

* Analysis of trusses:

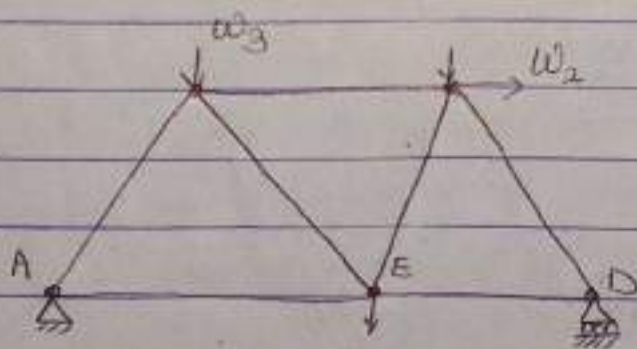
Concept

① **Truss**: It is a structure in which all members are subjected to either tension or compression in them B.M is zero everywhere in the truss.

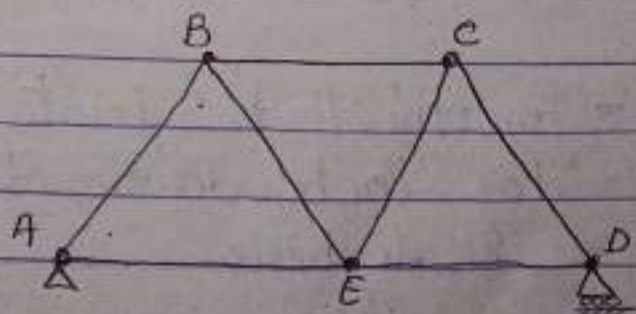
② **Assumptions (Precautions) in the analysis of trusses**:-

(a) All the load must be applied only of the joints (otherwise, if the loads are applied at the intermediate towards of the members then they will bend & we can not call them as trusses. They are called as frames.)

i.e.,

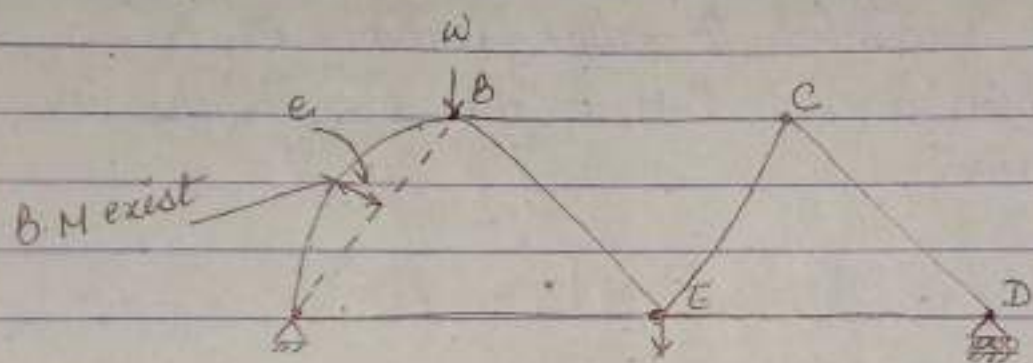


Truss



Frame (Members BC bends)

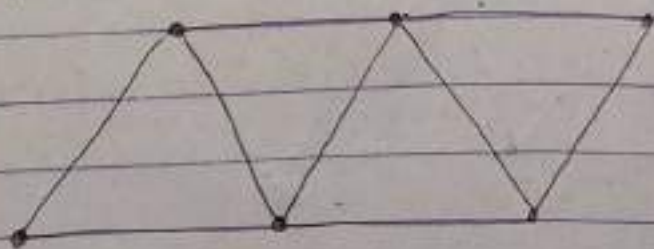
- (b) All the members must be straight & connected by smooth pins, otherwise if the member are curved, ^{then} they will bend & we cannot call that structure as a truss.



Not a truss (it is a frame)

3. In a truss, the total number of members (m) & the total number of joints (j) are related by

$$m = 2j - 3$$



- For the first 3 joints we require 3 members
- For each additional joints we require 2 members

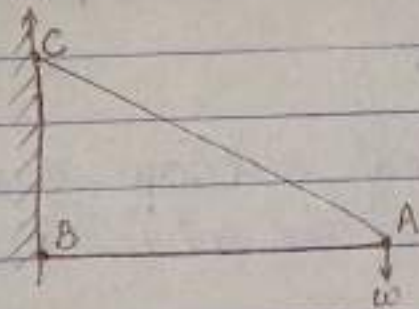
So,

$$m = 2j - 3$$

If the above condition is satisfied, then we get a stable triangulated, Determinate truss.

