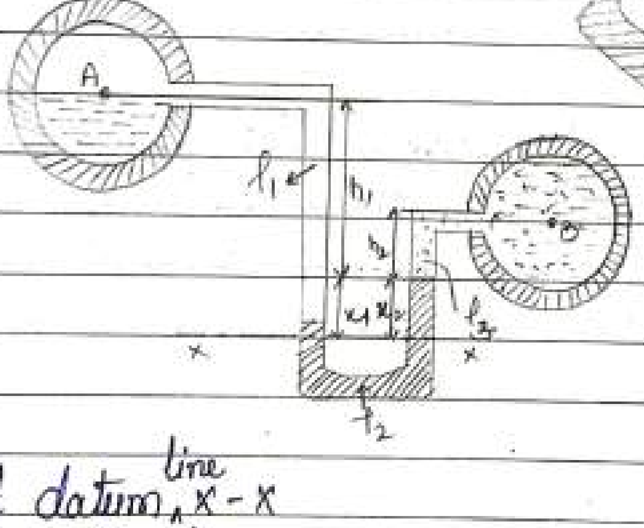
θ	Sensitivity
$0°$	∞
$30°$	2
$40°$	1.15
$90°$	1

Differential Manometer:

It is a device used to measure the difference of pressure between any two points that may lie in the single pipe or in different pipe lines.

$P_A - P_B = ?$





At datum line, x - x

$P_1 = P_2$

$$P_A + \rho_1 g(h_1 + x) = P_B + \rho_3 g(h_2) + \rho_2 g x$$

$$\boxed{P_A - P_B = \rho_3 g h_2 - \rho_1 g h_1 - \rho_2 g x - \rho_1 g x}$$

If the points A & B lie in the same horizontal line and having same fluid.

$h_1 = h_2$

$$P_A - P_B = (\rho_2 - \rho_1) g x$$

$$\therefore h_A - h_B = \frac{(\rho_2 - \rho_1)}{\rho_1 g} g x$$

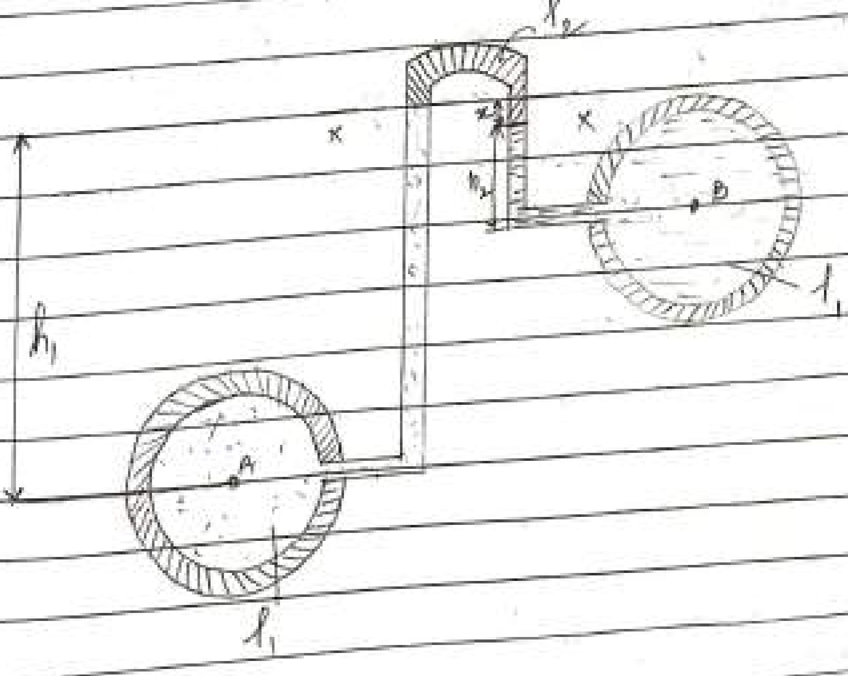
$$\boxed{h_A - h_B = \left(\frac{\rho_2}{\rho_1} - 1\right) x}$$

