K K COLLEGE OF ENGINEERING AND MANAGEMENT, DHANBAD

DEPARTMENT OF PHYSICS

LECTURE NOTES :::COURSE-B.Tech

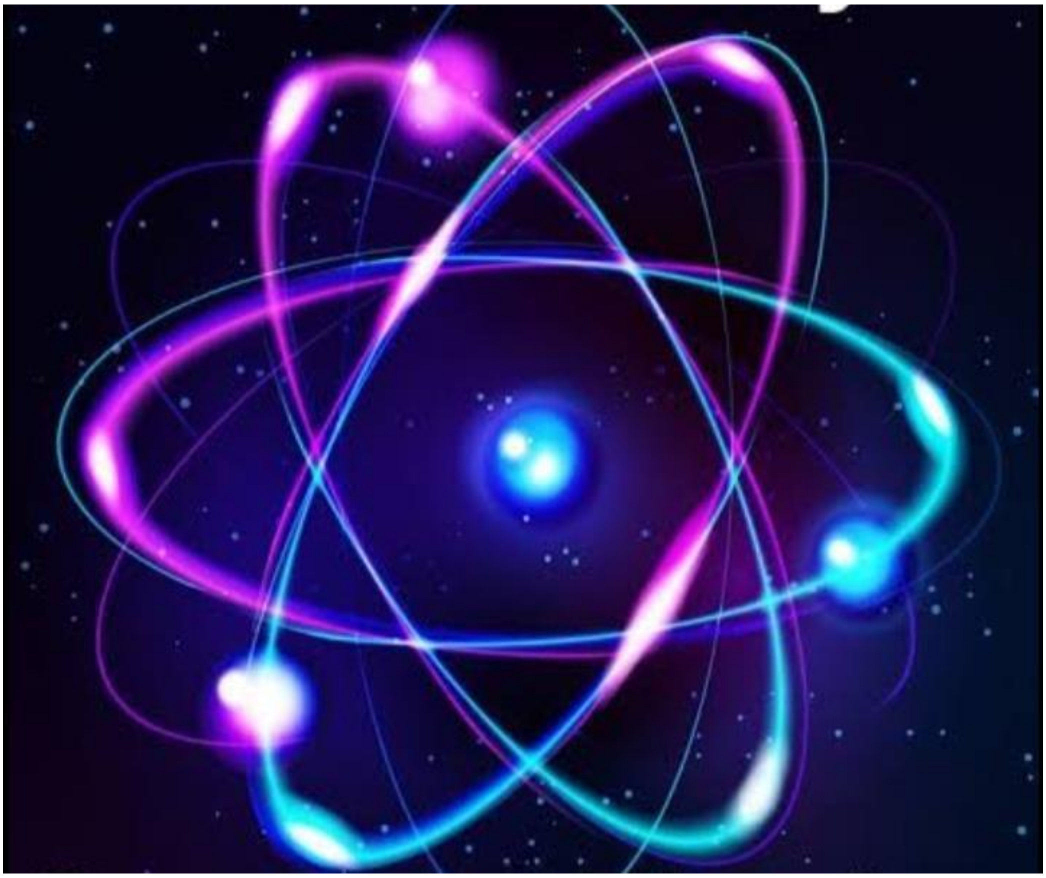
SEMESTER-SECOND

SUBJECT-PHYSICS II

For All branch

Module 1

Basic Quantum Mechanics



INADEQUACY OF CLASSICAL MECHANICS

The development of classical mechanics is based on Newton's three laws

(i)The law of inertia or Galileo law,

(ii)The law of force and

(iii)The law of action and reaction.

These laws include the concept of absolute time. Classical mechanics explains correctly the motion of celestial bodies like planets, stars and macroscopic as well as microscopic terrestrial bodies moving with nonrelativistic speeds (i.e. v<<c,c being the speed of light in vaccum).The inadequacies of classical mechanics are:

(i)It does nothold in the region of atomic dimensions i.e. it could not explain the non-relativistic motion of atoms, electrons, protons etc.

(ii)It could not explain the stability of atoms.

(iii) It could not explain observed spectrum of black body radiations.

(iv) It could not explain the observed variation of specific heat of metals and gases.

(v)It could explain the origin of discrete spectra of atoms since, to classical mechanics, the energy changes are always continuous.

In addition to this, classical mechanics could not explain a large number of observed phenomena like photoelectric effect, Compton effect, Raman effect etc.

The inadequacy of classical mechanics led to the development of Quantum Mechanics.

According to quantum mechanics the energy changes are not continuous but discrete. The absorption and emission of energy in atoms is in bundles of energy hv, where v is frequency and h is Plank's constant. By the aid of this hypothesis Plank was able to explain law of distribution of energy in the spectrum of black body. This theory was given by Plank in his historic paper titled "Theory of Law of distribution of energy in a Normal spectrum" presented before the Berlin Academy of Science on Dec. 14, 1990. This day, infact, may be considered as the birthday of quantum mechanics.

Quantum mechanics is able to explain stability of atoms, observed spectrum of black body radiation, phenomena of photoelectric effect, Compton effect , theory of α-decay, existence magnetic momentum etc.

INTRODUCTION TO QUANTUM PHYSICS

Quantum physics is the branch of physics that deals with small objects and the quantization of various entities, including energy and angular momentum. Just as with classical physics ,quantum physics has several subfields, such as mechanics and the study of electromagnetic forces.

PLANK'S QUANTUM THEORY

Plank's proposed the quantum theory for explanation of energy distribution in a black body radiation. According to this theory energy is not emitted not absorbed continuously but in a discrete units or packets.

These energy packets are called as photons or quanta.

If v be the frequency of light (photon), the energy E associated with photon is directly proportional to v.

$E \alpha\, v$

$E = \text{Constant}\, v$

$E = h\, v$

Where h = Plank's constant = 6.63 x 10* -34 js

According to this theory, energy is always emitted or absorbed in intergral multiple of hv and not in fraction of hv.

$E = h\, v$

Where n = 1,2,3,4………. n

= integer value

CONCEPT OF PHOTON:-

Acoording to Plank's theory, the quantum of light wave of frequency *v*

has the energy .

E = hv

Where, E=Photon energy

h = Plank's constant

Thus light wave has minimum energy = hv and Id it has more energy , the energy of light wave is integral multiple of hv.

Einstein proposed that when light falls on metal , the energy E = hv of photon is absorbed by the atom ,i.e. the energy is transferred from atom to atom. This is called as absorption of energy .

When light of frequency 'v' is emitted by an atom , an energy hv is transferred from atom to light, this is called Emission.

Thus , In this way Photon absorption and Photon emission takes place.

BLACK BODY RADIATION

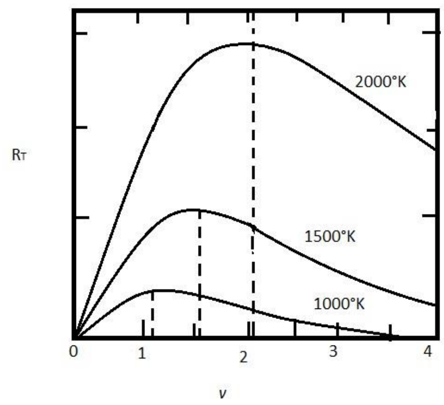
The spectrum of the thermal radiation emitted by a hot body depends upon the composition of the body. However, experiment shows that there is one class of hot bodies that emits thermal spectra of a universal character. These are called blackbodies, i.e. bodies that have surface which absorb all the thermal radiation incident upon them. The name is appropriate because such bodies do not reflect light and appear black when their temperature are low enough that they are not self-luminous.

All clack bodies at the same temperature emit thermal radiation with the same spectrum.

The spectral distribution of blackbody radiation is specified by the RT (v), d spectral radiancy , which is defined so that

RT (v) dv is equal to the energy emitted peer unit time in radiation of frequency in the interval v to v + dv from a unit area of the surface at absolute temperature, T.

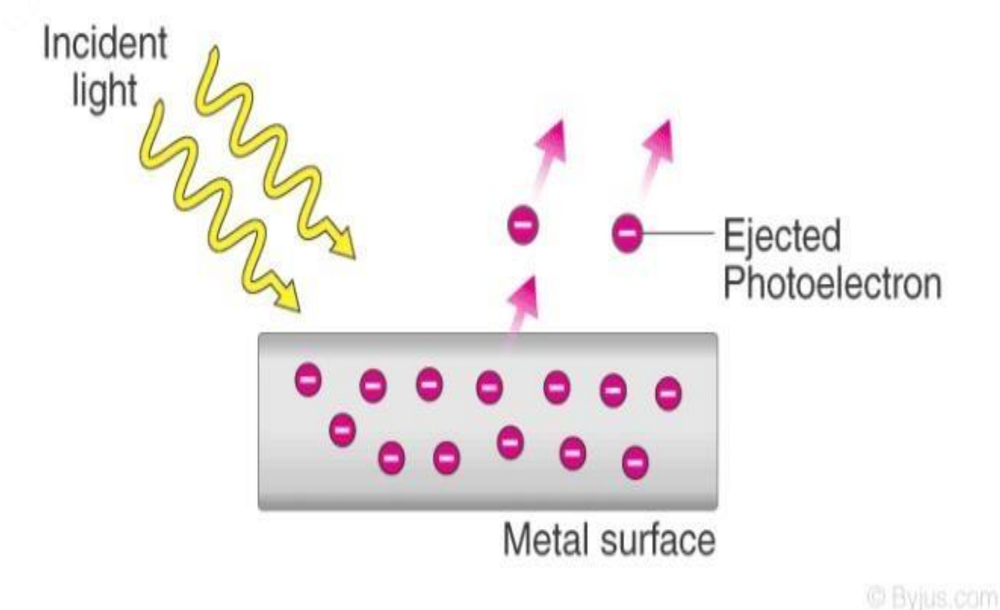
The experimentally observed dependence of RT (v) on v and T is shown in figure below:



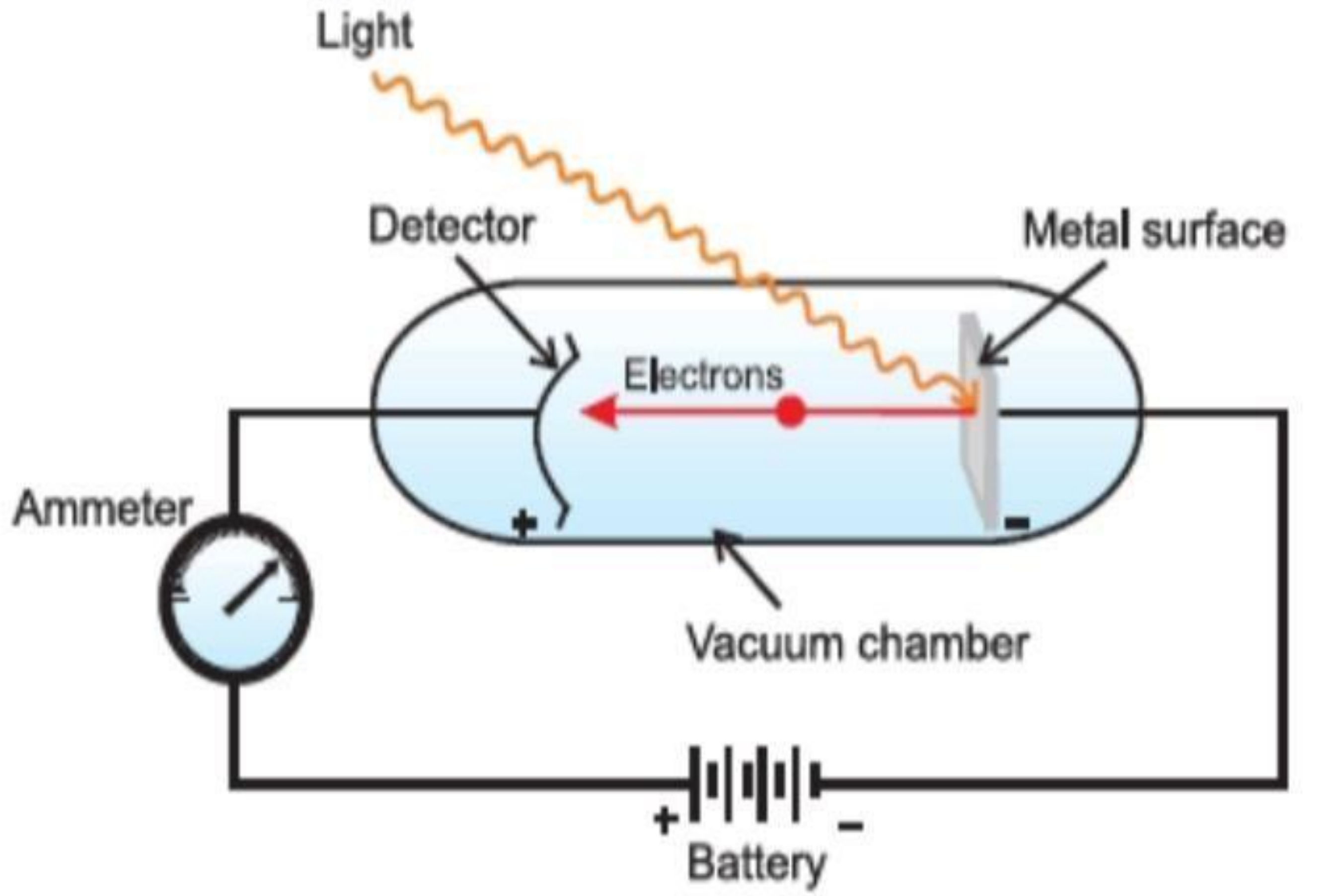
Photoelectric effect

The photoelectric effect is the emission of electrons when electromagnetic radiation, such as light, hits a material. Electrons emitted in this manner are called photoelectrons. The phenomenon is studied in condensed matter physics, and solid state and quantum chemistry to draw inferences about the properties of atoms, molecules and solids. The effect has found use in electronic devices specialized for light detection and precisely timed electron emission.The emission of electrons from a metal plate caused by light quanta – photons.

The experimental results disagree with classical electromagnetism, which predicts that continuous light waves transfer energy to electrons, which would then be emitted when they accumulate enough energy. An alteration in the intensity of light would theoretically change the kinetic energy of the emitted electrons, with sufficiently dim light resulting in a delayed emission. The experimental results instead show that electrons are dislodged only when the light exceeds a certain frequency—regardless of the light's intensity or duration of exposure. Because a lowfrequency beam at a high intensity could not build up the energy required to produce photoelectrons like it would have if light's energy was coming from a continuous wave, Albert Einstein proposed that a beam of light is not a wave propagating through space, but a swarm of discrete energy packets, known as photons.



.Emission of conduction electrons from typical metals requires a few electron-volt (eV) light quanta, corresponding to shortwavelength visible or ultraviolet light. In extreme cases, emissions are induced with photons approaching zero energy, like in systems with negative electron affinity and the emission from excited states, or a few hundred keV photons for core electrons in elements with a high atomic number.[1] Study of the photoelectric effect led to important steps in understanding the quantum nature of light and electrons and influenced the formation of the concept of wave–particle duality.[2] Other phenomena where light affects the movement of electric charges include the photoconductive effect, the photovoltaic effect, and the photoelectrochemical effect.



Stopping potential-Stopping potential is defined as the potential required to stop ejection of electron from a metal surface when incident beam of energy greater than the work potential of metal is directed on it.

Work function-The minimum energy required to eject an electron from the surface is called the photoelectric work function. The threshold for this element corresponds to a wavelength of 683 nm. Using this wavelength in the Planck relationship gives a photon energy.

Einstein's photoelectric equation

Einstein's view of light was magnificent as well as revolutionary. He proposed a weird but effective model of radiation. Light consisted of very small particles. These particles were not matter but pure energy. He called each of these a quantum of radiation. Therefore, light must be made up of these quantas or packets of energy or quantum energy. We call them photons and they carry the momentum and energy from our source of light.

According to the Einstein-Plank relation, we have $E = hv$ ...(1)

Where 'h' is the Plank's constant and 'v' is the frequency of the radiation emitted.

Also from the experiment on Photoelectric effect, we see that there is a threshold frequency below which the electrons won't come out of the metallic surface. In equation (1) we see that Energy is a function of frequency. Hence this observation is explained by equation (1). This also explains the instantaneous nature of the photoelectric emission.Since there is no electric field outside the metal surface, the energy of an electron will be purely Kinetic in nature. The quantum energy absorbed from the photon will be partly used to overcome the attraction of the metallic surface.

So, we have K.E. of the photo-electrons = (Energy obtained from the Photon) – (The energy used to escape the metallic surface)

This energy is a constant for a given surface. We denote it by Φ. We call it the work function and it is constant for a given substance. Thus we can write:

$$K.E. = hv - \Phi \quad \ldots (2)$$

This is the Einstein's Photoelectric equation.

COMPTON EFFECT

In 1920 Prof. A.H. COMPTON discovered that when a monochromatic beam of light frequency radiation (x-rays ,y –rays etc.) is scattered by a substance then the scattered radaition contains two components one is having a lower frequency or greater wavelength and the other is having the same frequency or wavelength .The radiation of unchanged frequency in the scattered beam is known as unmodified radiation while the radiation of lower is called as modified radiation. This phenomenon known as compton effect .this compton scattering result is giving (i)modified frequencies (ii)unmodified frequency and (iii)recoil-electrons.

An adequate explanation of this effect was provided by compton in 1922 on the basis of quantum theory of radiation. Acorrding to quantum concept of radiation , the radiation is constituted by energy packets called photons. The energy of the photon is ‘hʃ’,where `h` is the Planck’s constant and `ʃ` is the frequency of radiation . The photons move with velocity of light `C` ,possess momentum hʃ/C and obey all the laws of conservation of energy and momentum when they strike the electron of scattering substance.

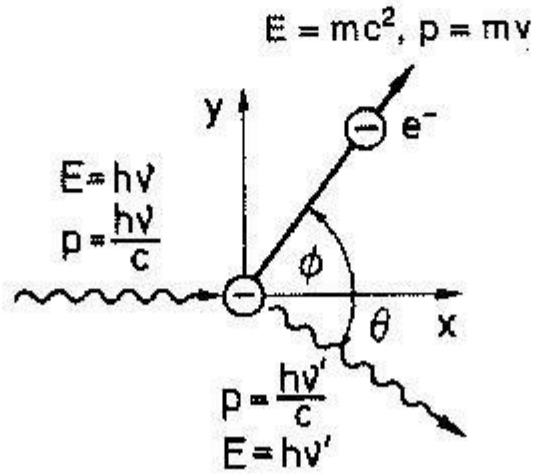
According to comton ,thde phenomenon of scattering is due to an elastic collision between two particles , the photon of incident radiation and the electron of scatterer.

When the photon of energy hʃ collides with the electron of the scatterer at rest ,it transfers some energy to the electron, oa smaller energy `hʃ` and consequently a lower frequency or greater wavelenght than of the incident photon .the observed change in frequency or wavelenght of the scattered radiation is known as compton effect .In the scattering process , the electron gains kinetic energy and thus recoils with velocity v .

THEORY

Considering the phenomenon of scattering as a collision between the photon and the electron ,and applying the laws of conservation 0f energy and momentum ,compton derived an expression for the change of wavelenght in the following manner .

Compton assumed that the electron is free and is at rest before collision with the photon. after collision the relativistic mass of the electron is considered . the assumption is justified for the collision by high energy photons because only a small fraction of the photon energy imparts sufficient energy to the recoiled electron .

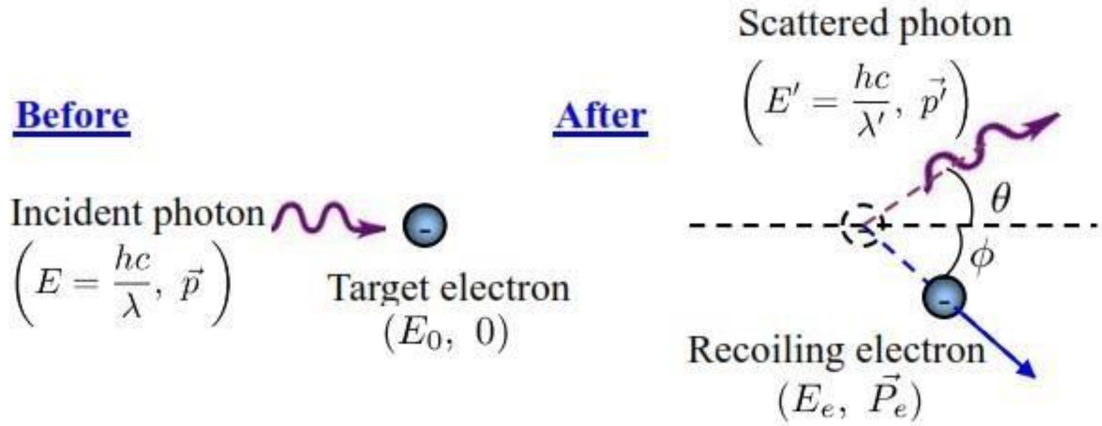


COMTON EFFECT

Photon interact with electron as shpwn in above .where Φ= scattering angle and Θ =recoiling angle. A photon is incident on graphite block emit two type of photon wavelength (ʎ and ʎ') and Δʎ= ʎ'- ʎ where ʎ' > ʎ and Δʎ=compton shift

Now, from law of conservation of energy ,

Energy before collision = energy after collision



Using the above diagram, derive the Compton Shift Equation.