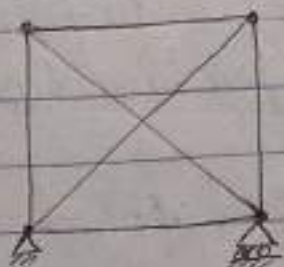
4. If $m = 2j - 3 \Rightarrow$ Perfect Stable truss
 $m < 2j - 3 \Rightarrow$ Unstable or deficient truss
 $m > 2j - 3 \Rightarrow$ Redundant or Indeterminate truss (Internally)

Ques 1: For the truss shown in fig. B.M exists in the members.



- (A) AB (B) AC (C) Both AB & AC (D) None

Ques 2: The truss shown in fig:



- (A) Perfect
(B) Deficient
(C) Redundant
(D) None

Sol:- $m = 6$, $J = 4$

$$m = 2J - 3$$

$$6 = 2 \times 4 - 3$$

$$6 > 5$$

⑤. Method of Analysis of trusses :-

- a) Method of joints : It is a special cases of MTD of sections only.
- b) Method of sections

a) Method of joints :-

Concepts:

- 1. Equilibrium of a joint is considered in MTD of joints.

2 Procedure :

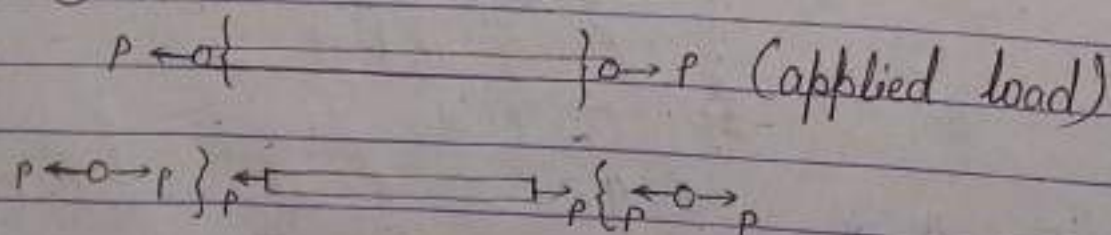
→ Step 1:

Find the support reactions, considering equilibrium of the entire truss.

→ Step 2:

Consider equilibrium of a joints where only two unknown forces are available and use $\sum x = 0$ & $\sum y = 0$ to find them, and proceed to other joints also,

Note ①

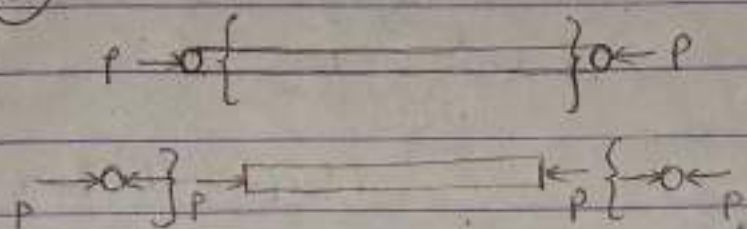


Conclusions :-

- If the arrow mark is away from the joint, then

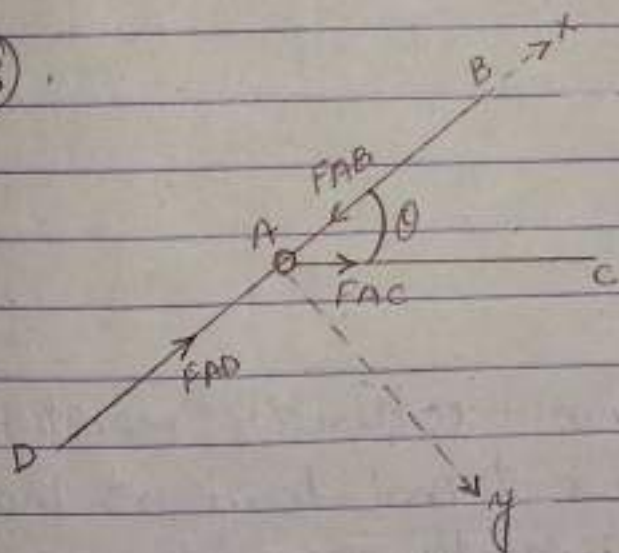
it means that force in the members is tensile.