$$\Rightarrow \left(\frac{\rho_2/\rho_{H_2O}}{\rho_1/\rho_{H_2O}} - 1\right) x$$

$$\therefore h_A - h_B = \left(\frac{S_2 - S_1}{S_1}\right) \cdot x$$

$$\therefore h_A - h_B = \left(\frac{S_m}{S_L} - 1\right) \cdot x$$

Inverted U-Tube Differential Manometer



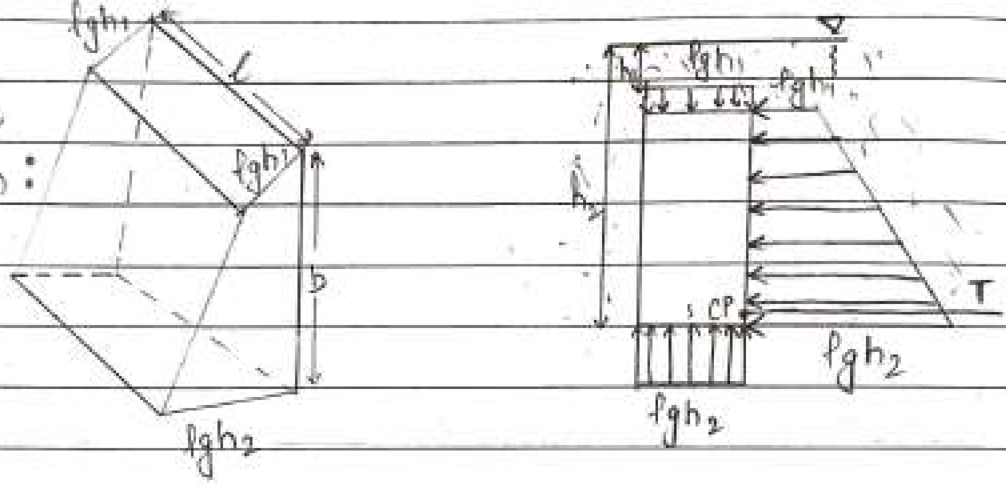
At the datum line x - x

$$P_A - \rho_1 g h_1 = P_B - \rho_1 g h_2 - \rho_2 g x$$

$$\therefore \boxed{P_A - P_B = \rho_1 g h_1 - \rho_1 g h_2 - \rho_2 g x}$$

Hydrostatic Forces on Submerged Surfaces.

Pressure Prism :



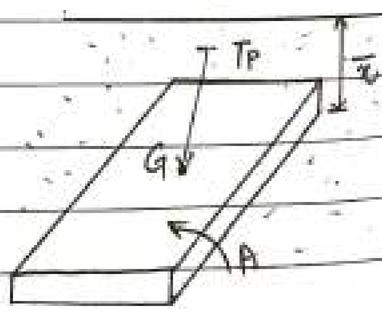
Total Pressure (TP) :

Whenever a surface either a plane or curve is complet submerged in the static fluid, the pressure force variations will take place across the surface. The resultant of all the pressure force variations is called "Total Pressure". It has units of 'Force' (N).

Centre of Pressure (CP) :

It is the point at which total pressure will act.

a) Horizontal Surface.



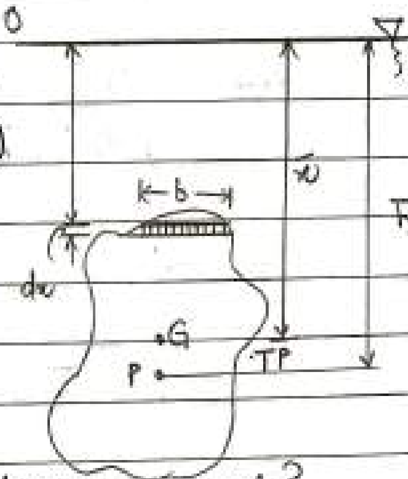
Let A → area of surface
w → sp. wt. of liquid

$\bar{x}$ = distance of C·G from free surface.