- **Conservation of energy**: equating total pre- and post-collisional energies, we have that

$$h\nu_0 + m_0c^2 = h\nu + mc^2 \,. \tag{1.45}$$

Using the expression $c = \nu\lambda$, (1.45) can be rewritten as

$$\frac{hc}{\lambda_0} + m_0c^2 = \frac{hc}{\lambda} + mc^2 \,, \tag{1.46}$$

or

$$\frac{h}{\lambda_0} - \frac{h}{\lambda} + m_0c = mc \,. \tag{1.47}$$

- **Conservation of Momentum**: equating the x- and y-components,

$$x - \text{component}: \qquad \frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\phi + \gamma m_0 v\cos\theta \tag{1.48}$$

$$y - \text{component}: \qquad 0 = \frac{h\nu}{c}\sin\phi - \gamma m_0 v\sin\theta \,. \tag{1.49}$$

In terms of λ, (1.47) and (1.48) can be written as

$$\frac{h}{\lambda_0} - \frac{h}{\lambda}\cos\phi = \gamma m_0 v\cos\theta \,, \tag{1.50}$$

and

$$\frac{h}{\lambda}\sin\phi = \gamma m_0 v\sin\theta \,, \tag{1.51}$$

respectively.

Squaring and adding (1.50) and (1.51) yields the result

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2\cos\phi}{\lambda_0\lambda} = \gamma^2 m_0^2 v^2 = \gamma^2 m_0^2 c^2 - m_0^2 c^2 \,. \tag{1.52}$$

Squaring (1.47) gives

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda_0\lambda} + 2m_0hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) + m_0^2c^2 = \gamma m_0^2c^2 \,. \tag{1.53}$$

Subtracting (1.52) from (1.53) gives the result

$$\frac{2h^2}{\lambda_0\lambda}(\cos\phi - 1) + 2m_0hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = 0 \,, \tag{1.54}$$

or equivalently,

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0 c}\left(1 - \cos\phi\right) \ . \tag{1.55}$$

Using the trigonometric identity $\cos 2A = 1 - 2\sin^2 A$ with $2A = \phi$, this result can be written in the alternative form

$$\Delta\lambda = 2\frac{h}{m_0 c}\sin^2\frac{\phi}{2} \ . \tag{1.56}$$

- So (1.55) or (1.56) provides an expression for the Compton Shift $\Delta\lambda$.
- The expression h/m_0c is called the **Compton wavelength**.
- Employing the modern values of the physical constants:

  Planck constant: $h = 6.626075 \times 10^{-34}$ J s

  Electron rest-mass: $m_0 = 0.9109390 \times 10^{-30}$ kg

  Speed of light: $c = 2.997925 \times 10^8$ m s^{-1}

  we find that the Compton Shift may be expressed by

$$\Delta\lambda = 0.024263\left(1 - \cos\phi\right) \ , \tag{1.57}$$

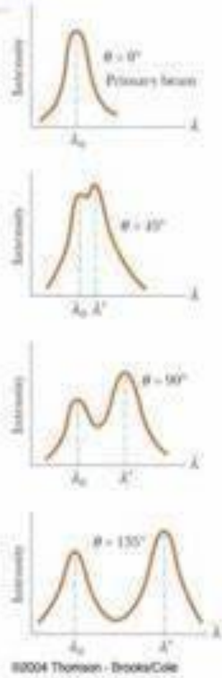
where we have expressed the Compton wavelength in units of Angstroms ($1\ \mathring{A} = 10^{-10}$ m).

Compton Shift Equation

- The graphs show the scattered x-ray for various angles
- The shifted peak, λ', is caused by the scattering of free electrons

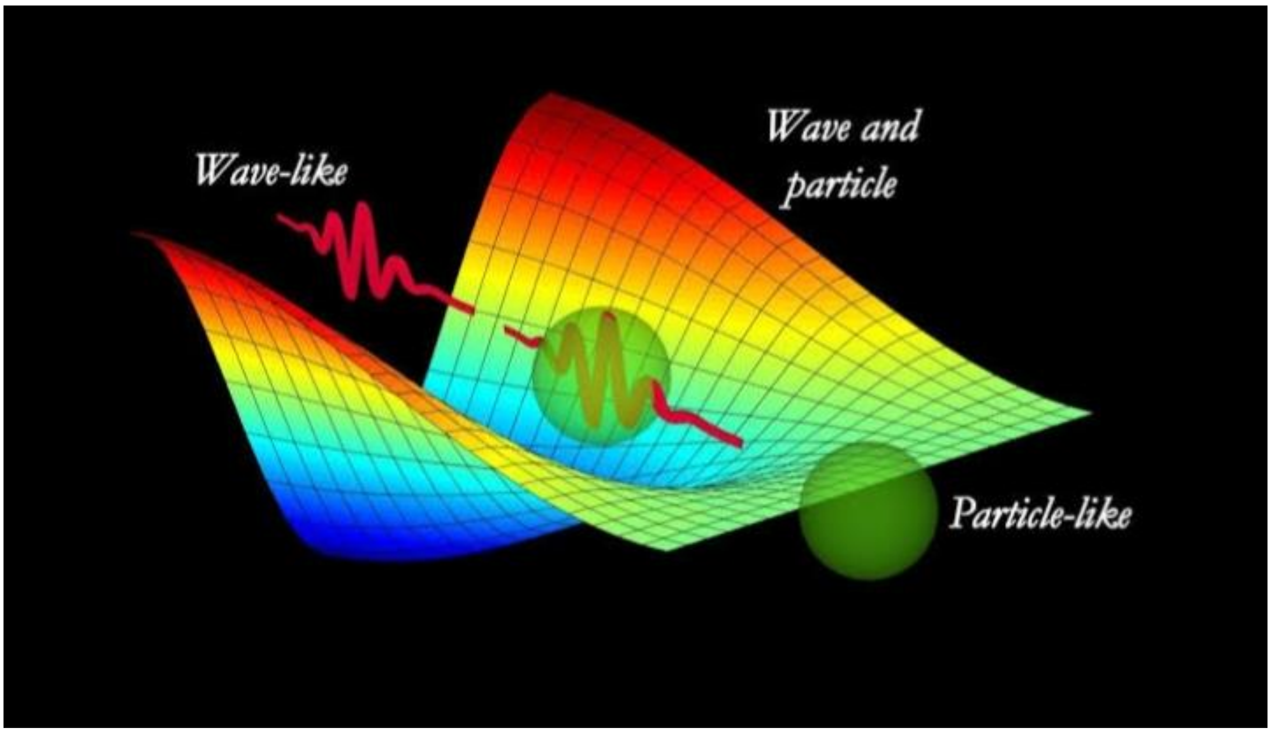
$$\lambda' - \lambda_o = \frac{h}{m_e c}(1 - \cos\theta)$$

  - This is called the **Compton shift equation**



Module 2

Wave Particle Duality and Bound States



Wave function and Born's interpretation of wave function

A wave function in quantum physics is a mathematical description of the quantum state of an isolated quantum system. The wave function is a complex-valued probability amplitude, and the probabilities for the possible results of measurements made on the system can be derived from it. The most common symbols for a wave function are the Greek letters ψ and Ψ.

According to the superposition principle of quantum mechanics, wave functions can be added together and multiplied by complex numbers to form new wave functions and form a Hilbert space. The inner product between two wave functions is a measure of the overlap between the corresponding physical states, and is used in the foundational probabilistic interpretation of quantum mechanics, the Born rule, relating transition probabilities to inner products. The Schrödinger equation determines how wave functions evolve over time, and a wave function behaves qualitatively like other waves, such as water waves or waves on a string, because the Schrödinger equation is mathematically a type of wave equation. This explains the name "wave function", and gives rise to wave–particle duality. However, the wave function in quantum mechanics describes a kind of physical phenomenon, still open to different interpretations, which fundamentally differs from that of classic mechanical waves.

In Born's statistical interpretation in non-relativistic quantum mechanics, the squared modulus of the wave function, $|\psi|2$, is a real number interpreted as the probability density of measuring a particle as being at a given place – or having a given momentum – at a given time, and possibly having definite values for discrete degrees of freedom. The integral of this quantity, over all the system's degrees of freedom, must be 1 in accordance with the probability interpretation. This general requirement that a wave function must satisfy is called the normalization condition. Since the wave function is complex valued, only its relative phase and relative magnitude can be measured—its value does not, in isolation, tell anything about the magnitudes or directions of measurable observables; one has to apply quantum operators, whose eigenvalues correspond to sets of possible results of measurements, to the wave function ψ and calculate the statistical distributions for measurable quantities.

Position-space wave functions---The state of such a particle is completely described by its wave function,

$$\Psi(x,t)\ ,$$

where x is position and t is time. This is a complex-valued function of two real variables x and t.

For one spinless particle in one dimension, if the wave function is interpreted as a probability amplitude, the square modulus of the wave function, the positive real number

$$|\Psi(x,t)|^2 = \Psi^*(x,t)\Psi(x,t) = \rho(x,t),$$

is interpreted as the probability density that the particle is at x. The asterisk indicates the complex conjugate. If the particle's position is measured, its location cannot be determined from the wave function, but is described by a probability distribution.

Schrodinger's wave equation -time dependent or independent

The Schrödinger equation is a linear partial differential equation that governs the wave function of a quantum-mechanical system. It is a key result in quantum mechanics, and its discovery was a significant landmark in the development of the subject. The equation is named after Erwin Schrödinger, who postulated the equation in 1925, and published it in 1926,

forming the basis for the work that resulted in his Nobel Prize in Physics in 1933.Schrödinger's equation inscribed on the gravestone of Annemarie and Erwin Schrödinger. (Newton's dot notation for the time derivative is used.)

Conceptually, the Schrödinger equation is the quantum counterpart of Newton's second law in classical mechanics. Given a set of known initial conditions, Newton's second law makes a mathematical prediction as to what path a given physical system will take over time. The Schrödinger equation gives the evolution over time of a wave function, the quantum-mechanical characterization of an isolated physical system. The equation can be derived from the fact that the time-evolution operator must be unitary, and must therefore be generated by the exponential of a self-adjoint operator, which is the quantum Hamiltonian.

The Schrödinger equation is not the only way to study quantum mechanical systems and make predictions. The other formulations of quantum mechanics include matrix mechanics, introduced by Werner Heisenberg, and the path integral formulation, developed chiefly by Richard Feynman. Paul Dirac incorporated matrix mechanics and the Schrödinger equation into a single formulation. When these approaches are compared, the use of the Schrödinger equation is sometimes called "wave mechanics".

Schrodinger Wave Equation Derivation (Time-Dependent)

The single-particle time-dependent Schrodinger equation is,

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t)$$

______(1)

Where

V represents the potential energy and is assumed to be a real function

Now, if we write the wave function as a product of temporal and spatial terms, then the equation will become,

$$\psi(\mathbf{r})i\hbar\frac{df(t)}{dt} = f(t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r})$$

_____________(2)

Or

$$\frac{i\hbar}{f(t)}\frac{df}{dt} = \frac{1}{\psi(\mathbf{r})}\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r})$$

Since the right-hand side is a function of r only and the left-hand side is of t only, the two sides should equal a constant. In cases where we designate the constant E, the two ordinary differential equation, namely

$$\frac{1}{f(t)}\frac{df(t)}{dt} = -\frac{iE}{\hbar}$$

And

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Here, the former equation is solved to get,

$$f(t) = e^{-iEt/\hbar}$$

However, the latter equation is the time-independent Schrödinger equation

Considering a complex plane wave:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}.$$

Now the Hamiltonian of a system is

H=T+V

Where T is the kinetic energy and V is the potential energy. As we know that H is the total energy, we can rewrite the equation as:

$$E = \frac{p^2}{2m} + V(x).$$

Now, by taking the derivatives, we get

$$\frac{\partial \Psi}{\partial t} = -i\omega A e^{i(kx-\omega t)} = -i\omega \Psi(x,t)$$
$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 A e^{i(kx-\omega t)} = -k^2 \Psi(x,t)$$

We know that

$$p = \frac{2\pi\hbar}{\lambda} \text{ and } k = \frac{2\pi}{\lambda}$$

Where

'λ' is the wavelength 'k' is the wave number.

Now, as we have

$$k = \frac{p}{\hbar}.$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\Psi(x,t).$$

Hence

Here,by multiplying the Hamiltonian to Ψ (x, t), we get,

$$E\Psi(x,t) = \frac{p^2}{2m}\Psi(x,t) + V(x)\Psi(x,t).$$

The above expression can also be written as:

$$E\Psi(x,t) = \frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t).$$

As the energy of a matter wave is

$$E = \hbar\omega,$$

So we can say that

$$E\Psi(x,t) = \frac{\hbar\omega}{-i\omega}\Psi(x,t).$$

Now, by combining the parts, we can get the Schrodinger Wave Equation.

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t).$$

This is the the Schrodinger Wave Equation (time-dependent).

Schrodinger's time independent equation

Consider a particle of mass “m” moving with velocity “v” in space. Suppose a system of stationary waves is associated with the particles at any point in space in the neighborhood of particle.

We know that:

Schrodinger time independent wave equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right]$$

Here Ψ(x,t) is a wave function.

Eigen value And Eigen function

If Ψ is a well behaved function, then an operator P may operator on Ψ in two different way depending on the nature of function Ψ.

1.The operator P operating on the function Ψ change the function into another function

$$P\Psi=\Phi\text{—}(1)$$

Then the new function Φ will in general be linearly independent of initial function Ψ.

2.The operator p operating on same function Ψ may leave the function unchanged but with a complex or real multiple.

$$P\Psi=\lambda\Psi$$

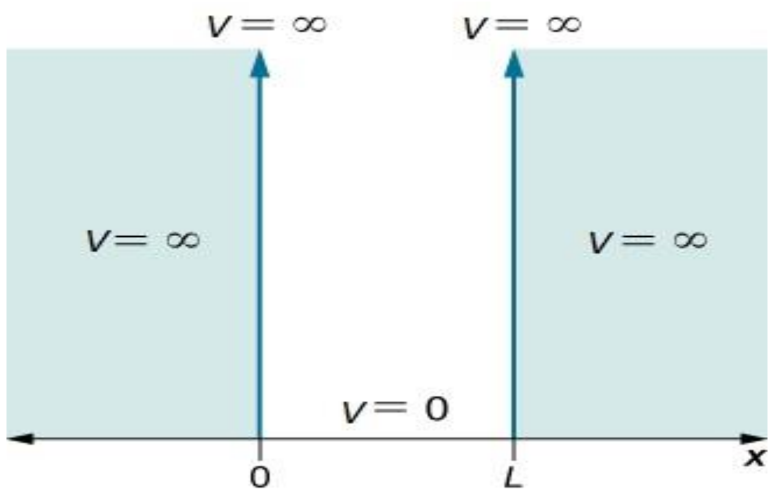
Where λ may be real or complex number. In this case the function Ψ is a member of the class of physically meaning full function called the Eigen function of the operator p. The number λ is called the Eigen value of the operator p associate with Eigen function Ψ equation (2) is called is Eigen value equation.

Remark: The Eigen function are select from a special class of function. In a bound state problem. For example, All wave function are required to be continuous derivative and to vanish at infinity in such a ways as to have an integral square. In the continuous state the wave function are not allowed to become infinite at large distance.

As an example to illustrate the Eigen vale of an operator, consider an operator (d/dx) operating on a well behaved function e-4x, then the result as d/dx(e-4x)=-4e-4x.

Comparing with standard Eigen value equation PΨ=λΨ we not that (-4) is the Eigen value of operator (d/dx) associate with Eigen function e-4x.

Particle in a box (motion in one dimension)



Let us consider a free particle in one dimensional box. The particle has a mass 'm' and is restrical to move in a straight line along the x-axis. The range of the particle is 0<x<a and the particle is reflected box whenever it reaches the end of the range. We have the general equation.

Δ2Ψ+(2m/ℏ2)(E-v)Ψ=0

For a free particle v=0 for 0<x<a and v=∞ for x<0 and x>0.

Also the wave function Ψ vanish at x=0 and x=a. For one dimension, the modified equation is given by,

Δ2y/Δx2+(2m/ℏ2)EΨ=0

Δ2Ψ/Δx2+(8π2mE/h2)Ψ=Ψ—(2)

Let us take,

$8\pi 2mE/h2=k2$

$E=k2h2/8\pi 2m$

So, equation (2) may be written as

$\Delta 2\Psi/\Delta x2+k2\Psi=0$—(3)

The general solution for equation is given bye

$\Psi(x)=$ A sine k x+ B cos k x —(4)

And from the boundary condition .

$\Psi=0$ at $x=0$

So from equation (4) 'B' must be zero

$\Psi(a) = A \sin k a = 0$

Since we cannot take a to be zero because it does not yield any solution, so we take

Sin k x=0

Or, ka =nπ

$K=n\pi/a$

Where, n=1,2,3,4………..

Hence, the only permissible solution of the wave equation are,

Ψ n(x)=A sin (nπ/a)x—(5)

And the value of energy is given by,

$E=k2h2/8\pi m$—(6)

Putting the value of k=nπ/a in equation (6)E n=n2h2π2/8π2ma2

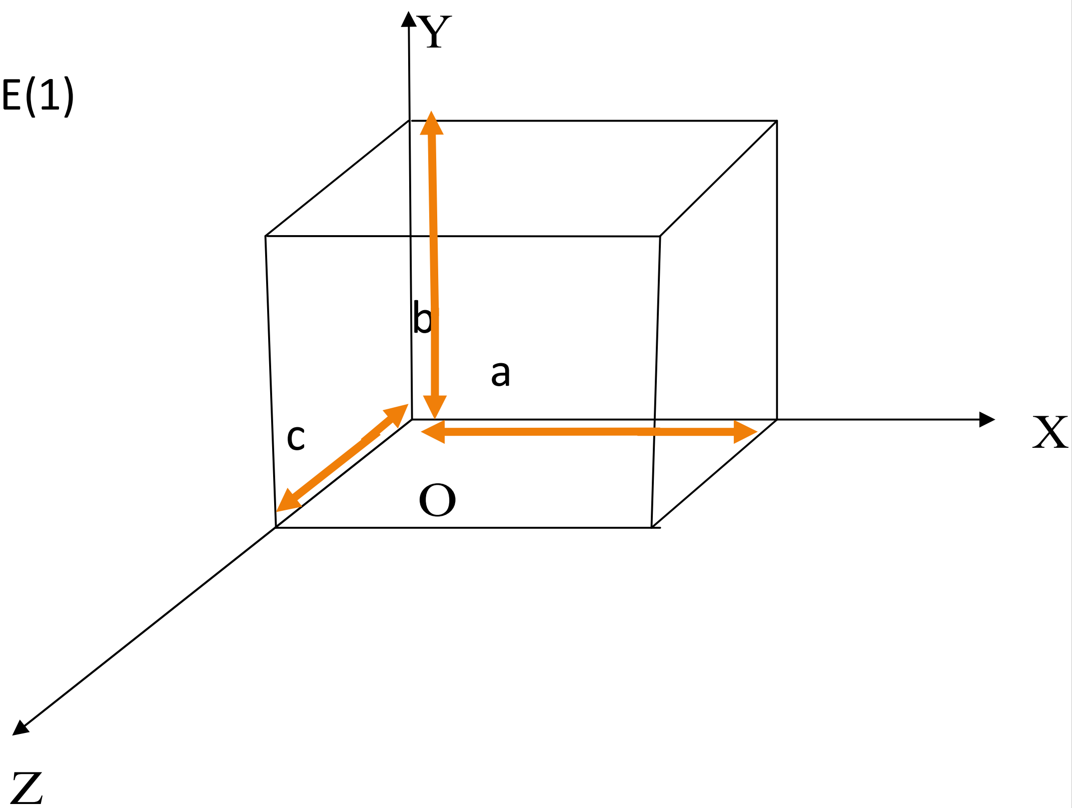
E n=n2h2/8ma2—(8)

It shown 'En' depend upon the value of n. Each value of En is called Eigen value or proper value and each value of Ψn is called an Eigen function an proper function.