Note (2)



→ If the arrow marks is towards the joints it means that force in the member is compressive.

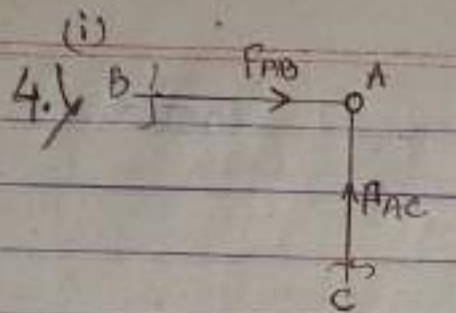
(3)



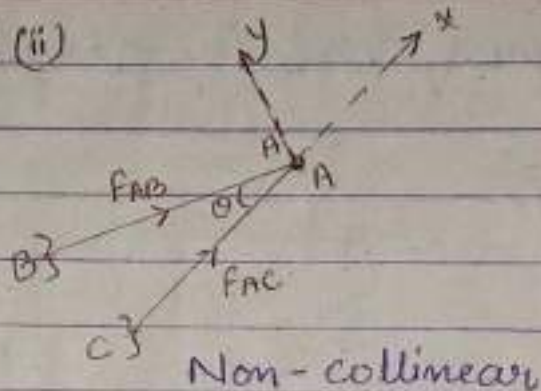
$$\begin{aligned}\sum y = 0 &\Rightarrow F_{AC} \sin \theta = 0 \\ \sin \theta &\neq 0 \\ F_{AC} &= 0\end{aligned}$$

Conclusions:-

At a joints if 3-members are meeting & two two members are collinear { Same line } then force in third member is always zero. { If no external load at the joint }.



Non-collinear

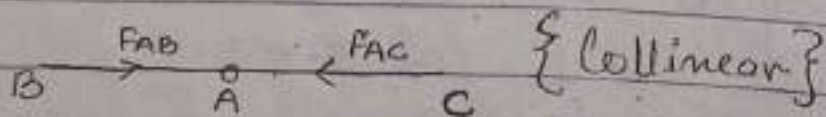


i) $\sum x = 0 \Rightarrow +F_{AB} = 0$
 $\sum y = 0 \Rightarrow +F_{AC} = 0$

ii) $\sum y = 0 \Rightarrow +F_{AC} \cdot \sin \theta = 0$
 $F_{AC} = 0$
 $\sum x = 0 \Rightarrow +F_{AB} = 0$

Conclusion :-

At a joint, if two non-collinear members are meeting with no external load at that joint, ~~force~~ forces in both members are zero.

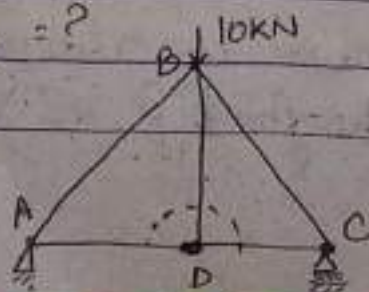


$$\sum x = 0 \Rightarrow F_{AB} - F_{AC} = 0$$

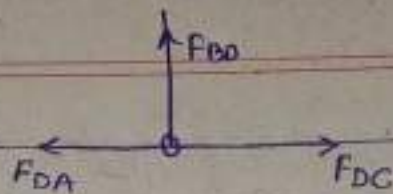
$$\Rightarrow F_{AB} = F_{AC}$$

$$F_{AB} \neq 0, F_{AC} \neq 0$$

Ques 1 :- FBD = ?