$$T.P = \text{pressure} \times \text{area}$$
$$= \rho g \bar{x} \times A = \omega A \bar{x}$$

$$\boxed{TP = \omega A \bar{x} \;\&\; CP = \bar{x}}$$

b) Vertical Surface



Pressure force acting on elemental area $= \rho g x \cdot (b\,dx)$

Sum of the pressure forces
$$= \int \rho g (b\,dx) \times x$$
$$= \rho g \int (b\,dx) x$$
{$(b\,dx)x$ = first area moment}

$$TP = \rho g A \bar{x} = \omega A \bar{x}$$

" Sum of the moments of individual forces is equal to the moment caused by the resultant force - Varignance Theorem."

Moment caused by pressure force acting on elemental area about free surface = Force $\times x$
$$= (P \times A) \times x$$
$$= \rho g x \cdot b\,dx \cdot x$$
$$= \rho g (b\,dx) x^2$$

Sum of moments $= \int \rho g (b\,dx) x^2$
$= \rho g \int (b\,dx) x^2$ [$(b\,dx)x^2$ = second area moment = I]

$= \rho g I_o$

Parallel Axis Theorem:

$$\rho g I_0 = \rho g \left(I_G + A\bar{x}^2\right) \quad \text{—— (i)}$$

Moment caused by TP about free surface

$$= TP \times \bar{h}$$
$$= \omega A\bar{x} \times \bar{h} \quad \text{—— (ii)}$$

From (i) and (ii)

$$\rho g A\bar{x} \times \bar{h} = \rho g \left(I_G + A\bar{x}^2\right) \qquad \left\{ \begin{array}{l} \rho g A\bar{x} \times \bar{h} = \rho g I_0 \\ \Rightarrow \bar{h} = \dfrac{I_0}{A\bar{x}} \end{array} \right\}$$

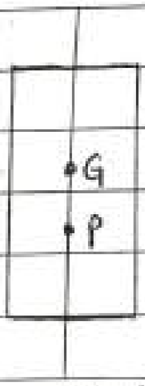
$$\therefore \bar{h} = \frac{I_G}{A\bar{x}} + \frac{A\bar{x}^2}{A\bar{x}}$$

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$$; $I_G \rightarrow$ MI of the surface about and axis passing through C·G.

$\therefore \bar{h} > \bar{x} \Rightarrow$ C·P always lies below C·G

$$\bar{h} - \bar{x} = \frac{I_G}{A\bar{x}}$$

* For symmetrical surfaces, C·G & C·P lie along the same line.



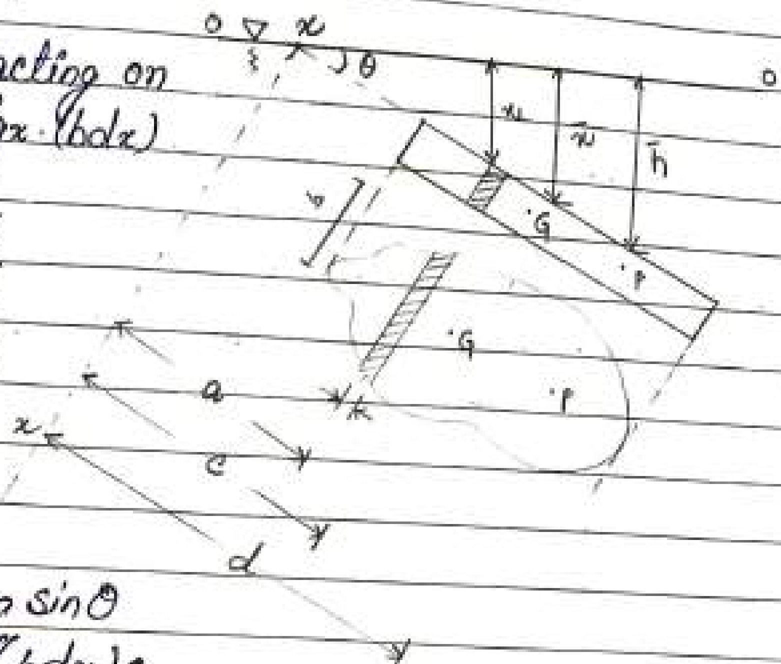
Symmetrical

Unsymmetrical

$$\boxed{\begin{array}{l} TP = \omega A\bar{x} \\ CP = \bar{x} + \dfrac{I_G}{A\bar{x}} \end{array}}$$