Particle in a 3-D box

Consider a single particle for example glass molecule of mass 'm' confined within a rectangular box with edges parallel to x , y & z axes as shown in figure(1).

FIGURE(1)



Let the sides of rectangular box be a,b & c.The particle can move freely within the region $0<x<a$

,$0<y<b$ & $0<z<c$, i;e inside the box where potential(v) is zero i;e

$V(x,y,z) = 0 \quad ; 0<x<a$

$V(x,y,z) = 0$; $0<y<b$

&, $V(x,y,z) = 0$; $0<z<c$

Consider that the potential rises suddenly to have a very large value at the boundaries i;e the potential outside the box is infinite.We have schrodinger wave equation inside the box is given by,

$$+ \quad + \quad + \quad E\Psi = 0$$

(i) $\hbar$

Which is a partial differentiation equation in three independent variables and may be solved by the method of sepration of variables.The solution of equation (i) , we have the form

$\Psi(x,y,z) = X(x)Y(y)Z(z)=X$ (ii)

Where $X(x)$,$Y(y)$ & $Z(z)$ is a function of x,y&z alone.

Putting the value of Ψ from eq^n (ii) in eq^n(i) and dividing by $X(x)Y(y)Z(z)$. we have,

$$- + - \quad - \quad + \quad +$$

$$E = 0$$

$\hbar$

$- = - - \quad - \quad = - \qquad E$

(iii) $\hbar$

The left hand side of eq^n (iii) is a function of x alone , while the right hand side is a function of y and z and is independent of x. since both sides are equal to each other.so,each side should separately be equal to a constant equally,i;e,

$- = k_x$

(iv)

$\therefore -\frac{1}{Y}\frac{\partial\ Y}{\partial y^2} - \frac{1}{Z}\frac{\partial\ Z}{\partial z^2} - \frac{2m}{\hbar^2}E = k_x$

Or,

$- = - - \quad = E - k_x$ (v)

$\hbar$

Again in eq^n (v),the left hand side is a function of y alone,while right hand side is a function of z and is independent of y. If above eq^n is to be satisfied,both sides must be equal to a constant say k_y i;e.,

$- = k_y$ (vi)

$- \quad = E - k_x = k_y$

$\hbar$

$- \quad = - \quad E - k_x - k_y \longrightarrow$

(vii) $\hbar$

Again we may have,

$- = k_z \longrightarrow$ (viii)

$- \; E - k_x - k_y = k_z$

$\hbar$

$- \; E = k_x + k_y + k_z \longrightarrow$

(ix) $\hbar$

For convenience,we introduce

$\frac{\hbar^2}{2m}$

$\frac{\hbar^2}{2m}$

$k_x = -E$,

$k_y = -E$ and

$k_z = -E \, \hbar$

Differential equations in x,y and z coordinates

$+ \hbar \; E \; X(x) = 0 \longrightarrow$ (x)

$$\overline{} + \overline{\hbar}\ E\ Y(y) = 0 \longrightarrow \quad \text{(xi)}$$

$$\overline{} + \overline{\hbar}\ E\ Z(z) = 0 \longrightarrow \quad \text{(xii)}$$

The general solution of equation (x) will be a sine function of arbitrary amplitude frequency and phase i;e.,

$$X(x)= A \sin(Bx+C) \longrightarrow \quad \text{(xiii)}$$

Where A,B&C are constants whose value are determined by boundary conditions.

Also,| Ψ| represents the probability of finding the particles at any point within the box. so, |X(x)| which is a function of x coordinate only represents the probability of finding the particles at any point along the x-axis.As the potential is very high at the walls of the box, the probability of finding the particles at the walls will be zero i;e.,

$$|X(x)| = 0 \quad \text{when } x = 0 \text{ and } x = a$$
$$X(x) = 0 \quad \text{when } x = 0 \text{ and } x = a$$

Using these boundary condition in equation (xiii),we get

$0=A \sin(0+C)$, $A\neq 0$

$\therefore$ sinC=0 and 0=Asin(Ba+C)

Which gives, sinBa=0

Or, $Ba = n_x\pi$

$B = \_\_$

Where, n_x is a positive integer

$\therefore X(x) = A \sin \_\_ \longrightarrow$ (xiv)

Applying the normalization condition between x=0 to x=a, we have,

$\int |X(x)| dx = 1$

Or,

$\int |A \sin \_\_ | dx = 1$

$A \int \sin \_\_ dx = 1$

Or,

$\frac{A\ a}{2} = 1$

$\sqrt{\left(\frac{2}{a}\right)}$ $A =$

$\therefore$ $\sqrt{\left(\frac{2}{-}\right)}$ $\left(\frac{n_x\ \pi x}{}\right.$ $X(x) =$ sin $\longrightarrow$ (xv)

Similarly, we can solve equation(xi) and(xii) to get,

$Y(x) = \sqrt{\left(\frac{2}{-}\right)}$ $\left(\frac{n_y \pi y}{}\right.$ $\longrightarrow$ sin (xvi)

And,

Z(x) = $\sqrt{\left(\frac{2}{\ }\right)}$ $\left(\frac{n_z \pi z}{\ }\right.$ sin $\longrightarrow$ (xvii)

Now,the complete wave function $\Psi_{,\ ,}$ (x, y, z) has the form

$\Psi_{,\ ,}(x, y, z)$ = X(x)Y(y)Z(z)

$=\sqrt{\left(\frac{2}{a}\right)}\left(\frac{n_x \pi x}{a}\right)\sqrt{\left(\frac{2}{b}\right)}\left(\frac{n_y \pi y}{b}\right)\sqrt{\left(\frac{2}{c}\right)}\left(\frac{n_z \pi z}{c}\right)$ sin sin sin

$=\frac{2\sqrt{2}}{\sqrt{abc}}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right)\sin\left(\frac{n_z \pi z}{c}\right)$ $\longrightarrow$ (xviii)

From equation (xv),we get

$\frac{\partial^2 X}{\ }$ $\left(\frac{n_x \pi}{\ }\right)^2\sqrt{\left(\frac{2}{\ }\right)}$ $\left(\frac{n_x \pi x}{\ }\right.$ $= -$ sin

$= -\frac{\ }{\ }$ X(x) $\longrightarrow$ (xix)

Putting the value from equation (xix) in equation (x),we get

$-\frac{\ }{\ }$ X(x) + $\frac{\ }{\hbar}E$ X(x) = 0

Or,

$$E = \frac{1}{}\left(\frac{n_x \pi}{\hbar}\right) = \frac{n_x h}{} \longrightarrow \quad (xx)$$

Similarly,we have

$$E = \longrightarrow \quad (xxi)$$

And,

$$E = \longrightarrow \quad (xxii)$$

The allowed values of total energy are given by,

E $= E + E + E$

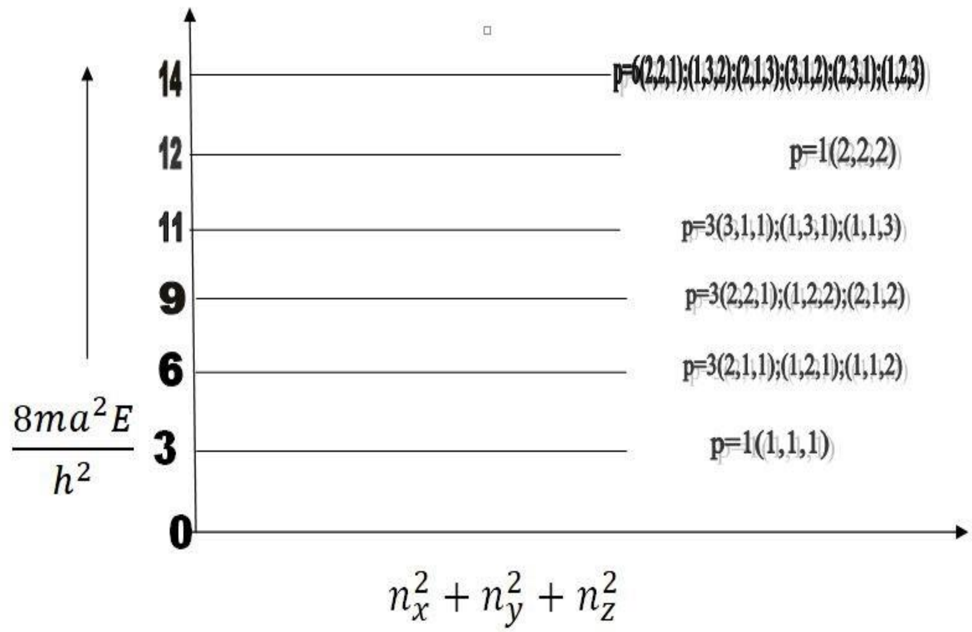
$$E = \frac{n_x}{} + \frac{n_y}{} + \frac{n_z}{}\Big] \longrightarrow \quad (xxiii)$$

Where n , n and n denote any set of three positive numbers.

When the box is a cube , i;e., a=b=c , then energy expression may be written as,

$$\boxed{E = \frac{}{} \quad n \;+ n \;+ n}$$

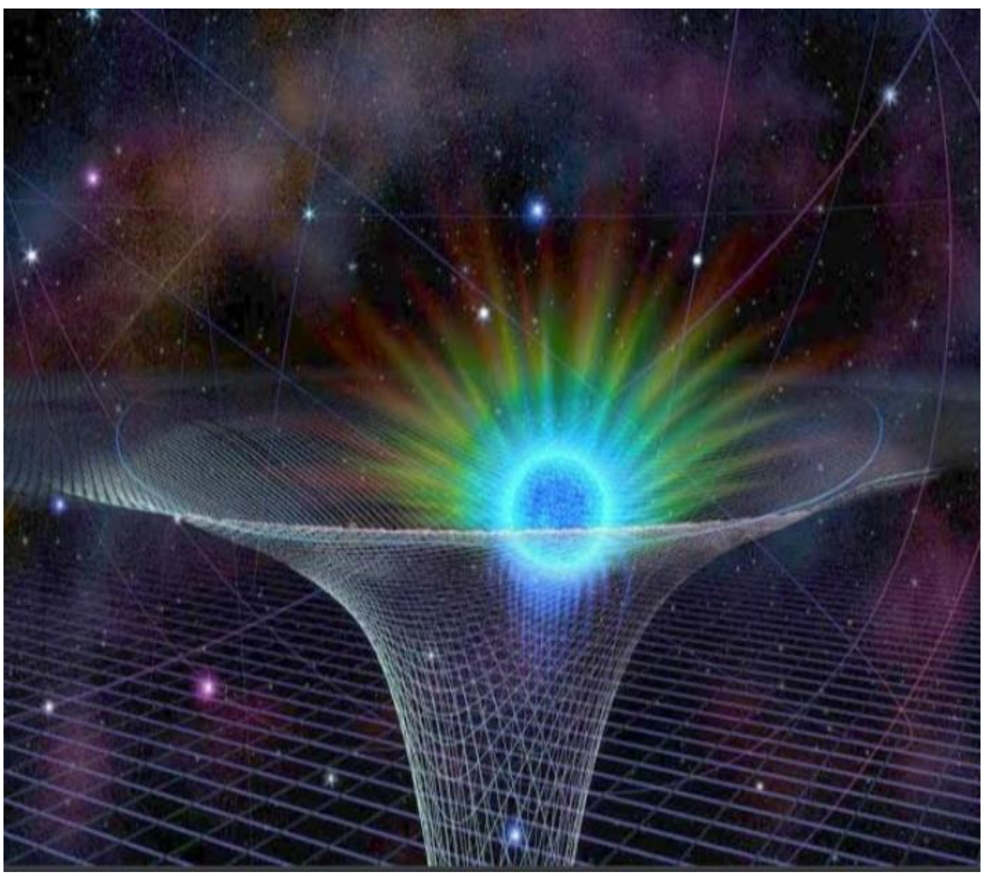
With n , n and n = 1,2,3,…………..



Above figure shows the energy level of few states of a particle enclosed in a cubical box.when there is only one wave function corresponding to a particular eigen value,the level is known as non-degenerate but when there are a number of wave functions corresponding to a single eigen value, the level is known as degenerate.

Module 3

Theory of Relativity



S K FATIMA, DEPT.OF PHYSICS, K.K.C.E.M

The theory of relativity usually encompasses two interrelated theories by Albert Einstein: special relativity and general relativity, proposed and published in 1905 and 1915, respectively.Special relativity applies to all physical phenomena in the absence of gravity. General relativity explains the law of gravitation and its relation to other forces of nature.It applies to the cosmological and astrophysical realm, including astronomy.

The theory transformed theoretical physics and astronomy during the 20th century, superseding a 200-year-old theory of mechanics created primarily by Isaac Newton. It introduced concepts including spacetime as a unified entity of space and time, relativity of simultaneity, kinematic and gravitational time dilation, and length contraction. In the field of physics, relativity improved the science of elementary particles and their fundamental interactions, along with ushering in the nuclear age. With relativity, cosmology and astrophysics predicted extraordinary astronomical phenomena such as neutron stars, black holes, and gravitational waves.

The theory which deals with the relativity of motion relativity and general theory of relativity. with respect to each other or at rest.the general theory of relativity deals with objects and systems which are accelerating or decelerating with respect to one another.

Frame of reference

A frame of reference (or reference frame) consists of an abstract coordinate system whose origin, orientation, and scale are specified by a set of reference points — geometric points whose position is identified both mathematically (with numerical coordinate values) and physically (signaled by conventional markers).

For n dimensions, n + 1 reference points are sufficient to fully define a reference frame. Using rectangular (Cartesian) coordinates, a reference frame may be defined with a reference point at the origin and a reference point at one unit distance along each of the n coordinate axes.

In Einsteinian relativity, reference frames are used to specify the relationship between a moving observer and the phenomenon or phenomena under observation. In this context, the phrase often becomes "observational frame of reference" (or "observational reference frame"), which implies that the observer is at rest in the frame, although not necessarily located at its origin. A relativistic reference frame includes (or implies) the coordinate time, which does not equate across different frames moving relatively to each other. The situation thus differs from Galilean relativity, where all possible coordinate times are essentially equivalent.

Inertial and non-inertial frames of reference

the frame of reference is selected in such a way that the laws of nature may become simpler in that frame of reference. There are two types of frame of reference-inertial frame of reference and non inertial frame of reference.

In an inertial frame, Newton's first law, the law of inertia, is satisfied: Any free motion has a constant magnitude and direction.

In Newton's time the fixed stars were invoked as a reference frame, supposedly at rest relative to absolute space. In reference frames that were either at rest with respect to the fixed stars or in uniform translation relative to these stars, Newton's laws of motion were supposed to hold. In contrast, in frames accelerating with respect to the fixed stars, an important case being frames rotating relative to the fixed stars, the laws of motion did not hold in their simplest form, but had to be supplemented by the addition of fictitious forces, for example, the Coriolis force and the centrifugal force. Two experiments were devised by Newton to demonstrate how these forces could be discovered, thereby revealing to an observer that they were not in an inertial frame: the example of the tension in the cord linking two spheres rotating about their center of gravity, and

the example of the curvature of the surface of water in a rotating bucket. In both cases, application of Newton's second law would not work for the rotating observer without invoking centrifugal and Coriolis forces to account for their observations (tension in the case of the spheres; parabolic water surface in the case of the rotating bucket).

A non-inertial reference frame is a frame of reference that undergoes acceleration with respect to an inertial frame. An accelerometer at rest in a non-inertial frame will, in general, detect a non-zero acceleration. While the laws of motion are the same in all inertial frames, in non-inertial frames, they vary from frame to frame depending on the acceleration.In classical mechanics it is often possible to explain the motion of bodies in non-inertial reference frames by introducing additional fictitious forces (also called inertial forces, pseudo-forcesand d'Alembert forces) to Newton's second law. Common examples of this include the Coriolis force and the centrifugal force. In general, the expression for any fictitious force can be derived from the acceleration of the non-inertial frame.As stated by Goodman and Warner, "One might say that F = ma holds in any coordinate system provided the term 'force' is redefined to include the so-called 'reversed effective forces' or 'inertia forces'.In the theory of general relativity, the curvature of spacetime causes frames to be locally inertial, but globally non-inertial. Due to the non-Euclidean geometry of curved space-time, there are no global inertial reference frames in general relativity. More specifically, the fictitious force which appears in general relativity is the force of gravity.

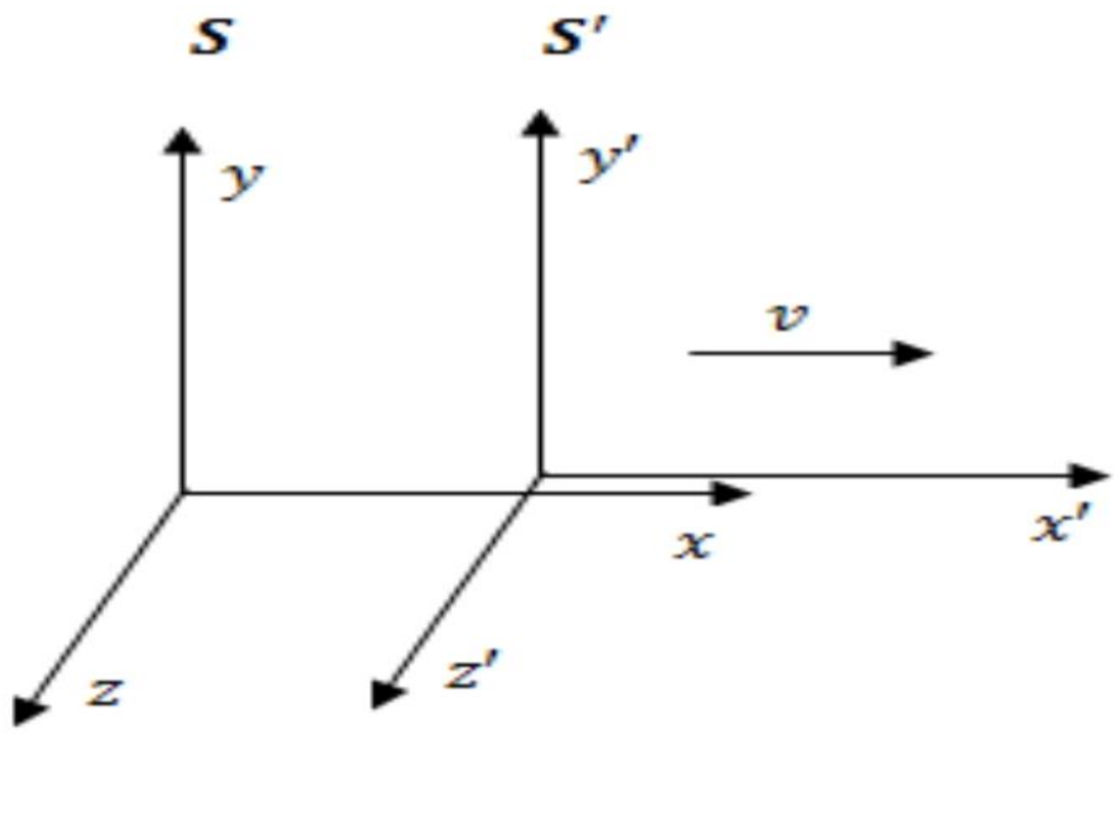
Postulates of special theory of Relativity

Two postulates underlie special theory of relativity. The first, principle of relativity States that "the laws of Physics are the same in all inertial frames of reference". This postulates follows from the absence of a universal frame of reference. If the laws of Physics where different for different observer in relative motion,the observer's could find from this difference is which of them where is stationary in space and which. But such a distinction does not exist, and the principle of relativity expresses this.

The second postulate is based on the result of many experiments."the speed of light in free space has the same value in all inertial frames of reference.".

Gallilian transformation

Galilean transformations, also called Newtonian transformations, set of equations in classical s that relate the space and time coordinates of two systems moving at a constant velocity relative to each other. Adequate to describe phenomena at speeds much smaller than the speed of light, Galilean transformations formally express the ideas that space and time are absolute; that length, time, and mass are independent of the relative motion of the observer; and that the speed of light depends upon the relative motion of the observer.