c) Inclined Surface

Pressure force acting on elemental area $= \rho g x \cdot (b\,dx)$



$\sin\theta = \frac{x}{a} = \frac{\bar{x}}{c} = \frac{\bar{h}}{d}$

$= \rho g (b\,dx) a \sin\theta$

Sum of pressure forces

$= \int \rho g (b\,dx) a \sin\theta$

$= \rho g (\sin\theta) \int (b\,dx) a$

$= \rho g \sin\theta \times A \times c$

$= \rho g \sin\theta \times A \times \frac{\bar{x}}{\sin\theta} = \rho g A \bar{x}$

$TP = A\omega\bar{x}$

Moment of pressure force acting on elemental area

$= \rho g x \cdot (b\,dx) a$

$= \rho g (a \sin\theta)(b\,dx) a$

Sum of the moments $= \int \rho g (b\,dx) a^2 \sin\theta$

$= \rho g \sin\theta \int (b\,dx) a^2$

$= \rho g \sin\theta \cdot I_x$

$= \rho g \sin\theta \,(I_G + Ac^2) \qquad c = \frac{\bar{x}}{\sin\theta}$

$= \rho g \sin\theta \left[I_G + A\left(\frac{\bar{x}}{\sin\theta}\right)^2 \right]$ —— (i)

Moment caused by total pressure about $x-x$, $= TP \times d$

$= \omega A \bar{x} \times d$ —— (ii)

From (i) & (ii)

$\omega A \bar{x}\, d = \omega A \bar{x} \frac{\bar{h}}{\sin\theta} = \rho g \sin\theta \left(I_G + A\frac{\bar{x}^2}{\sin^2\theta} \right)$

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

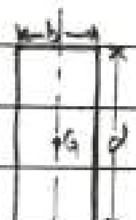
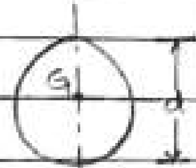
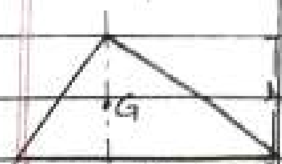
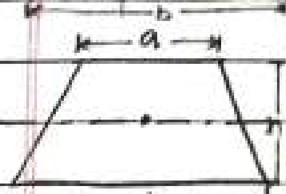
$$\boxed{\begin{array}{l} TP = \omega A\bar{x} \\ CP = \dfrac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} \end{array}}$$

- For horizontal surface, $\theta = 0°$,

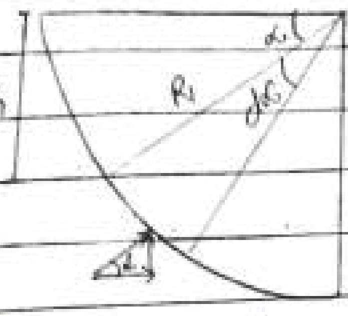
$$\bar{h} = \bar{x}$$

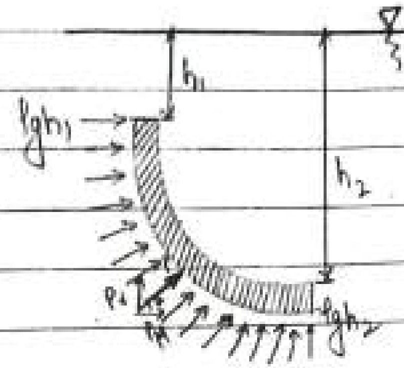
- For vertical surface, $\theta = 90°$,

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$$

Shape	C.G from base	Area	I_G parallel to base
	$d/2$	$b \times d$	$\frac{bd^3}{12}$
	$d/2$	$\frac{\pi}{4} d^2$	$\frac{\pi}{64} d^4$
	$h/3$	$\frac{1}{2} bh$	$\frac{bh^3}{36}$
	$\left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{1}{2}(a+b)h$	$\frac{a^2+4ab+b^2}{36(a+b)} \times h^3$

d) Curved Surface





$$dA = R \times \alpha \times 1$$

Pressure force acting on $dA = \rho g h \times dA$

$$= \rho g h \times R d\alpha$$

$$\sin\alpha = \frac{h}{R}$$

$$h = R\sin\alpha$$

$\therefore$ TP acting on $dA = \rho g (R\sin\alpha) R d\alpha = \rho g R^2 \sin\alpha \, d\alpha$

Horizontal component of TP $= \rho g R^2 \sin\alpha \, d\alpha \times \cos\alpha$

Sum of horizontal components $= \int_{\alpha=0}^{\pi/2} \rho g R^2 \sin\alpha \cos\alpha \, d\alpha$

$$= \rho g R^2 \int \sin\alpha \cos\alpha \, d\alpha$$

$$= \rho g R^2 \left(\frac{-\cos 2\alpha}{2} \right)_0^{\pi/2}$$

$$= \frac{\rho g R^2}{2}$$

$$= \rho g (R \times 1) \frac{R}{2} = \omega A \bar{x}$$

$P_H = \omega A \bar{x}$

$P_V = W$ of liquid supported by curved surface

Buoyancy

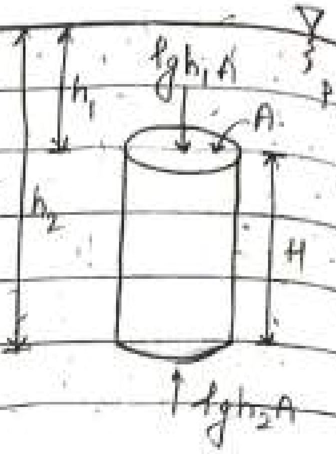
Buoyancy Force $= \rho g h_2 A - \rho g h_1 A$

$= \rho g A (h_2 - h_1)$

$= \rho g AH = \rho g V_{body}$

$= \rho g V_{fluid\ displaced}$

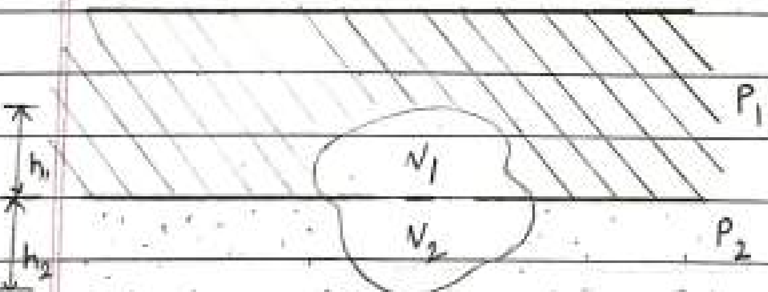
$=$ Weight of fluid displaced

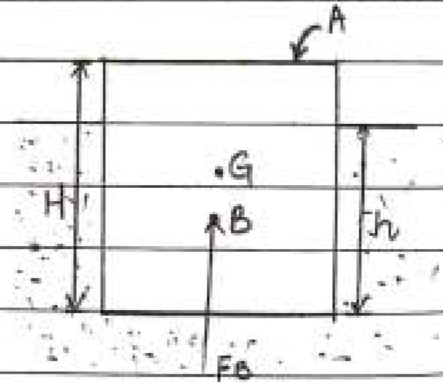


Whenever an object submerged partially or completely, it will be subjected to the net resultant vertical upward force and is known as 'Buoyancy force' or 'Force of Buoyancy'.