Let there are two inertial frames of references S and S'. S is the stationary frame of reference and S' is the moving frame of reference. At time $t=t'=0$ that is in the start, they are at the same position that is Observers O and O' coincides. After that S' frame starts moving with a uniform velocity v along x axis.

Let an event happen at position P in the frame S'. The coordinate of the P will be x' according to the observer in S' and it will be x according to O in S.

The frame S' has moved a distance "vt" in time t (refer figure).

What should be the relation between x and x'. As we can see from the figure that

$x = x' + vt'$

But here the $t = t'$ thus

$$x = x' + vt \qquad (1)$$

Where t and t' are the time measured from S and S' frames respectively.

We have

$$x' = x - vt \qquad (2)$$

It can be achieved by just exchanging the sides of the equation (1).

If we see equation 1, we will find that it is the position measured by O when S' is moving with +v velocity. But if the same thing is measured by O' then velocity of S should be –v. (For example, when we travel in a train, then according to the outside observers, we are travelling in x direction (suppose), but the outside objects, according to me travel in the opposite direction with the same but negative velocity).For y coordinate

$y = y'$ (3)

or $y' = y$ (4)

because there is no movement of frame along y-axis.

Similarly $z = z'$ (5)

And $z' = z$ (6)

And here $t = t'$ (7)

And $t' = t$ (8)

Equations 1, 3, 5 and 7 are known as Galilean inverse transformation equations for space and time.

Or Equations 2, 4, 6 and 8 are known as Galilean transformation equations for space and time.

Michelson - Morley experiment

Michelson-Morley experiment, an attempt to detect the velocity of Earth with respect to the hypothetical luminiferous ether, a medium in space proposed to carry light waves. First performed in Germany in 1880–81 by the physicist A.A. Michelson, the test was later refined in 1887 by Michelson and Edward W. Morley in the United States.The procedure depended on a Michelson interferometer, a sensitive optical device that compares the optical path lengths for light moving in two mutually perpendicular directions. Michelson reasoned that, if the speed of light were constant with respect to the proposed ether through which Earth was moving, that motion could be detected by comparing the speed of light in the direction of Earth’s motion and the speed of light at right angles to Earth’s motion. No difference was found. This null result seriously discredited the ether theories and ultimately led to the proposal by Albert Einstein in 1905 that the speed of light is a universal constant.

Sound waves require some medium through which these waves can travel. Maxwell in 1864 showed that light is an electromagnetic wave and hence was supposed that there is an ether which propagates light ray. By observing how light propagates through the ether, one can determine an absolute reference frame. Hence, the Michelson Morley experiment was accomplished to detect ether that was assumed to be the carrier of light waves. The purpose of the Michelson and Morley experiment was to detect the velocity of the Earth to ether. The procedure was based on the optical device named interferometer that compares the path lengths for light rays travelling in perpendicular directions.

According to Michelson’s experiment theory, the light should travel at different speeds through ether. The speed at which light moves depends on the relative motion through space. Michelson Morley designed an interferometer to spot the minute differences in the arrival time of light beams. Out of all these beams, one can take a long time to reach the sensor while travelling through ether.

The experiment performed compared the speed of light to notice the relative motion of Earth through ether. However, the conclusion of the Michelson Morley experiment comes out to be negative. It means that they found no difference between the speed of light while travelling through ether.

Michelson Morley interferometer sent white light for the actual observations and yellow light from a sodium flame through a half-transparent mirror. The mirror was used to split the coming light beam into two separate beams travelling perpendicular to each other. After leaving this mirror, beams moved out to the long arms end where they faced back reflection into the middle. These two beams then recombine to produce a pattern of constructive and destructive interference.

Procedure of Michelson Morley Experiment

Michelson claimed that if the speed of light was constant concerning the ether medium through which the Earth moves, then that motion can be detected. It can be sensed by comparing the speed of light perpendicular to and in the direction of the Earth's motion. The beam of light gets incident at a half-silvered glass plate. This plate acts as a beam splitter, which splits the light beam into two coherent beams. One beam transmits, and the other reflects. The beam transmitted strikes the mirror, say, M1, and gets reflected. The beam reflected strikes the mirror, say, M2, which again gets reflected. The returned beams reach the telescope, which is used for interference patterns produced by these two rays. The above picture shows that incident light from the coherent source of light strikes the half-silvered mirror. The separation between the plate and two mirrors is the same, which refers to the arm's length. The light reflected from two mirrors interfere with the mirror.

Now, from the Michelson Morley experiment notes, it can be noticed that the apparatus and light both are moving in the same direction. Thus, the relative velocity will be c - v. After reflection, the apparatus, and light both move in the opposite direction. Hence, in this case, relative velocity will become c + v.

Let Us Calculate the Time Taken by the Transmitted Ray to Travel to the Mirror:

$t1 = 1/(c–v) + 1/(c+v)$

$t1 = l*(c+v+c–v)/ (c2 -v2)$

$t1 = l*[2c/ (c2 -v2)]$

$t1 = 2lc/c2 *[1/1 - (v2/c2)]$

$t1 = 2l/c *[1/1 - (v2/c2)]$

$t1 = 2l/c *[1 - (v2/c2)]-1$

Applying Binomial Theorem on the above equation and neglecting higher power terms gives:

$t1 = 2l/c *[1 + (v2/c2)]$

Now, time taken by the reflected ray to travel to mirror:

t2 = [1/ (c2 – v2)1/2] + [1/(c2 + v2)1/2]

t2 = 2l/ (c2 – v2)1/2

t2 = 2l/c * [1/ (1– (v2/c2)1/2]

t2 = 2l/c * [1 - (v2/c2)]-1/2

Similarly, applying Binomial Theorem:

t2 = 2l/c * [1 + (v2/2c2)]

Michelson Morley experiment derivation indicates the time difference between two rays:

Δt = t2 – t1

Using the values of t1 and t2:

$\Delta t = 2l/c * [1 + (v2/c2) - 1 - (v2/2c2)]$

$\Delta t = l/c * (v2/c2)$

After the first attempt, the apparatus is rotated clockwise to 90-degree so that two mirrors can exchange their position. Now the time difference between two mirrors can be given by:

$\Delta t = -l/c * (v2/c2)$

Due to the rotation of apparatus, there is a delay in time, which is given by:

$\Delta t - \Delta t' = 2l/c * (v2/c2)$

This time delay causes the fringe pattern to move. Let N denote the total amount of fringe shift, which can be calculated as:

$N = \Delta\delta/2\pi$

$N = 2l/\lambda * (v2/c2)$

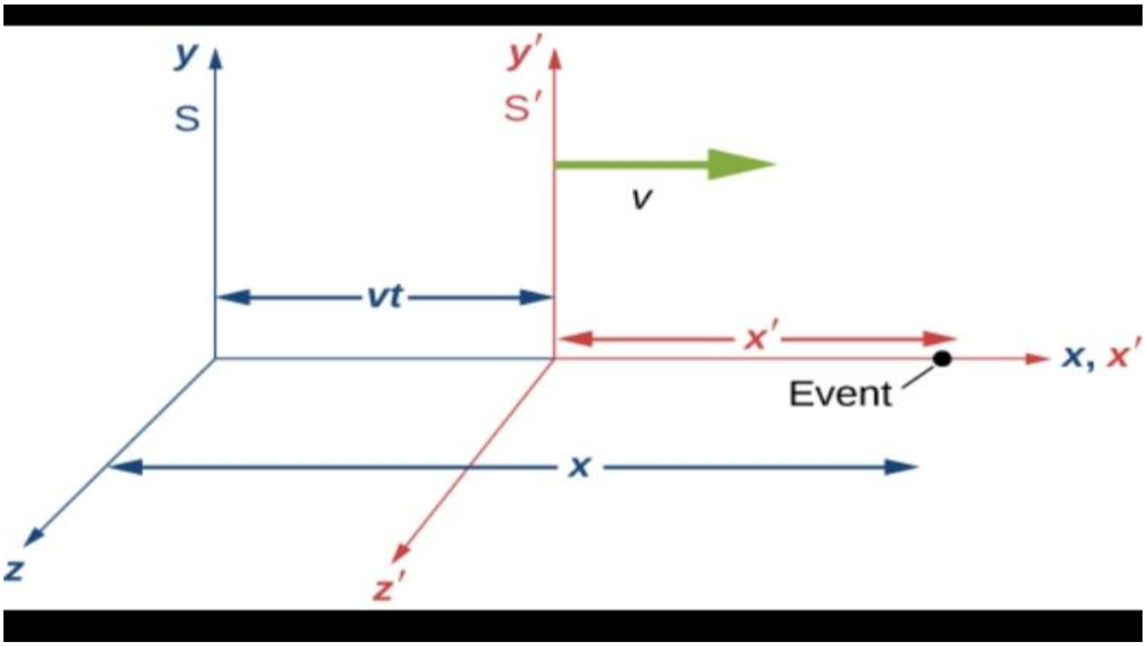
Conclusion

The major objective of the Michael Morley experiment was to verify the ether hypothesis. The experiment has been repeated several times but there was no particular conclusion of the Michelson Morley experiment.

Lorentz transformation

H A Lorentz algebraically searched for this transformation equations which relate the observation of position and time made by two observers sitting and two different inertial frames and are known as Lorentz transformation equations.Results of Galilean Transformation equations can not be applied for the objects moving with a speed comparative to the speed of the light.

Therefore new transformations equations are derived by Lorentz for these objects and these are known as Lorentz transformation equations for space and time



Let there are two inertial frames of references S and S'. S is the stationary frame of reference and S' is the moving frame of reference. At time t=t'=0 that is in the start, they are at the same position that is Observers O and O' coincides. After that S' frame starts moving with a uniform velocity v along x axis.

Let an event happen at position P in the frame S'. The coordinate of the P will be x' according to the observer in S' and it will be x according to O in S.The frame S' has moved a distance "vt" in time t.

As we can see from the figure that from frame S'

$x' \propto x - vt$

or $x' = k\,(x - vt)$ (1)

where k is constant of proportionality that we will determine.

Similarly from frame S

$x = k(x' + vt')$ (2)

Put equation (1) in (2)

$x = k[k(x - vt) + vt']$

or $x/k = kx - kvt + vt'$

or $vt' = x/k - kx + kvt$

or $t' = x/kv - kx + kvt$

or $t' = kt - kx (1 - 1/k2)/v$ (3)

Similarly from frame S, time t will be

$t = kt' + kx' (1 – 1/k2)/v$ (4)

(This equation can be derived by putting equation 2 in 1 and then solving.)

Calculation of k:

Let us suppose a flash of light is emitted from the common origin of S and S' at time t=t'=0. From Einstein's 2nd second postulate, the flash of light travels with the velocity of light c and which remains same in both the frames.

After sometime, the position of the flash of the light as seen from observer O will be

$x = ct$

And as seen from O' will be

$x = ct'$ (Here the form of Physics law is same that is position = (velocity)(time) from Einstein 1st postulate)

Put these two values in equation (1) and (2) respectively, we get

$ct' = k (ct – vt) = kt (c –v)$

and $ct = kt'(c + v)$

Multiply above two equations

$c^2tt' = k^2tt'(c^2 - v^2)$

or $k^2 = c^2/(c^2 - v^2)$

or $k^2 = 1/ (1 - v^2/c^2)$ (5)

or $k = 1/\sqrt{(1 - v^2/c^2)}$ (6)

The k is known as relativistic factor.

Substitute equation (6) in (1), we get

$x' = (x - vt)/(\sqrt{1 - v^2/c^2})$ (7)

As it is assumed that frame S' is moving only along x direction, therefore along y and z direction

$y' = y$ (8)

And $z' = z$ (9)

Equations 7-9 are known as Lorentz transformation equations for space.

Let us derive Lorentz transformation equation for time:

Cross-multiply equation (5)

$1/k2 = 1 – v2/c2$

Or $1 – 1/k2 = v2/c2$

Put the above equation in equation (3)

$t' = kt – kx(v2/c2)/v$

or $t' = k (t – kxv/c2)$

Put value of k from equation 5 in above equation, we get

t' = (t – kxv/c2)/ (√1 – v2/c2) (10)

Equation (10) is Lorentz transformation equation for time.

Equations 7 -10 are known as Lorentz transformation equations for space and time. These are again rewritten below:

x' = (x – vt)/(√1 – v2/c2)

y' = y

z' = z

t' = (t – xv/c2)/(√1 – v2/c2)

If the frame is changed (that is from S), then the equations are known as Lorentz inverse transformation equations for space and time. These are given as:

$x = (x' + vt')/(\sqrt{1 - v2/c2})$

$y = y'$

$z = z'$

$t = (t' + x'v/c2)/(\sqrt{1 - v2/c2})$

Special case:

If $v <<< c$

Then Lorentz equations will become Galilean by neglecting v2/c2 or v/c2 wherever necessary as shown below:

$x' = x - vt$

$y' = y$

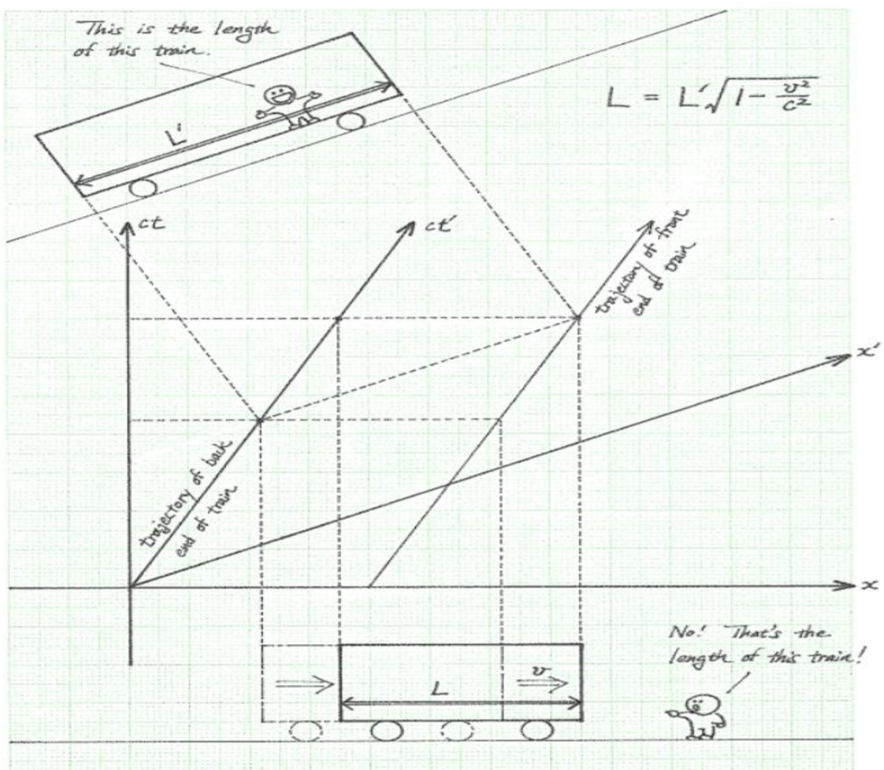
$z' = z$

$t' = t$

Length contraction

Length contraction is the phenomenon that a moving object's length is measured to be shorter than its proper length, which is the length as measured in the object's own rest frame. It is also known as Lorentz contraction or Lorentz–FitzGerald contraction (after Hendrik Lorentz and George Francis FitzGerald) and is usually only noticeable at a substantial fraction of the speed of light. Length contraction is only in the direction in which the body is travelling. For standard objects, this effect is negligible at everyday speeds, and can be ignored for all regular purposes, only becoming significant as the object approaches the speed of light relative to the observer.Length contraction was postulated by George FitzGerald (1889) and Hendrik Antoon Lorentz (1892) to explain the negative outcome of the Michelson–Morley experiment and to rescue the hypothesis of the stationary aether (Lorentz–FitzGerald contraction hypothesis).Although both FitzGerald and Lorentz alluded to the fact that electrostatic fields in motion were deformed ("Heaviside-Ellipsoid" after Oliver Heaviside, who derived this deformation from electromagnetic theory in 1888), it was considered an ad hoc hypothesis, because at this time there was no sufficient reason to assume that intermolecular forces behave the same way as electromagnetic ones. In 1897 Joseph Larmor developed a model in which all forces are considered to be of electromagnetic origin, and length contraction appeared to be a direct consequence of this model. Yet it was shown by Henri Poincaré (1905) that electromagnetic forces alone cannot explain the electron's stability. So he had to introduce another ad hoc hypothesis: non-electric binding forces (Poincaré stresses) that ensure the electron's stability, give a dynamical explanation for length contraction, and thus hide the motion of the stationary aether.

Eventually, Albert Einstein (1905) was the first to completely remove the ad hoc character from the contraction hypothesis, by demonstrating that this contraction did not require motion through a supposed aether, but could be explained using special relativity, which changed our notions of space, time, and simultaneity.Einstein's view was further elaborated by Hermann Minkowski, who demonstrated the geometrical interpretation of all relativistic effects by introducing his concept of four-dimensional spacetime.



To develop an equation relating distances measured by different observers, we note that the velocity relative to the Earth-bound observer in our muon example is given by

v=L0/Δt

The time relative to the Earth-bound observer is Δt, since the object being timed is moving relative to this observer. The velocity relative to the moving observer is given by

$v=L/\Delta t0$

The moving observer travels with the muon and therefore observes the proper time $\Delta t0$. The two velocities are identical; thus,

$L0/\Delta t=L\Delta/t0$

We know that $\Delta t = \gamma\Delta t0$. Substituting this equation into the relationship above gives

$L=L0/\gamma$

Substituting for γ gives an equation relating the distances measured by different observers.

Length contraction L is the shortening of the measured length of an object moving relative to the observer's frame.

$L=L0\sqrt{}1-v^2/c^2$

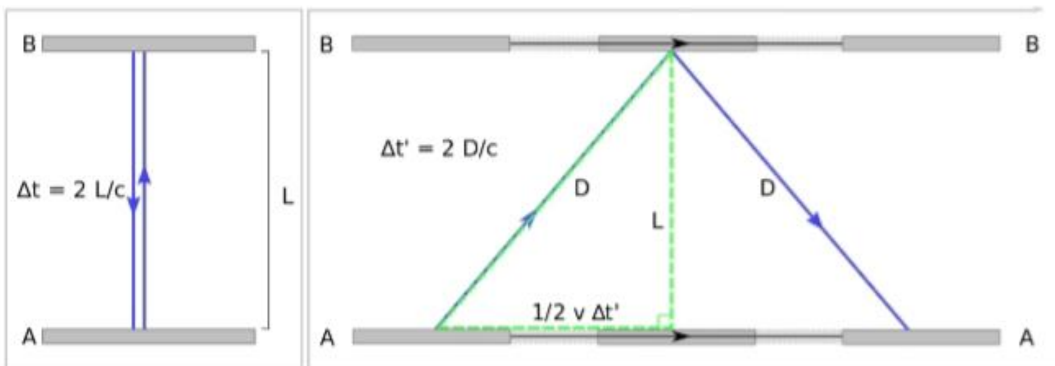
If we measure the length of anything moving relative to our frame, we find its length L to be smaller than the proper length L0 that would be measured if the object were stationary. For example, in the muon's reference frame, the distance between the points where it was produced and where it decayed is shorter. Those points are fixed relative to the Earth but moving relative to the muon. Clouds and other objects are also contracted along the direction of motion in the muon's reference frame.

Time dilation

time dilation, in the theory of special relativity, the “slowing down” of a clock as determined by an observer who is in relative motion with respect to that clock. In special relativity, an observer in inertial (i.e., nonaccelerating) motion has a well-defined means of determining which events occur simultaneously with a given event. A second inertial observer, who is in relative motion with respect to the first, however, will disagree with the first observer regarding which events are simultaneous with that given event. (Neither observer is wrong in this determination; rather, their disagreement merely reflects the fact that simultaneity is an observer-dependent notion in special relativity.) A notion of simultaneity is required in order to make a comparison of the rates of clocks carried by the two observers. If the first observer’s notion of simultaneity is used, it is found that the second observer’s clock runs slower than the first observer’s by a factor of Square root of$\sqrt{(1 - v2/c2)}$, where v is the relative velocity of the observers and c equals 299,792 km (186,282 miles) per second—i.e., the speed of light. Similarly, using the second observer’s notion of simultaneity, it is found that the first observer’s clock runs slower by the same factor. Thus, each inertial observer determines that all clocks in motion relative to that observer run slower than that observer’s own clock.