The point at which the force of buoyancy acts is known as 'Centre of Buoyancy'. The Centre of Buoyancy is the centroid of the volume of fluid displaced (volume of body immersed)

Volume of body immersed $= A \times h$



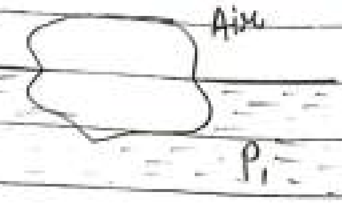


$$\boxed{\text{Buoyancy force} = \rho_1 g V_1 + \rho_2 g V_2}$$

At $p = 1$ bar, $T = 20 + 273 = 293K$

$P = \rho RT$

$\therefore \rho = \frac{P}{RT} = \frac{100}{0.287 \times 293} = 1.18 \, kg/m^3$



F_B = weight of liquid displaced
+ weight of air displaced.

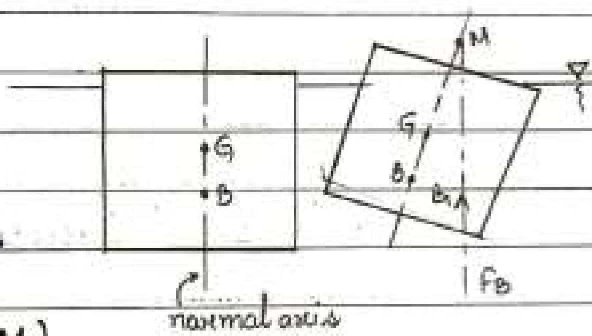
Since ρ of air is very small in magnitude, weight of air displaced will be neglected.

$\therefore$ For floating bodies,

$$\boxed{F_B = \text{weight of liquid displaced}}$$

Meta centre (M)

It is defined as the point about which the body will try to oscillate.

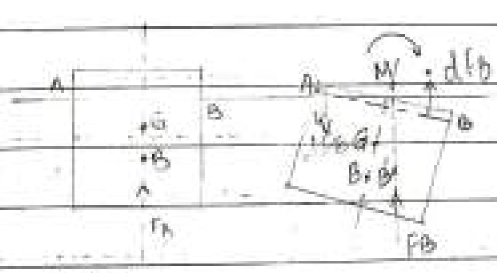


* Metacentric Height (GM)

It is the distance from metacentre to centre of gravity.

①. Analytical Method

$$\boxed{GM = \frac{I_{y-y}}{V} - BG}$$



I_{YY} → moment of inertia about y-y axis

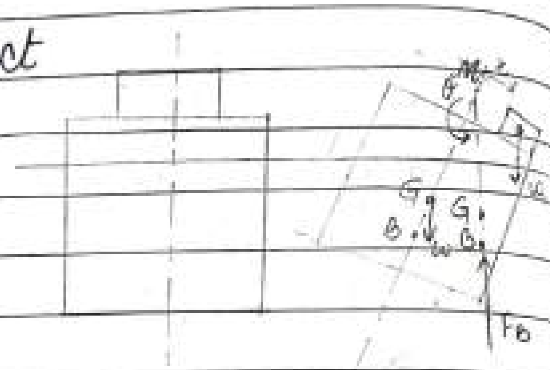
V → volume of liquid body immersed.

Due to change of centre of buoyancy, F_B will induce a moment.

This moment will be exactly equal in magnitude to the moment caused by the forces of F_B.

② Experimental Method

Let w be the weight of object and w_1 be the weight of small object.



$$F_B \times GG_1 = W_1 \times x$$

$$W \times GG_1 = W_1 \times x$$

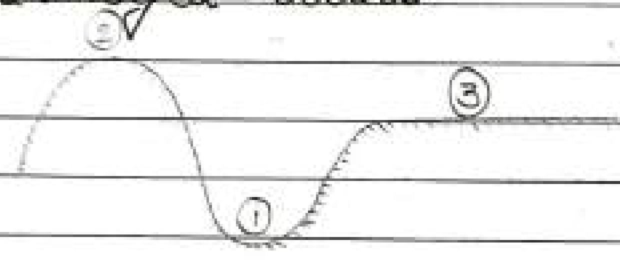
$$w \times Gm \times \tan\theta = w_1 \times x$$