A closely related phenomenon predicted by special relativity is the so-called twin paradox. Suppose one of two twins carrying a clock departs on a rocket ship from the other twin, an inertial observer, at a certain time, and they rejoin at a later time. In accordance with the time-dilation effect, the elapsed time on the clock of the twin on the rocket ship will be smaller than that of the inertial observer twin—i.e., the non-inertial twin will have aged less than the inertial observer twin when they rejoin.

The time-dilation effect predicted by special relativity has been accurately confirmed by observations of the increased lifetime of unstable elementary particles traveling at nearly the speed of light. The clock paradox effect also has been substantiated by experiments comparing the elapsed time of an atomic clock on Earth with that of an atomic clock flown in an airplane. The latter experiments, furthermore, have confirmed a gravitational contribution to time dilation, as predicted by the theory of general relativity.



Left: Observer at rest measures time $2L/c$ between co-local events of light signal generation at A and arrival at A.

Right: Events according to an observer moving to the left of the setup: bottom mirror A when signal is generated at time $t'=0$, top mirror B when signal gets reflected at time $t'=D/c$, bottom mirror A when signal returns at time $t'=2D/c$

Time dilation can be inferred from the observed constancy of the speed of light in all reference frames dictated by the second postulate of special relativity.

This constancy of the speed of light means that, counter to intuition, speeds of material objects and light are not additive. It is not possible to make the speed of light appear greater by moving towards or away from the light source.

Consider then, a simple vertical clock consisting of two mirrors A and B, between which a light pulse is bouncing. The separation of the mirrors is L and the clock ticks once each time the light pulse hits mirror A.

In the frame in which the clock is at rest (diagram on the left), the light pulse traces out a path of length 2L and the period of the clock is 2L divided by the speed of light:

$$\Delta t = \frac{2L}{c}$$

From the frame of reference of a moving observer traveling at the speed v relative to the resting frame of the clock (diagram at right), the light pulse is seen as tracing out a longer, angled path. Keeping the speed of light constant for all inertial observers requires a lengthening of the period of this clock from the moving observer's perspective. That is to say, in a frame moving relative to the local clock, this clock will appear to be running more slowly. Straightforward

application of the Pythagorean theorem leads to the well-known prediction of special relativity:

The total time for the light pulse to trace its path is given by:

$$\Delta t' = \frac{2D}{c}$$

The length of the half path can be calculated as a function of known quantities as:

$$D = \sqrt{\left(\frac{1}{2}v\Delta t'\right)^2 + L^2}$$

Elimination of the variables D and L from these three equations results in:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which expresses the fact that the moving observer's period of the clock in the frame of the clock itself.Because all clocks that have a common period in the resting frame should have a common period when observed from the moving frame, all other clocks—mechanical, electronic, optical (such as an identical horizontal version of the clock in the example)—should exhibit the same velocity-dependent time dilation.

Mass-energy equivalence

Mass-energy equivalence implies that, even though the total mass of a system changes, the total energy and momentum remain constant. Consider the collision of an electron and a proton. It destroys the mass of both particles but generates a large amount of energy in the form of photons. The discovery of mass-energy equivalence proved crucial to the development of theories of atomic fusion and fission reactions.

Mass-energy equivalence states that every object possesses certain energy even in a stationary position. A stationary body does not have kinetic energy. It only possesses potential energy and probable chemical and thermal energy. According to the field of applied mechanics, the sum of all these energies is smaller than the product of the mass of the object and square of the speed of light.

Mass-energy equivalence means mass and energy are the same and can be converted into each other. Einstein put this idea forth but he was not the first to bring this into the light. He described the relationship between mass and energy accurately using his theory of relativity. The equation is known as Einstein’s mass-energy equation and is expressed as,

E=mc^2

Where E= equivalent kinetic energy of the object,

m= mass of the object (Kg) and

c= speed of light (approximately = 3 x 108 m/s)

Derivation of Einstein’s Equation

The simplest method to derive Einstein’s mass-energy equation is as follows,

Consider an object moving at a speed approximately of the light.A uniform force is acting on it. Due to the applied force, energy and momentum are induced in it.

As the force is constant, the increase in momentum of the object= mass x velocity of the body.

We know,

Energy gained= Force x Distance through which force acts

E= F x c .... (1)

Also,

The momentum gained = force x Duration through which force acts

As, momentum = mass x velocity,

The momentum gained = m x c

Hence, Force= m x c .... (2)

Combining the equation (1) and (2) we get,

E= m c^2

Applications of Einstein's Equation

1.Einstein's theory was used to understand nuclear fission and fusion reactions. Using the formula, it was revealed that a large amount of energy is liberated during nuclear fission and fusion processes. This phenomenon is used in creating nuclear power and nuclear weapons.

2.To find out binding energy in an atomic nucleus, the equation is used. By measuring the masses of various nuclei and subtracting it from the sum of masses of protons and neutrons, Binding energy is calculated. Measurement of binding energy is used to calculate the energy released during nuclear reactions.

3.Einstein's equation is used to find out the change in mass during the chemical reactions. Whenever there is a chemical reaction, breakage and formation of new bonds take place. During the exchange of molecules, change in mass takes place. For chemical energy, Einstein's equation can be written as

$E = \Delta m \times c^2$

Where Δm- change in mass

4.The radioactivity of various elements is based on the theory of mass-energy equivalence. Radioactivity produces X-rays, gamma rays. So in many radiotherapy equipments, the same principle is used.

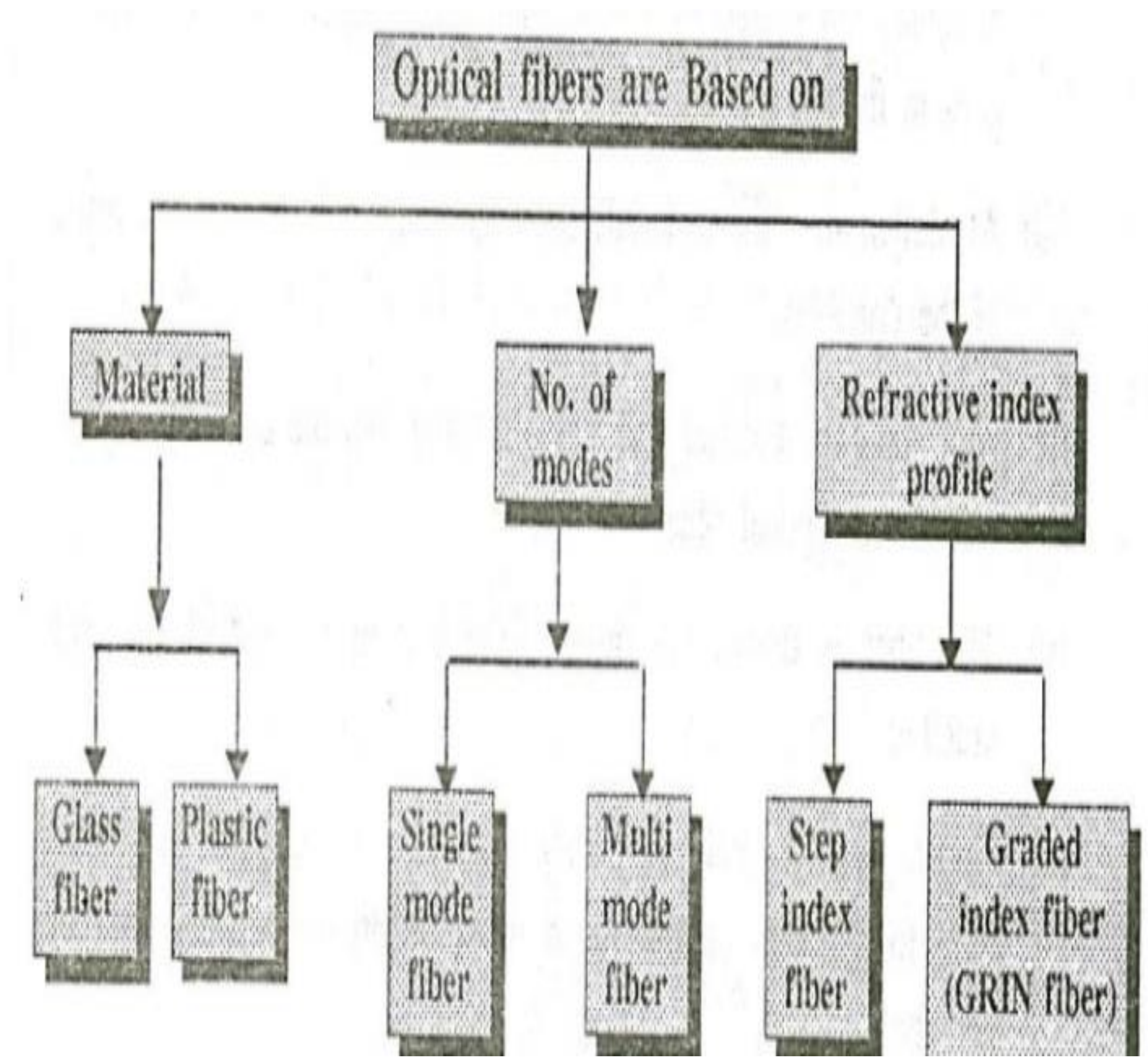
5.To understand the effect of gravity on all-stars, moon and planet, and to measure age of fossil fuels.

6.In many surgeries, where opening and stitching of body parts is not done,Cath lab is used. It works on Einstein's equation

.7.To understand the universe, its constituents and age of planets, The equation is used.

Module 4

OPTICAL FIBRE



Optical fibre

optical fibre is a waveguide through which light can be transmitted with very little leakage through the side walls. Optical fibres transport light signals from place to place as metallic conductors transportlelectrical signals.

John tyndall a British physicist demonstrated to royal society in 1870 that light can be guided along a curved stream of water. Owing to total internal reflection light gets confined to the the water stream and the the stream appears luminous. That results into enchanting display of colourful luminous fountain of water in public places or garden. A luminous water stream was the the precursor of an optical fibre.

An optical fibre is a glass or plastic conduits as thin as thin human hair, designated to guide light waves along their length. Usually optical fibre is a cylindrical wave guide system through which the the optical wave can propagate.when light enters one end of the fibre, it undergoes successive total internal reflection from sidewalls and travels down the length of the fibre along a zigzag path as shown in the figure.(1) a small fraction of light may escape through side walls but a major fraction emerges out from the other end of the fibre. It works on the principle of total internal reflection

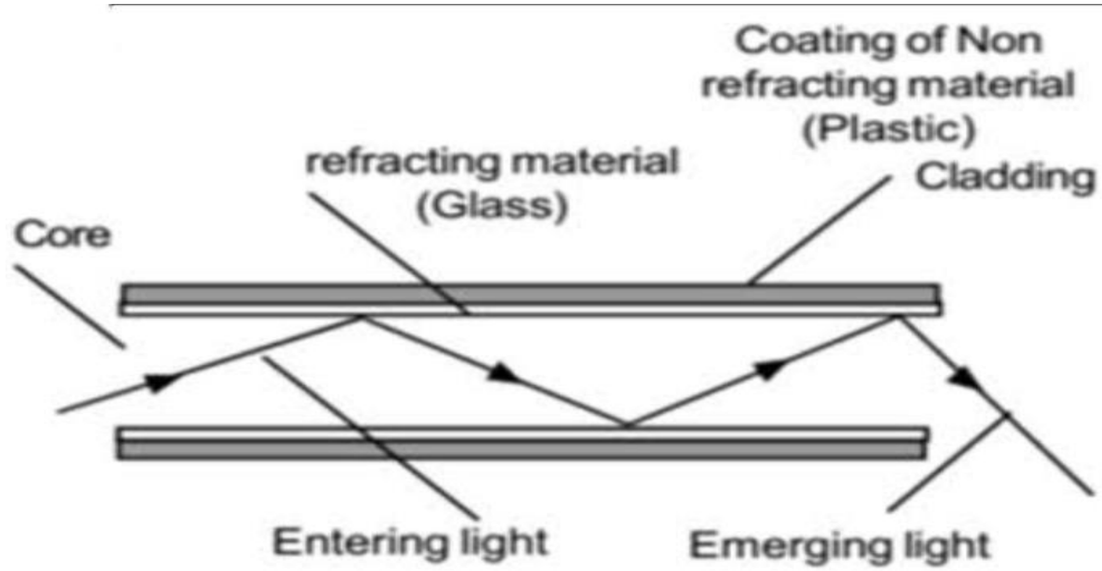


Figure-(1)

An important structure used in optical system is the layered structure or the waveguide. As the name implies,these structure are used to confine the optical waves in a variable defined region and guide their. The layered structure is made either from none crystaline materials or from crystalline materials. For example,optical fibres used in optical communication, whereas semiconductor waveguides are are used in semiconductor laser.

A practical fibre has in general three coaxial regions as shown in figure(2). These are core, cladding and sheath or jacket. Core is the innermost region which is the light guiding. It is surrounded by a coaxial middle region known as cladding.the refractive index of cladding is always lower than that of the core.the purpose of cladding is to make the light to be confined to the core.angle will be refracted back into the core. Since the angle of incidence is equal to to the angle of refraction, the light continuous to rebound and propagate through the fibre. Here,the interface between the core and cladding acts as a mirror at which total internal reflection of the transmitted light takes place.

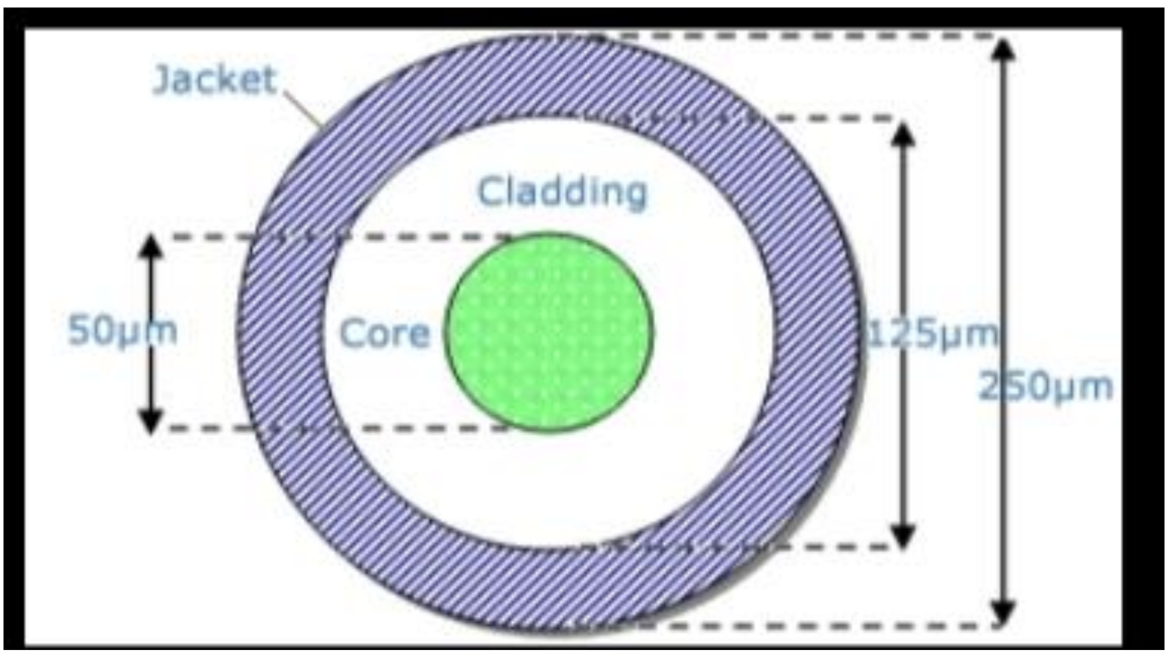


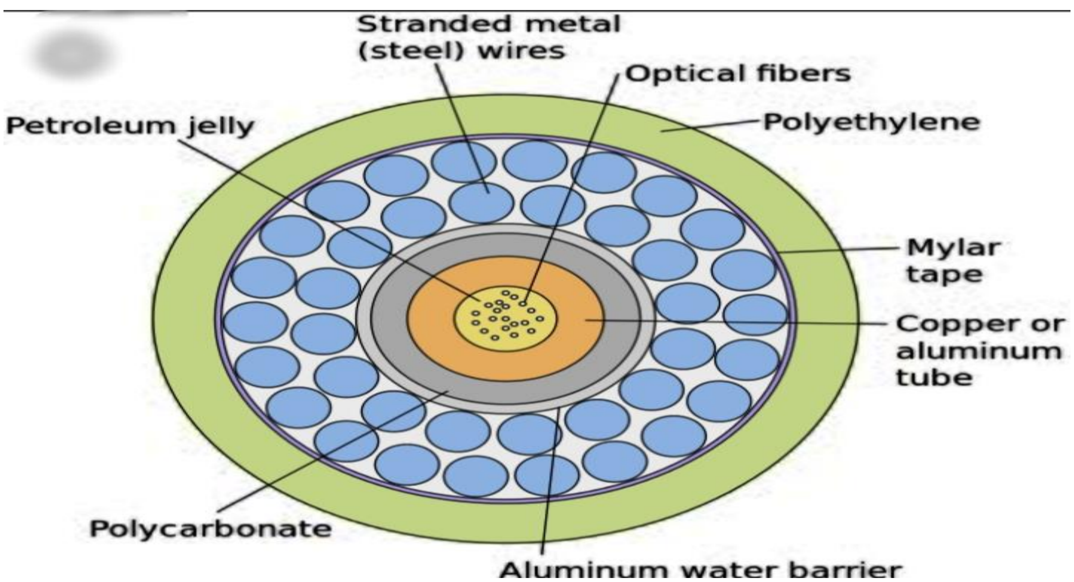
Figure (2)-cross sectional view of an optical fibre.

To prevent energy losses via absorption and, the cladding should be at least a few wavelength thick. Here,it can be noted that the dielectric cladding on glass core reduces scattering loss, protects from observing external optical disturbances and provides mechanical strength to main c

ore glass fibre. Sometimes, there is buffer coating over cladding which adds further strength to main fibre and protects fibre from Mechanical vibration and impact.

The outermost section of the fibre is known as the jacket.jacket is made of plastic or polymer and other materials and this protects the fibre structure from moisture, abrasion, mechanical shocks, and other environmental hazards. In addition it increases the mechanical strength of the fibre.

Optical fibre are constructed, either as a single fibre or flexible bundle or a cable. A fibre bundle is a number of fibres in a single jacket. Is fibres carries light independently. The cross-sectional view of typical telecommunication cable is shown in figure (3) below.

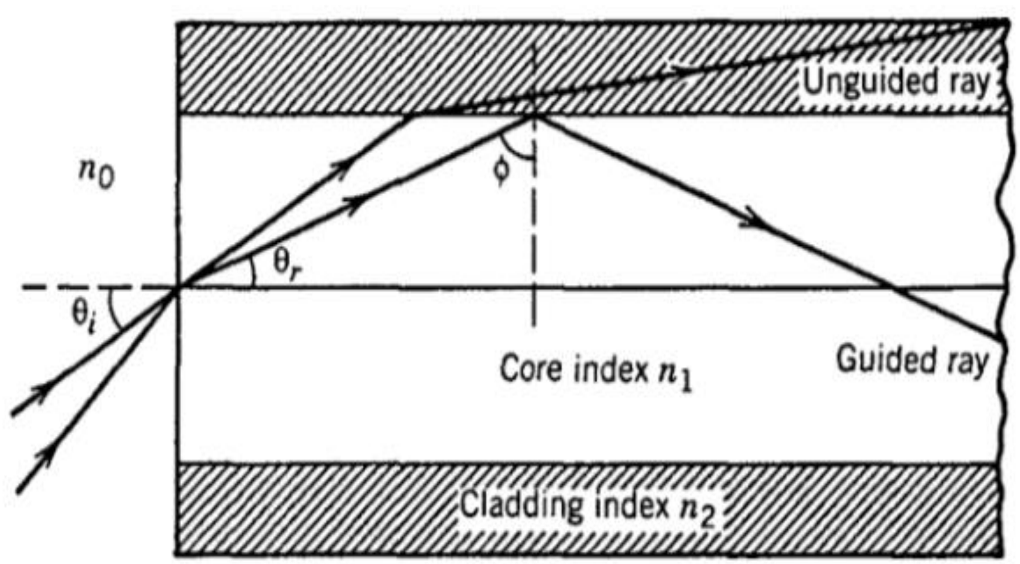


it contains 6 fibres and has an insulated Steel cables at the centre for providing tensile strength.each optical fibre consists of a core surrounded by a cladding which in turn is coated with an insulating jacket. The fibres are does individually buffered and strengthened.6 insulated Corpus are distributed in the space between the fibres.they are used for electrical transmission if required. The fibres are wrapped with mylar tape to bind the assembly. The assembly is then filled in corrugated aluminium sheath which acts as a sheath. A polyethylene jacket is applied over the top.