$$\therefore \boxed{Gm = \frac{w_1 \times x}{w \tan\theta}}$$

$$\tan\theta = \frac{GG_1}{Gm}$$

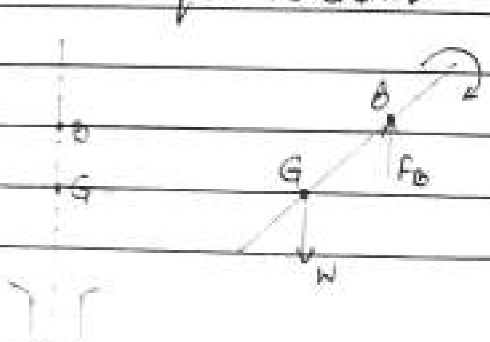
Conditions of Equilibrium

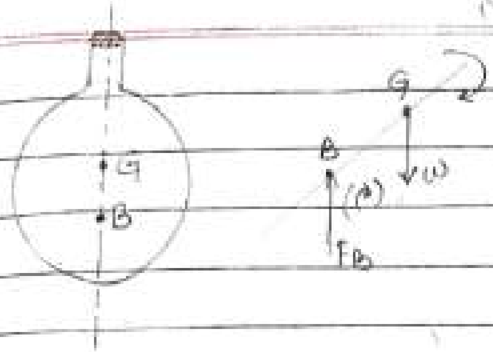
a) Submerged Bodies



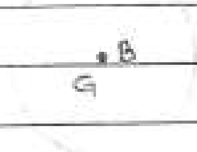
① → Stable Equilibrium
② → Unstable Equilibrium
③ → Neutral Equilibrium

Stable Equilibrium
(B is above G)



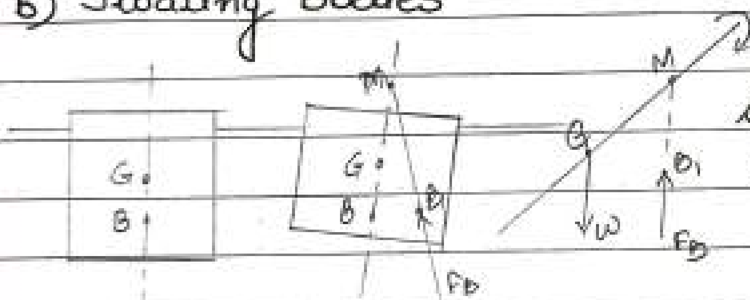


Unstable Equilibrium
(B is below G)

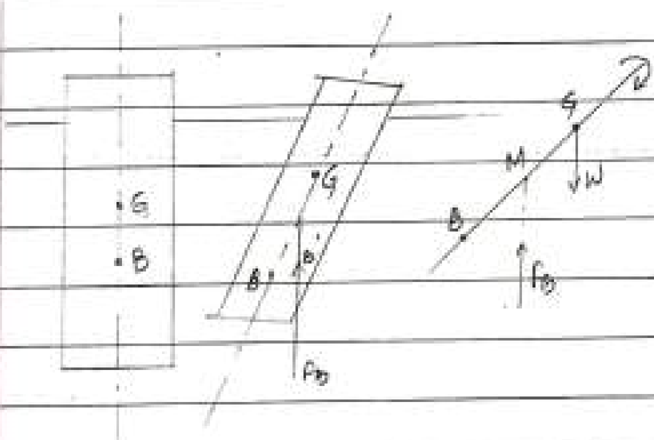


Neutral Equilibrium
(B coincides with G)

b) Floating Bodies



Stable Equilibrium
(M is above G)



Unstable Equilibrium
(M is below G)

Neutral Equilibrium (M coincides with G)

Time period of oscillation, $T = 2\pi\sqrt{\dfrac{K^2}{GM \times g}}$

GM → Metacentric height

When ship is loaded vertically as a stack, CG shifts upward and chances of M coming below G are more. So, for the safer side, GM is increased, but it cannot be increased randomly considering the comfort factors. ($T \propto \frac{1}{\sqrt{GM}}$)