PROPAGATION OF LIGHT THROUGH A CLADDED FIBRE

The main function of an optical fibre is to accept and transmit as much light from the source as possible.the light gathering ability of a fibre depends on two factors- core size and numerical aperture (NA).the numerical aperture of the fibre is determined by the acceptance angle and the fractional refractive index change.

Acceptance angle and Acceptance cone



Light confinement through total internal reflection in step-index fibers. Rays for which $\phi < \phi_c$ are refracted out of the core.

Hear refractive index distribution in the transverse direction as

$n(r) = n_1 \quad 0 < r \leq a$ core

$= n_2 \quad a < r \leq b$. Cladding

Where a- radius of core,b- radius of cladding.

Let n be the refractive index of the medium from which light is launched into the. Let a Ray of light enter the fibre at angle θ_i to the axis of the fibre. Dari refracts at an angle θ_r and strikes the core cladding interface at an angle Φ. If Φ is greater than critical angle Φ_c , the ray undergoes total internal reflection at the interface because refractive index of core is greater than refractive index of the cladding. As long as the angle Φ is greater than critical angle, the light will stay within the fibre.

Let us compute the angle of incidence for which $\Phi > \Phi_c$ such that light rebounds within the fibre.

Applying snell's law to the launching phase of the fibre ,we get

$\sin\theta_i / \sin\theta_r = n_1 / n$ __________(1)

If angle of incidence is increased angle of refraction will increase. Due to increase in angle of refraction Φ will decrease. If angle of incidence increased beyond the limit,Φ will drop below the critical value Φ_c and the ray will be refracted in the cladding region. 8 mile scappe from the side walls of the fibre. So, the largest value of the angle of incidence occurs when $\Phi=\Phi_c$.

We can write

$\theta_r = 90° - \Phi$

$\sin\theta_r = \sin(90° - \Phi) = \cos\Phi$. ___________(2)

From equation (1)

Sin θi =n1 /n (sin θr)

sin θi =n1 /n (cosΦ)

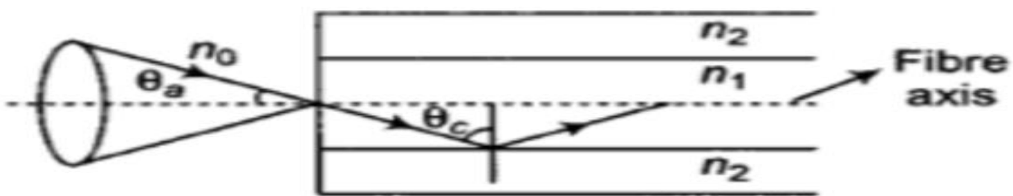
When Φ=Φc, then θi=θi(maximum)

Therefore,

Sin[θi(max)] =n1 /n (cosΦ) ____________(3)

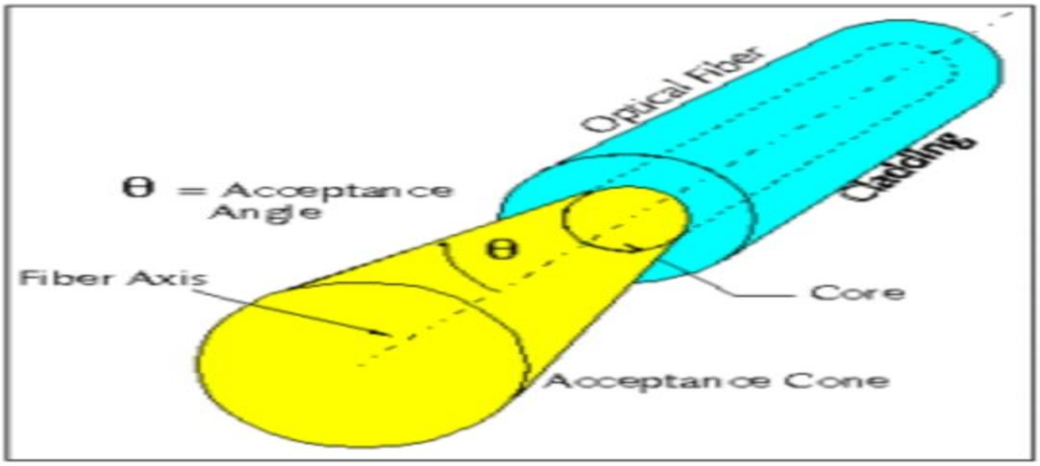
Here, sin Φc = n2/n1

Or. Cos Φc = root under 1-sin^2Φc



Acceptance angle of fibre

The acceptance angle of an optical fiber is defined based on a purely geometrical consideration (ray optics): it is the maximum angle of a ray (against the fiber axis) hitting the fiber core which allows the incident light to be guided by the core. The sine of that acceptable angle (assuming an incident ray in air or vacuum) is called the numerical aperture, and it is essentially determined by the refractive index contrast between core and cladding of the fiber, assuming that the incident beam comes from air or vacuum:acceptance angle of fiber-Here, ncore and ncladding are the refractive indices of core and cladding, respectively, and n is the refractive index of the medium around the fiber, which is close to 1 in case of air.



acceptance cone

Acceptance cone: It is the cone in which the light incident at acceptance angle or less than the acceptance angle and then the light can propagate through the fiber after total internal reflection.

Fractional Refractive index change (Δ)= $\Delta = n1 - n2/n1$

Relation NA and Δ:

$NA = n1\sqrt{2}\,\Delta$

Numerical Aperture (NA): NA is the light gathering ability or capacity of an optical fiber. More the NA. the more efficient will be fiber. It is also known as figure of merit.

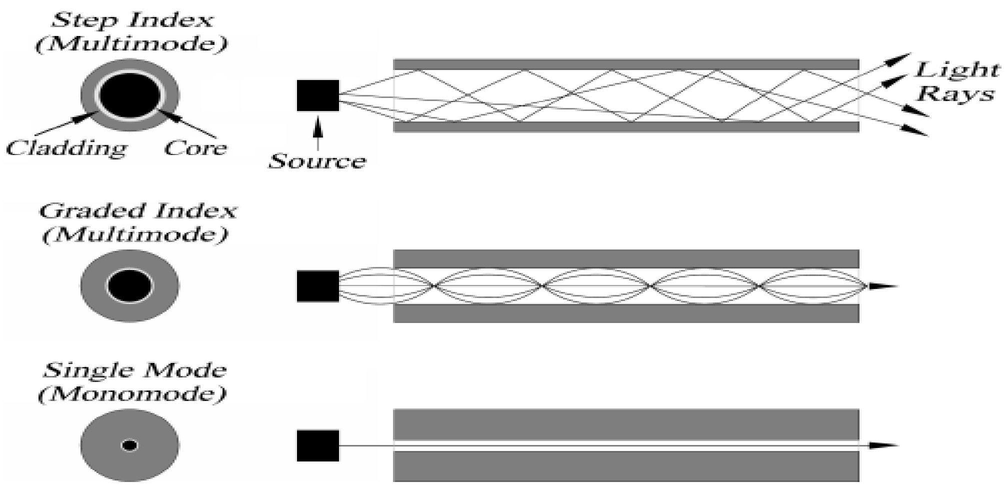
NA is related to refractive index of core (n1), cladding (n2) and outside medium (n0) as

NA = √n12 – n22/n

If the medium is air then n=1, then

NA = √n1^2– n2^2

Types of optical fibre



Step-Index Multimode Fiber Working Principles and Applications

In optical fibers, a step-index fiber is a fiber where a uniform refractive index exists within the core and a sharply decreased refractive index exists in the core-cladding interface because of the lower refractive index in cladding. For step-index multimode fiber, the light propagates in the shape of a zigzag along the fiber/core axis according to the principle of total reflection. Light entering the fiber at different angles of incidence will go through different paths. Although the incident lights propagate at the same speed simultaneously at the input, the time to reach the output of the fiber is different, resulting in a temporal dispersion called modal dispersion.

Since digital communications use light pulses to transmit signals down the length of the fiber, the modal dispersion causes the pulse to widen severely and spread out when they travel along with the fiber. The more modes the fiber transmits, the more pulses spread out. This significantly limits the bandwidth of step-index multimode fibers. The modal dispersion is not good for optical fiber communication as well. For digital optical fiber systems, when the dispersion is serious, it will cause pulses to overlap with each other, causing inter-symbol interference (ISI) and increasing the bit error rate (BER). Therefore, fiber dispersion not only affects the transmission capacity of the fiber, but also limits the relay distance of optical fiber communication systems. Because of that limitation, the step-index multimode fiber is normally used in short-distance (within a few kilometers) and low-speed (8 Mb/s or less) communication systems with a relatively lower cost. However, things differ in step-index single mode fibers. In a step-index single mode fiber, it can only transmit light of one mode. Therefore, the modal dispersion is very small, causing less impact on the transmission distance. Normally, single mode fibers are all step-index fibers.

Graded-Index Multimode Fiber Working Principles and Applications

Graded index multimode fiber is a type of optical fiber where the refractive index is higher at the axis of the core and then it decreases gradually towards the core-cladding interface. That is to say, the refractive index of a graded-index fiber gradually decreases from its center, and eventually decreases to the same value as the cladding at the core edge. The change in refractive index causes refraction rather than total internal reflection. When light passes through a layer with a lower refractive index, the light will fold back to the fiber axis. Total internal reflection does not occur because refraction folds the light back into the fiber axis before it reaches the cladding boundary.

For graded-index multimode fiber, the light travels forward in the form of sinusoidal oscillation. Like step-index multimode fibers, different lights in a graded-index multimode fiber travel along different paths. However, the speed of light propagation in graded-index multimode fibers is different because the speed of guided light varies with the refractive index of the fiber core. The farther the light goes from the center of the fiber, the faster its speed is. The speed difference compensates for the longer paths followed by the light rays that go farthest from the center of the fiber. This equalization of the transmission time of different modes reduces the mode dispersion greatly, making a higher bandwidth in graded-index fiber than step-index fiber. Therefore, most of the multimode fiber today is graded-index fiber. Compared to step-index fiber, the graded-index fiber is usually used in medium-distance (10~20 km) and relatively higher-speed (34~140 Mb/s) communication systems with higher cost.

Single mode fibre

Single-mode fiber is a common type of optical fiber that is used to transmit over longer distances. It is one of the two optical fiber types, the other being multi-mode fiber. A single-mode fiber is a single glass fiber strand used to transmit a single mode or ray of light.Single-mode fiber features only one transmission mode. Compared with multi-mode fiber, it can carry higher bandwidths; however, it needs to have a light source having a narrow spectral width.Single-mode fiber is also known as a single-mode optical fiber, uni-mode fiber, mono-mode optical fiber and single-mode optical waveguide.

Single-mode fiber is a common type of optical fiber that is used to transmit over longer distances. It is one of the two optical fiber types, the other being multi-mode fiber. A single-mode fiber is a single glass fiber strand used to transmit a single mode or ray of light.Single-mode fiber features only one transmission mode. Compared with multi-mode fiber, it can carry higher bandwidths; however, it needs to have a light source having a narrow spectral width.

Single-mode fiber is also known as a single-mode optical fiber, uni-mode fiber, mono-mode optical fiber and single-mode optical waveguide.Single-mode fiber provides users with a greater transmission rate in addition to nearly 50 times longer distance as opposed to multi-mode fiber. However, single-mode fiber is more expensive than multi-mode fiber. Among all the differences between single-mode and multi-mode fibers, the most basic is the size difference in the fibers' core as well as the associated loss or attenuation and fiber bandwidth.

The optical fiber itself includes three fundamental parts: the core, the cladding and the coating or buffer. The most central part of the optical fiber is the core. This is the place through which the light travels. The core of single-mode fiber is much smaller than that of multi-mode fiber. There are three fundamental diameter sizes for fiber cores.Single-mode fiber features a core diameter of nominally 9 µm, whereas multi-mode fiber often features a 62.5-µm or 50-µm core diameter. The small core, coupled with a single light wave, eradicates any distortion caused by overlapping light pulses, offering a minimal signal attenuation plus the highest transmission speed.In contrast, multi-mode fiber provides users with high bandwidth at high speeds across moderate distances.It is not advisable to mix and match fibers. If you try to connect a single-mode fiber with a multi-mode fiber, it may result in a 20-dB loss, which is 99% of the total power.

Application of optical fibre

Optical Fibres are basically thin strands made up of pure glass. The fundamental use of optical fibre is in the area of transmission of information in light form. The application of optical fibre is genuinely proven to be more beneficial in comparison to the conventional use of metallic wires.These cables comprise of a thousand of tiny optical fibres encapsulated in an insulated and protective enclosure. Optical fibre was introduced four decades back in the market. Since the time of their inception in the market, fibre optic cables have revolutionized the entire world of telecommunication.These cables use metallic wires. It has eliminated conventional modes of networking. A few of the highly popular applications of fibre optic cables have been discussed below.

Use of Optical Fibre in the Medical industry

Due to the highly flexible and thin nature of fibre optic cables, it is used in several types of instruments. Fibre optic wires are heavily used in the area of both medicines as well as research. It helps in getting an inside view of the body.With the help of optical fibre, a doctor can view the internal parts of the body. This is done by inserting these fibres into the empty areas in the body. Optical fibre serves as a laser during performing a wide range of medical treatments that include surgeries, biomedical research, and microscopy.Optical communication forms an essential element of any non-intrusive surgical method. It is mostly called as endoscopy. In these types of applications, a bright and tiny light is used for lighting up the area of surgery in the body. It makes it possible to lessen the size and number of cuts or slits made on the skin.

Use of optical fibre in Communication

Fibre optic cables have the ability to send bulk amounts of information at blistering speeds. This technology is largely used in the formation of internet cables. More and more communication industries are opting for optical fibres over traditional wires. This is for the reason that in contrast to copper wires, optic fibre wires are less heavy, carry more data and more flexible.One of the most important uses of optical fibre is that it is used as a “medium” for computer networking, and telecommunication. This is due to its flexibility and can easily be combined together as wires. Optical fibre is very beneficial for long-distance communications. This is because light spans through it at a significantly low attenuation when compared to an electrical wire.Optical fibre makes networking between devices in the same building or across close by structures faster and convenient. Rapid transmission of data is another important property of optical fibre because of which it is the top choice for communication industries. Users can notice a notable reduction in the amount of time taken for the transfer of information and files across networks.

In the area of communication system, optical fibre has been widely used for transmission and receiving of signals in various networking fields. The use of this communication method enhances the precision and speed of the transmitted data. In comparison to conventional copper-based wires, optical fibres are significantly lighter, sturdier, a lot more flexible and come with improved data-carrying abilities.Another significant advantage of optical fibre is that calling within or outside the nation has become a lot easier with it. The use of fibre optic communication helps you connect quickly and enjoy clear conversations without any sort of interruptions or time lag.The use of optical fibre cables has grown tremendously for sending and receiving signals over the last few years. Their greater speed and bandwidth makes them a perfect way to transmit signals for HDTV (high definition television set). Another important advantage of fibre optic wire is its cost-effectiveness in comparison to traditional copper wire.

Optical Fibres used in Defense Purpose

Defense is another significant area where optical fibres are extensively used. These cables are suitable for transmission of data in aerospace and military applications that require high data security. Optical fibres present a sturdy and robust communication medium that makes it useful in creating wirings in aeroplanes, hydrophones for Seismics and SONARs purposes.

Optical Fibres are used in Industries

There are several industries that have got benefitted with the installation of optical fibres. One of the areas where optical fibres are used extensively is “imaging”. It is most appropriate for places that are tough to reach. These fibres are used for lighting purposes and safety measures in designing the interior and exterior of automobiles. The optical fibre has an immense ability to send information at a high speed. This makes them useful in traction control and airbags. In most industries, flexibility, accuracy, and speed make it ideal for performing, testing and research objectives in industries.

Optical Fibres used for Broadcasting

Broadcasting is another area where optical fibre is widely used. These cables have the ability to broadcast television signals that are characterized by high speed and greater bandwidth. Companies that are into broadcasting of signals makes use of optical fibre for purposes that include video-on-demand, wiring CATV, HDTV, and several other applications.

Uses of Optical Fibre for Lightening and Decorations

Optical fibre is seen to be a lucrative, convenient and cost-efficient means to lighten the space. It is mostly used to decorate areas and illuminate objects such as the Christmas trees.The application of fibre optics has increased in the field of decoration and illumination. It has increased over the past years. These cables offer an easy, attractive and economical solution to various types of lighting projects.

Optical Fibres used in Mechanical Inspections

Another noteworthy advantage of Optical Fibre is in the area of mechanical inspection. Onsite review engineers widely use optical fibre to figure out the fault or damage that is usually difficult to locate by simply looking at the object. The plumbing industry is also not untouched by optical fibres. A lot of present-day plumbers and engineers use them for the evaluation of sewer, drain, and pipes.With the help of these cables, they can easily detect any crack, damage, leakage, etc in the plumbing system quickly and easily. It is an effective and economical way to detect damage signs as compared to costly specialized tools.

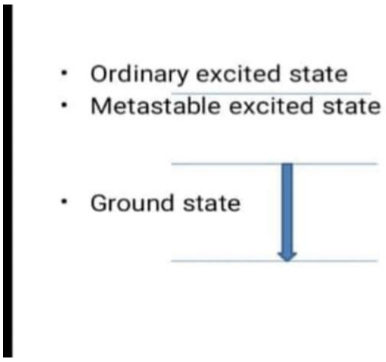
Module 5

LASERS

Concept of LASERS

the word laser stands for light amplification by stimulated emission of radiation. It is a device to produce a strong intense, monochromatic, collimated, uni-directional Kairali coherent beam of visible light.the most significant feature of light emitted by laser is the microscopic mechanism of emission in effect synchronises the emission of many atoms,so that the emitted light maintains its definite phase relationship for time interval much longer than corresponding to the emission of a single atom.

the principle involved in the laser is phenomenon of stimulated emission which was predicted by Einstein in 1917.the key to the laser is the presence in many atoms of one or more excited energy levels having lifetime of the order of 10^-3s or more instead of the usual 10^-8s (life time is defined as the time for which the atom can remain in the energy state,the lifetime for the ground energy state level is infinite). Search a relative Lee long-lived states are are called metastable States temporary stable.here is two wheeler today mission takes place between the metastable state and the lower energy state.



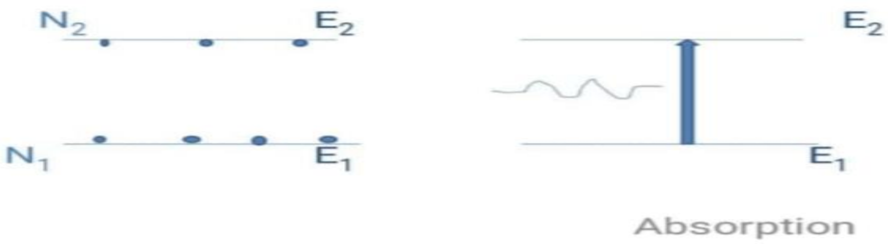
n 1960,Maiman built the first laser using Ruby as the active medium.since then laser action has been obtained in various material or solids including liquid, gases, ionized gases, dyes, semiconductor etc.

Interaction of radiation with matter- quantum mechanical view

to understanding of working principle of a laser requires an appreciation of quantum processes that takes place in a material when it is exposed to radiation.characterized by a set of discrete allowed energy levels.an atom can move from one energy level to another energy level when it receives or released an amount of energy equal to the energy difference between two energy levels. It is called a Quantum jump or transition.

: **Absorption or induced absorption**

When an atom in the ground state E1 absorb an incident photon its energy increases by an amount hν and it goes to the excited state E2. This process is called absorption.



A+ hν → A*

where A = ground state and A* excited state photon.

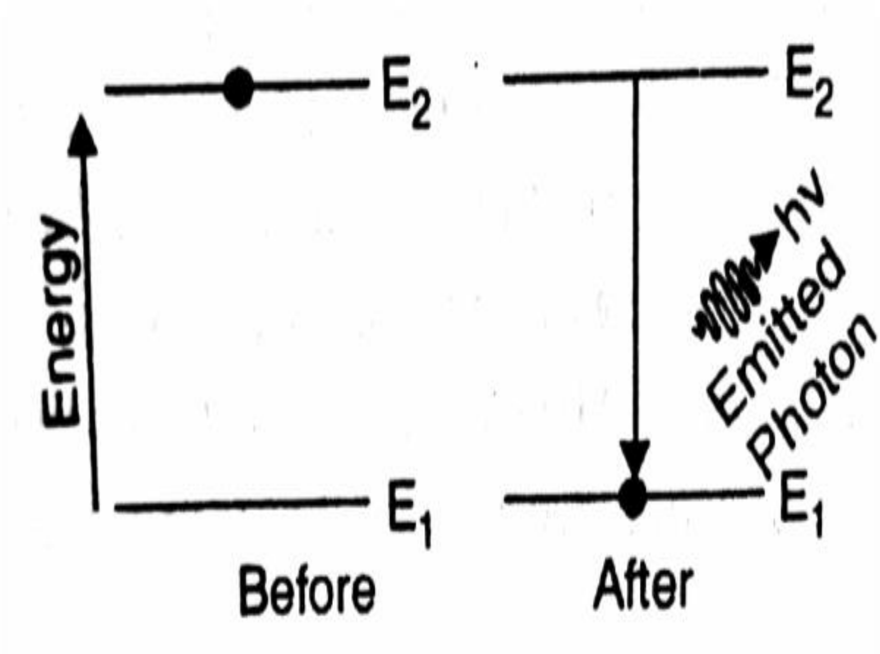
The number of absorption transition occurring in the material at any instant will be proportional to number of atoms present in state E1 and incident radiation density QNab α N1 Q Δt

Nab = B12 N1 Q Δt---(1)

Where B12 is probability of absorption transitions

Spontaneous emission:

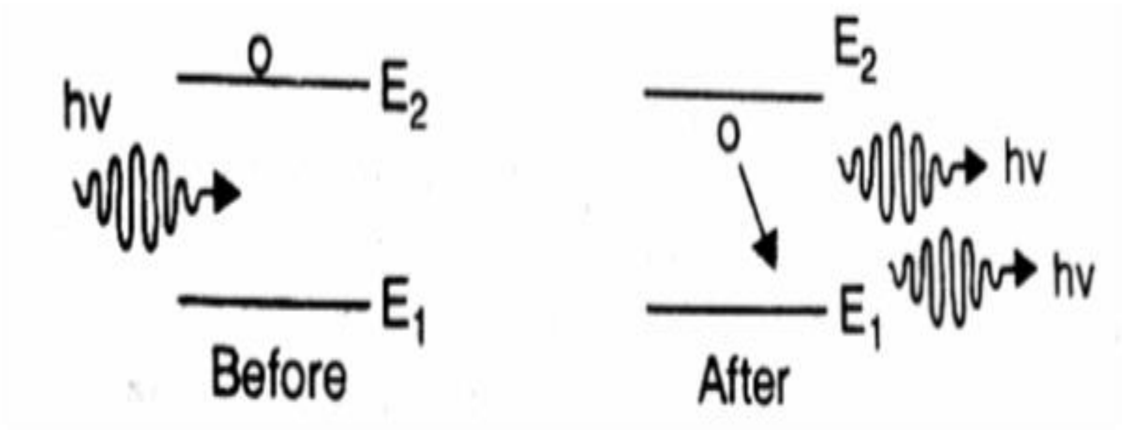
Normally the excited state is an unstable state where the life time of an atom is very short around 10−8 sec. Hence the atom in the excited state, E2 returns to the ground state spontaneously by releasing one photon of energy hν. This process is called spontaneous emission which can be represented by A*→ A + hν. The light emitted in this process is highly incoherent.



The number of spontaneous emissions at any instant will be proportional to the number of atoms in excited state Nsp α N2Δt Nsp = A21 N2Δt A21 is probability of spontaneous transitions

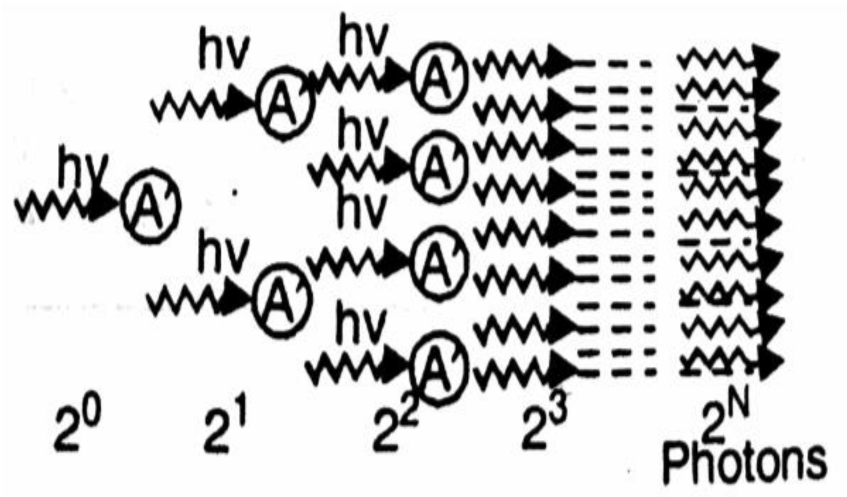
Stimulated emission:

In this process an incident photon is absorbed by an excited atom as a result of which the atom becomes unstable in the state E2 and makes a transition to the ground state releasing two photons. This process is called stimulated emission which can be written as A* + hν → A + 2hν



The number of stimulated transitions will be proportional to number of atoms in the excited state and radiation density Nst α N2 QΔt Nst = B21 N2QΔt where B21 is probability of stimulated transitions. The interesting feature of this process is that emitted photon is identical to incident photon in all respect i.e frequency, direction polarization and multiplication of photons take place in this process. One photon induces an atom to emit two photons, these two photons deexcite two more atoms thereby emitting four photons and so on, the number of photons builds up in an avalanche like manner.

The combined effect of all emission from a large sample of atoms leads to amplification of light. The constructive interference of many waves traveling in same direction with same phase produce an intense coherent beam. This is key for the operation of laser.



Relation among Einstein's Coefficient

Let us consider an assembly of atoms in thermal equilibrium at temperature T with radiation of frequency υ. Let us consider an atom having two energy states E_1 & E_2 and let N_1 & N_2 be the number of atoms per unit volume in the energy states 1 and 2 respectively. E_1 be the lower energy state and E_2 the higher energy state.

Let $u(\upsilon)$ represent the energy density of radiation i.e. $u(\upsilon)d\upsilon$ represents the energy density in the frequency interval υ & $\upsilon + d\upsilon$. An atom in the lower energy state E_1 can absorb radiation and get excited to the level E_2. This excitation process can occur only in the presence of radiation. Such a process is called absorption. The rate of absorption of radiation of frequency υ ($h\upsilon = E_2 - E_1$) which lead to transition 1 to 2 is proportional to N_1 & $u(\upsilon)$.

Thus the number of absorption per unit volume per unit time = $B_{12} N_1 u(\upsilon)$.

Where B_{12} is a proportional constant called Einstein's Coefficient of absorption of radiation.

Let us consider the reverse process i.e. the emission of radiation at a frequency υ when the atom de-excites from the level E_2 to E_1. Einstein postulated that an atom in an excited state can make radiative transition to a lower energy state either through spontaneous emission or through stimulated emission. In spontaneous emission, the probability per unit time of the atom making a downward transition is independent of the energy density of radiation field and depends on the number of atoms in the excited state E_2.

Therefore, number of spontaneous emission per unit volume per unit time = $A_{21}N_2$ where A_{21} is a proportional constant called Einstein's Coefficient of spontaneous emission.

In the case of stimulated emission, the number of stimulated emission is proportional to N_2 & C). Therefore, no of stimulated emission per unit volume per unit time = $B_{21} N_2 u(\upsilon)$, where B_{21} is a proportional constant called Einstein's Coefficient of stimulated emission.

At thermal equilibrium,

Total downward transition = Total upward transition

$$B_{12} N_1 u(\upsilon) = A_{21}N_2 + B_{21} N_2 u(\upsilon)$$

$$u(\upsilon) [B_{12} N_1 - B_{21} N_2] = A_{21}N_2$$

$$u(\upsilon) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$u(\upsilon) = \frac{A_{21}}{B_{12}\frac{N_1}{N_2} - B_{21}} \qquad (1)$$

From Boltzmann's law at thermal equilibrium, N_1 & N_2 are related as

$$\frac{N_2}{N_1} = \exp[-\frac{E_2 - E_1}{k_B\ T}] = \exp\ [-\frac{h\upsilon}{k_B T}] \qquad (2)$$

where k_B is the Boltzmann's constant and T represents the temperature of the system. Substituting equ.(2) in equ.(1) , we have

$$u(\upsilon) = \frac{A_{21}}{B_{12}\exp\left[\frac{h\upsilon}{k_B T}\right] - B_{21}}$$

$$u(\upsilon) = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}}{B_{21}}\exp\left[\frac{h\upsilon}{k_B T}\right] - 1} \qquad (3)$$

On the other hand, from Planck's law

$$u(\upsilon) = \frac{8\pi h\upsilon^3/c^3}{\exp\left[\frac{h\upsilon}{k_B T}\right] - 1} \qquad (4)$$

Comparing equations (3) & (4), we have

$$\frac{B_{12}}{B_{21}} = 1$$

i.e $\qquad B_{12} = B_{21} \qquad (5)$

and $\qquad \frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3} \qquad (6)$

Equation (5) & (6) are known as Einstein relations. And show that the Einstein's coefficients are related to each other. It is clear that if stimulated emission had not been assumed, it would not be possible to arrive at an expression for u(υ) which is similar to Planck's law. In order to obtain the correct form for u(υ), Einstein predicted the existence of stimulated emission, which was later confirmed by rigorous quantum theory.

From equ(5), it is clear that the stimulated emission rate per atom is the same as the absorption rate per atom. From equ,(6), it is clear that the ratio between spontaneous emission coefficient and the stimulated emission coefficient is proportional to υ^3. It means that at thermal equilibrium, the probability of spontaneous emission increases rapidly with the energy difference between states.

Condition For Light Amplification

At thermal equilibrium, the ratio of the stimulated to spontaneous transitions is generally very small and the stimulated emission is negligible. The ratio is given by

$$\frac{stimulated\ transition}{spontaneous\ transition} = \frac{B_{21}N_2\,\mathrm{u}(\upsilon)}{A_{21}N_2} = \frac{B_{21}}{A_{21}}\mathrm{u}(\upsilon) \qquad (1)$$

$$\frac{stimulated\ transition}{absorption} = \frac{B_{21}N_2\,\mathrm{u}(\upsilon)}{B_{12}N_1 u(\upsilon)} = \frac{N_2}{N_1} \qquad (2)$$

Because, $B_{12} = B_{21}$ i.e. probability of stimulated emission = probability of absorption (3)

(i) It is clear that the equ.(i) suggests that in order to enhance the number of stimulated transition, the radiation density u(ʋ) is to be made large.This is achieved by enclosing the radiation in an enclosure (resonator) and preventing the radiation from escaping from it. The radiation from the excited atoms in resonator adds up in phase, thereby increasing u(ʋ).

(ii) The equ(2) indicates that stimulated emission will be larger than absorption only when $N_2 > N_1$. As long as $N_1 > N_2$ absorption dominates stimulated emission and the medium will absorb the incident light rather than amplify it. Thus only when the above conditions are fulfilled, the medium amplifies light passing through it.

Population Inversion/ Inverted Population

Let us suppose that a sample containing a large number of atoms in thermal equilibrium at temperature T. Before any radiation is directed at the sample, let N_1 be the number of atoms in the lower energy state E_1 and N_2 be the number of atoms in the upper energy state E_2, then N_2 and N_1 are related as (given by Bolzmann)

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B\,T}\right) \qquad (3)$$

Where k_B is a Bolzmann constant, k_BT is the mean kinetic energy of an atom at temperature T. The higher the temperature, the more atoms will have been bumped up by thermal agitation (by atom-atom collisions) to the higher energy state E_2.

Since, $E_2 - E_1 = +ve$, $N_2 < N_1$ (figure)

Normally more atoms are in lower energy state. Now if the atoms are flooded with photons of energy $E_2 - E_1 = h\upsilon$, photons will disappear via absorption by lower energy state atoms and photons will be generated via stimulated emission of excited state atoms. As Einstein showed that $B_{12} = B_{21}$ and here $N_1 > N_2$, therefore from equ(2) the net effect will be absorption of photons.

To produce laser light, we must have more photons emitted than absorbed i.e. there must be a situation in which stimulated emission dominates. So, N_2 should greater than N_1. Therefore, a non-equilibrium state is to be produced in which the population of the upper energy level exceeds to a large extent the population of the lower energy state. When this situation occurs, the population distribution between the levels E_1 & E_2 is said to be inverted and the medium is said to have gone into state of population inversion. **Thus population**

inversion is a state in which the number of atoms in the higher energy state is greater than that in the lower energy state. It is clear from equ(3) that N_2 can exceed N_1 only if temperature were negative. Therefore**, the state of population inversion is sometime referred to as a negative temperature state.**

Active Medium/ System

A system in which population inversion is to be created is called an active medium. Light gets amplified in this medium. The medium may be solid, liquid or gas. Out of the different atoms in the medium, only a small fraction of atoms of a particular species are responsible for stimulated emission and consequent light amplification. They are called active centres. The remaining bulk of the medium plays the role of host and supports active centres.

Pumping

For realising and maintaining the condition of population inversion, atoms have to be raised continuously from lower energy state to the excited state. **The method of raising molecules or atoms from lower energy state to the higher energy state is called pumping**. In another word it can be defined as “the process of supplying energy to the medium with a view to transfer it into the state of population inversion is known as pumping.

Types of Pumping

(i) **Optical Pumping:** In this process, the predominant population of the upper level is achieved by means of light energy delivered from approximately selected sources such as gaseous discharged flash tubes, or continuously burning tubes.

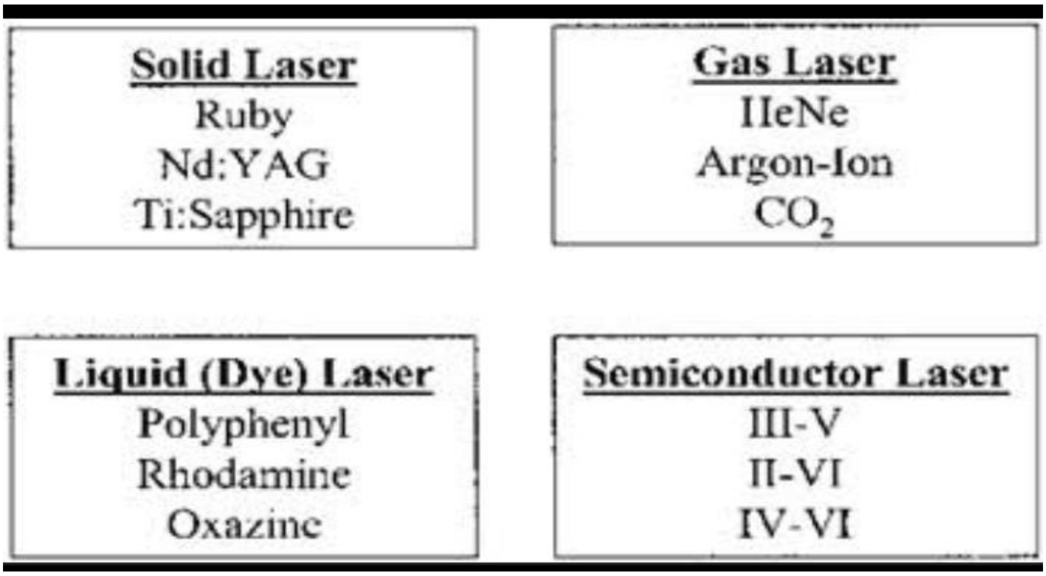
(ii) **Electrical Pumping:** This is accomplished by means of a sufficiently intense electrical discharge in the medium and is particularly suited to gas media. The discharge converts the gas into plasma where active centres collide inelastically with free electrons and cause the predominant populations of the upper pumping level. Atoms and molecules purposefully introduced into the gas are also of importance for pumping as they provide resonance energy exchange.

(iii) **Chemical Pumping:** This raises active centres into the higher level by means of suitable exothermal chemical reactions in the active material.

(iv) **Heat Pumping:** In this scheme of pumping, the active material at first brought to a high temperature and then rapidly cooled down.

Types of laser

Solid laser ruby laser



Solid laser-Ruby laser

Ruby laser is a solid-state laser that was developed by Maiman in 1960 using Ruby as an active medium. Ruby is a crystal of Aluminium oxide. In which a part of the aluminum ion is substituted by chromium ion.The active material in the Ruby is chromium ion. That is the energy level of chromium Ion takes part in the lasing action.

A ruby laser consists of three main parts:

(1) An active material (or laser medium).

(2) A present and system made of two parallel plates with a reflecting coating applied on them.

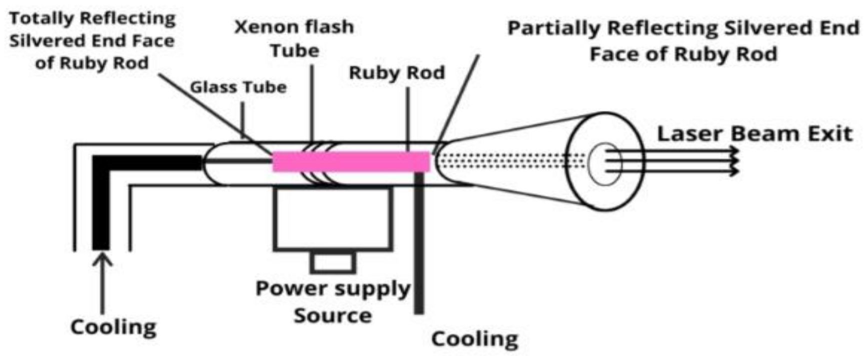
(3) An exciting system usually made up of helical xenon. Flash tubes for achieving population inversion and a power supply source.

Ruby Laser Construction and Working

Construction of Ruby Laser

Ruby is a crystal of Aluminium oxide. In which a part of the aluminium ion is substituted by chromium ion. The active material in the Ruby is chromium ion. That is the energy level of chromium Ion takes part in the lasing action. The colour (pinkor red) of a Ruby crystal depends upon the amount of chromium in it.A crystal of Aluminium oxide is specially grown with about 0.05% of the aluminium atom replace by chromium. Ruby crystals are grown in special furnaces, then annealed and shaped into rods.Such Rods are 2 to 30 cm in length

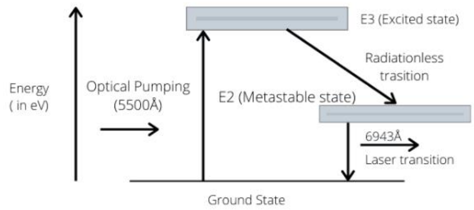
and 0.5 to 2 cm in diameter. Flat end faces of the rod are made strictly parallel, ground and polished to a high degree of precision.



Sometimes reflecting coating is applied not on special plates. But directly on the end faces of the Ruby Rod. The end faces of the rod are silver so that the Surface of the one end face becomes fully reflecting.And that of the other end face becomes partially reflecting. Usually the light transmission coefficient of the partially reflecting end face of the ruby rod is about 10 to 25 per cent. Thus , Ruby Rod with its one end only reflecting and the other end partially reflecting acts as a resonant cavity.

Working of Ruby Laser

The Ruby rod is arranged along the axis of a helical xenon flash tube. In such a manner that the will of the helix encloses the rod. The flash of the tube lasts several milliseconds.During this period of time, the tube absorbs energy amounting to Several thousand joules and most of the energy is spent on heating the Apparatus.This heat produced is removed by liquid nitrogen circulating around the Ruby rod. The remaining part of the energy in the form of blue and green radiation is absorbed by the Ruby. This energy ensures the excitation of chromium Ion from the ground state to the excited energy state for achieving population inversion.



The optical pumping result when incident photons of wavelength 5500Å. Raise the chromium Ion from ground state E1 to higher excited state E3. These iron interact with the crystal lattice and decay to metastable state E2 by spontaneous emission where they can stay for a longer period (3 x 10–3sec) Producing population inversion.That is, the number of chromium ions in the energy state E2 (metastable state) is more than the number of chromium ions in excited energy state E3. Hence, population inversion is achieve by optical pumping source.

Production of a Laser Beam

The some of chromium Ion jumps from the metastable state E2 to the ground state E1 through spontaneous emission of radiation. During this transition, a photon of wavelength 6943Å is emitted.This Photon Travels through the Ruby rod in a direction parallel to the axis of the rod and reflects back and forth (or to and fro) by the reflecting ends of the optical resonator until it Stimulates an excited chromium ion. Stimulated excited chromium Ion emits a photon exactly in phase with the stimulating photon.The Stimulated transition of chromium Ion from metastable state E2 to the ground state E1 is known as laser transition. These in-phase photons stimulate more chromium ions and hence the number of photons emitted increases.This process repeats and hence the in-phase photos get multiplied. Thus, a strong and coherent laser beam is obtained. Which comes out of the optical resonator through the partially reflecting face.

Output of Ruby Laser

In Ruby laser energy for excitation for population, inversion is supplied in the form of short flasher or pulse of light. So ruby laser has a pulse output and hence called pulse laser.

Spiking in Ruby Laser

In a ruby laser, an optical pumping source provides energy in the form of a basis of life of a few nanosecond duration. When a crystal or rod absorbs the flash or pulse of light, population inversion takes place.Ruby laser determinate at the ground state. By emitting laser radiation (or photo) of wavelength 6943Å and hence metastable state disappears.

Therefore lasing action stops till another flash of light is available after a few nanoseconds for population inversion.Thus, the output of a ruby laser is in the form of energetic pulses of very small duration (of the order of nanosecond). Before the output power attains steady-state value through damped relaxation oscillations. These energetic pulses of very short duration are spikes and the phenomena of generating these pulses are spiking.The output power of the ruby laser is not continuous. But it is in the form of pulses of short duration. In other words, the ruby laser operates in pulse mode and hence called pulsed laser. It is all about the Ruby laser construction and working. Now we have to study the advantages and disadvantages.

Advantages of Ruby Laser

It has large power output,The pumping efficiencies can be increased by using cylindrical Mirrors,It has a narrow linewidth.

Disadvantages of Ruby laser

The output laser beam is not continuous but the light is emitted in pulses.

The Monochromaticity may be affected by crystalline Imperfection. Thermal distortion and scattering.

Frequent cooling is required as a lot of heat energy is produced during its operation.

A large amount of energy is required to Trigger laser oscillations.

Applications of Ruby Laser

It is used as a high power source of Pulse coherent radiation in interferometry and in pulsed holography.

It is used for drilling brittle material, soldering, welding, and in-range finding. The Ruby rod is arranged along the axis of a helical xenon flash tube. In such a manner that the will of the helix encloses the rod. The flash of the tube lasts several milliseconds. During this period of time, the tube absorbs energy amounting to Several thousand joules and most of the energy is spent on heating the Apparatus.

A gas laser is a type of laser in which a mixture of gas is used as the active medium or laser medium. Gas lasers are the most widely used lasers.Gas lasers range from the low power helium-neon lasers to the very high power carbon dioxide lasers. The helium-neon lasers are most commonly used in college laboratories whereas the carbon dioxide lasers are used in industrial applications.The main advantage of gas lasers (eg: He-Ne lasers) over solid state lasers is that they are less prone to damage by overheating so they can be run continuously.

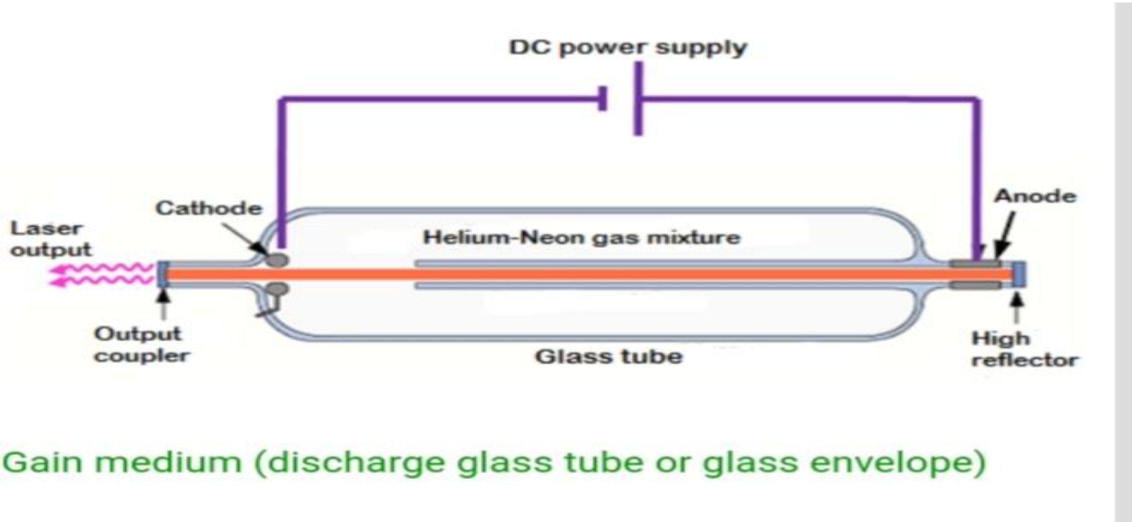
Helium-Neon laser(Gas laser)

Helium-Neon laser is a type of gas laser in which a mixture of helium and neon gas is used as a gain medium. Helium-Neon laser is also known as He-Ne laser.At room temperature, a ruby laser will only emit short bursts of laser light, each laser pulse occurring after a flash of the pumping light. It would be better to have a laser that emits light continuously. Such a laser is called a continuous wave (CW) laser.The helium-neon laser was the first continuous wave (CW) laser ever constructed. It was built in 1961 by Ali Javan, Bennett, and Herriott at Bell Telephone Laboratories.Helium-neon lasers are the most widely used gas lasers. These lasers have many industrial and scientific uses and are often used in laboratory demonstrations of optics.In He-Ne lasers, the optical pumping method is not used instead an electrical pumping method is used. The excitation of electrons in the He-Ne gas active medium is achieved by passing an electric current through the gas.The helium-neon laser operates at a wavelength of 632.8 nanometers (nm), in the red portion of the visible spectrum.

Helium-neon laser construction

The helium-neon laser consists of three essential components:

Pump source (high voltage power supply),Gain medium (laser glass tube or discharge glass tube),Resonating cavity,High voltage power supply or pump source.



In order to produce the laser beam, it is essential to achieve population inversion. Population inversion is the process of achieving more electrons in the higher energy state as compared to the lower energy state.In general, the lower energy state has more electrons than the higher energy state. However, after achieving population inversion,

more electrons will remain in the higher energy state than the lower energy state.In order to achieve population inversion, we need to supply energy to the gain medium or active medium. Different types of energy sources are used to supply energy to the gain medium.In ruby lasers and Nd:YAG lasers, the light energy sources such as flashtubes or laser diodes are used as the pump source. However, in helium-neon lasers, light energy is not used as the pump source. In helium-neon lasers, a high voltage DC power supply is used as the pump source. A high voltage DC supplies electric current through the gas mixture of helium and neon.In helium-neon lasers, light energy is not used as the pump source.

Gain medium (discharge glass tube or glass envelope)The gain medium of a helium-neon laser is made up of the mixture of helium and neon gas contained in a glass tube at low pressure. The partial pressure of helium is 1 mbar whereas that of neon is 0.1 mbar.The gas mixture is mostly comprised of helium gas. Therefore, in order to achieve population inversion, we need to excite primarily the lower energy state electrons of the helium atoms.

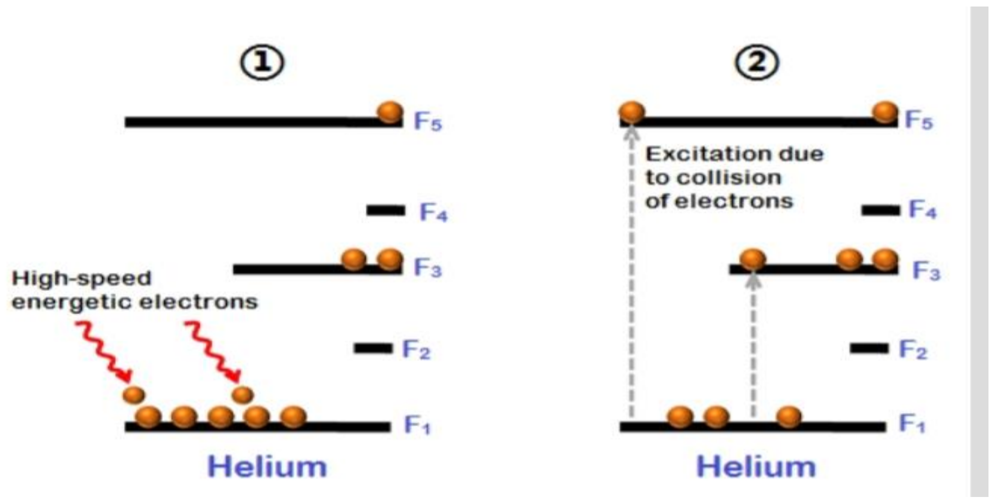
In He-Ne laser, neon atoms are the active centers and have energy levels suitable for laser transitions while helium atoms help in exciting neon atoms.Electrodes (anode and cathode) are provided in the glass tube to send the electric current through the gas mixture. These electrodes are connected to a DC power supply.

Resonating cavity

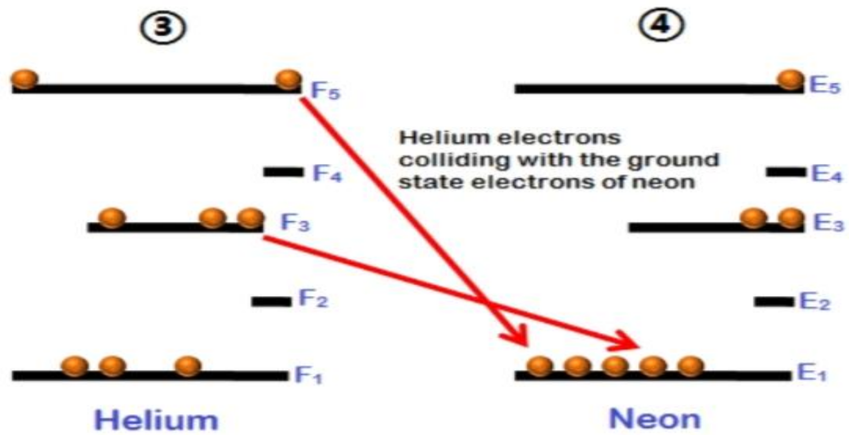
The glass tube (containing a mixture of helium and neon gas) is placed between two parallel mirrors. These two mirrors are silvered or optically coated.Each mirror is silvered differently. The left side mirror is partially silvered and is known as output coupler whereas the right side mirror is fully silvered and is known as the high reflector or fully reflecting mirror.The fully silvered mirror will completely reflect the light whereas the partially silvered mirror will reflect most part of the light but allows some part of the light to produce the laser beam.

Working of helium-neon laser

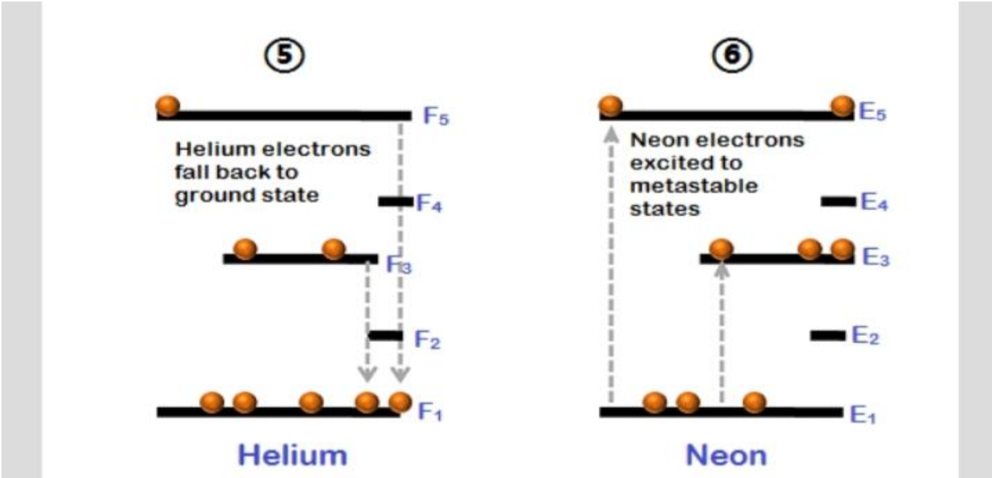
In order to achieve population inversion, we need to supply energy to the gain medium. In helium-neon lasers, we use high voltage DC as the pump source. A high voltage DC produces energetic electrons that travel through the gas mixture.



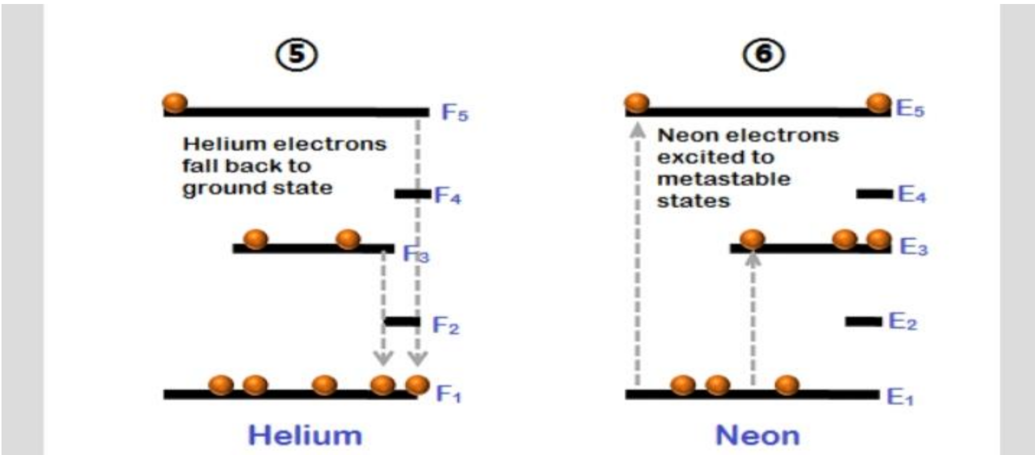
The gas mixture in helium-neon laser is mostly comprised of helium atoms. Therefore, helium atoms observe most of the energy supplied by the high voltage DC.When the power is switched on, a high voltage of about 10 kV is applied across the gas mixture. This power is enough to excite the electrons in the gas mixture. The electrons produced in the process of discharge are accelerated between the electrodes (cathode and anode) through the gas mixture.In the process of flowing through the gas, the energetic electrons transfer some of their energy to the helium atoms in the gas. As a result, the lower energy state electrons of the helium atoms gain enough energy and jumps into the excited states or metastable states. Let us assume that these metastable states are F3 and F5.



In the process of flowing through the gas, the energetic electrons transfer some of their energy to the helium atoms in the gas. The metastable state electrons of the helium atoms cannot return to ground state by spontaneous emission. However, they can return to ground state by transferring their energy to the lower energy state electrons of the neon atoms.

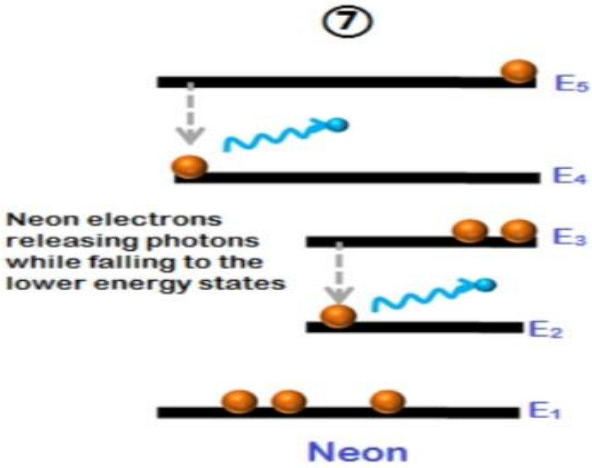


The energy levels of some of the excited states of the neon atoms are identical to the energy levels of metastable states of the helium atoms. Let us assume that these identical energy states are F3 = E3 and F5 = E5. E3 and E5 are excited states or metastable states of neon atoms.When the excited electrons of the helium atoms collide with the lower energy state electrons of the neon atoms, they transfer their energy to the neon atoms.



Unlike the solid, a gas can move or flow between the electrodes. Hence, when the excited electrons of the helium atoms collide with the lower energy state electrons of the neon atoms, they transfer their energy to the neon atoms. As a result, the lower energy state electrons of the neon atoms gain enough energy from the helium atoms and jumps into the higher energy states or metastable states (E3 and E5) whereas the excited electrons of the helium atoms will fall into the ground state. Thus, helium atoms help neon atoms in achieving population inversion.Likewise, millions of ground state electrons of neon atoms are excited to the metastable states. Likewise, millions of ground state electrons of neon atoms are excited to the metastable states. The metastable states have

the longer lifetime. Therefore, a large number of electrons will remain in the metastable states and hence population inversion is achieved.



After some period, the metastable states electrons (E3 and E5) of the neon atoms will spontaneously fall into the next lower energy states (E2 and E4) by releasing photons or red light. This is called spontaneous emission.The neon excited electrons continue on to the ground state through radiative and nonradiative transitions. It is important for the continuous wave (CW) operation.After some period, the metastable states (E3 and E5) electrons of the neon atoms will spontaneously fall into the next lower energy states (E2 and E4) by releasing photons or red lightThe light or photons emitted from the neon atoms will moves back and forth between two mirrors until it stimulates other excited electrons of the neon atoms and causes them to emit light. Thus, optical gain is achieved. This process of photon emission is called stimulated emission of radiation.The light or photons emitted due to stimulated emission will escape through the partially reflecting mirror or output coupler to produce laser light.

Advantages of helium-neon laser
Helium-neon laser emits laser light in the visible portion of the spectrum,High stability,Low cost,Operates without damage at higher temperatures
Disadvantages of helium-neon laser
Low efficiency,Low gain,Helium-neon lasers are limited to low power tasks
Applications of helium-neon lasers
Helium-neon lasers are used in industries.
Helium-neon lasers are used in scientific instruments.
Helium-neon lasers are used in the college laboratories.

Application of LASER

Lasers in Medicine

Lasers are used for bloodless surgery.
Lasers are used to destroy kidney stones.
Lasers are used in cancer diagnosis and therapy.
Lasers are used for eye lens curvature corrections.
Lasers are used in fiber-optic endoscope to detect ulcers in the intestines.
The liver and lung diseases could be treated by using lasers.
Lasers are used to study the internal structure of microorganisms and cells.
Lasers are used to produce chemical reactions.
Lasers are used to create plasma.
Lasers are used to remove tumors successfully.
Lasers are used to remove the caries or decayed portion of the teeth.
Lasers are used in cosmetic treatments such as acne treatment, cellulite and hair removal.

Lasers in Communications

Laser light is used in optical fiber communications to send information over large distances with low loss.
Laser light is used in underwater communication networks.
Lasers are used in space communication, radars and satellites.

Lasers in Industries

Lasers are used to cut glass and quartz.
Lasers are used in electronic industries for trimming the components of Integrated Circuits (ICs).
Lasers are used for heat treatment in the automotive industry.
Laser light is used to collect the information about the prefixed prices of various products in shops and business establishments from the bar code printed on the product.
Ultraviolet lasers are used in the semiconductor industries for photolithography. Photolithography is the method used for manufacturing printed circuit board (PCB) and microprocessor by using ultraviolet light.
Lasers are used to drill aerosol nozzles and control orifices within the required precision.

Lasers in Science and Technology

A laser helps in studying the Brownian motion of particles.
With the help of a helium-neon laser, it was proved that the velocity of light is same in all directions.

With the help of a laser, it is possible to count the number of atoms in a substance.
Lasers are used in computers to retrieve stored information from a Compact Disc (CD).
Lasers are used to store large amount of information or data in CD-ROM.
Lasers are used to measure the pollutant gases and other contaminants of the atmosphere.
Lasers helps in determining the rate of rotation of the earth accurately.
Lasers are used in computer printers.
Lasers are used for producing three-dimensional pictures in space without the use of lens.
Lasers are used for detecting earthquakes and underwater nuclear blasts.
A gallium arsenide diode laser can be used to setup an invisible fence to protect an area.

Lasers in Military

Laser range finders are used to determine the distance to an object.
The ring laser gyroscope is used for sensing and measuring very small angle of rotation of the moving objects.
Lasers can be used as a secretive illuminators for reconnaissance during night with high precision.
Lasers are used to dispose the energy of a warhead by damaging the missile.
Laser light is used in LIDAR's to accurately measure the distance to an object.