Solⁿ :-

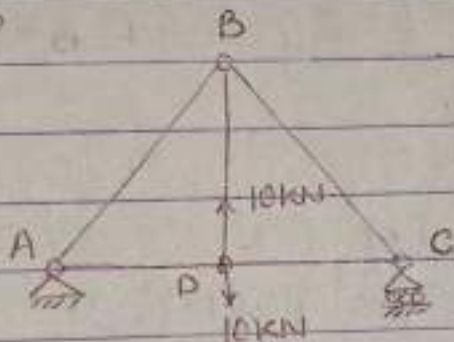


FBD of joint 'D'

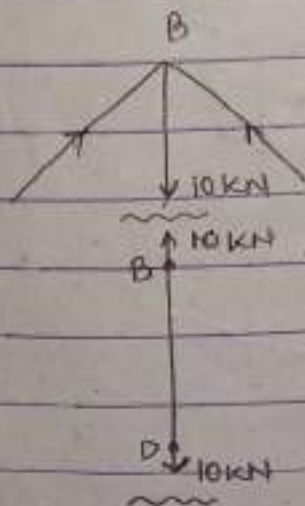
$$\sum F_y = 0 \Rightarrow +F_{BD} = 0$$

Ques :- (2)

$F_{BD} = ?$

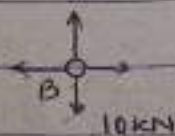


$F_{BD} = 10 \text{ kN (T)}$ { bcz arrow line is away from joints }
or it can also set from F.B.D



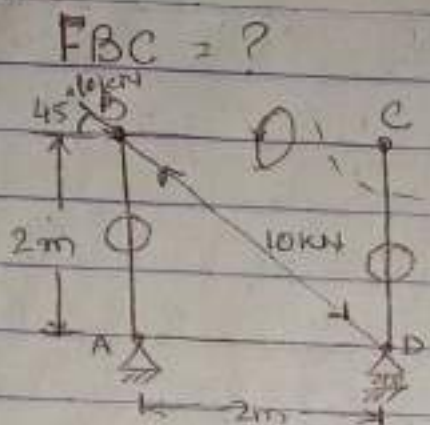
F.B.D of joint B

F.B.D of BD

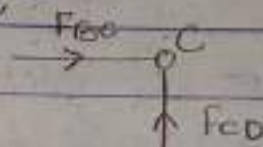


F.B.D of 'D'

Ques(3)



Sol:-



F.B.D of joint 'C'

$$\sum x = 0 \Rightarrow F_{BC} = 0$$

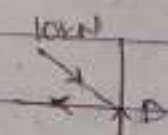
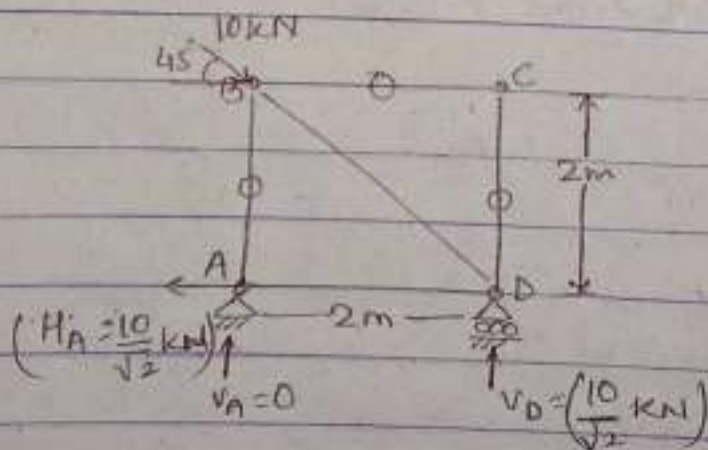
$$\sum y = 0 \Rightarrow F_{CD} = 0$$

NOTE:-

$$1. F_{AB} = 0$$

$$2. F_{BD} = 10 \text{ kN (C)}$$

Ques 4! Support reactions are



Sol:- $\sum x = 0 \Rightarrow -H_A + 10 \cos 45^\circ = 0$

$\left[\begin{array}{c} \rightarrow \leftarrow \\ \text{+ve} \quad \text{-ve} \end{array} \right]$

$$H_A = \frac{10}{\sqrt{2}} \text{ kN } (\leftarrow)$$

$$\sum M_D = 0 \Rightarrow V_A \times 2 + 10 \times 0 = 0$$

$\left[\begin{array}{c} \curvearrowright \curvearrowleft \\ \text{+ve} \quad \text{-ve} \end{array} \right]$

$$V_A = 0$$

$$\sum y = 0$$

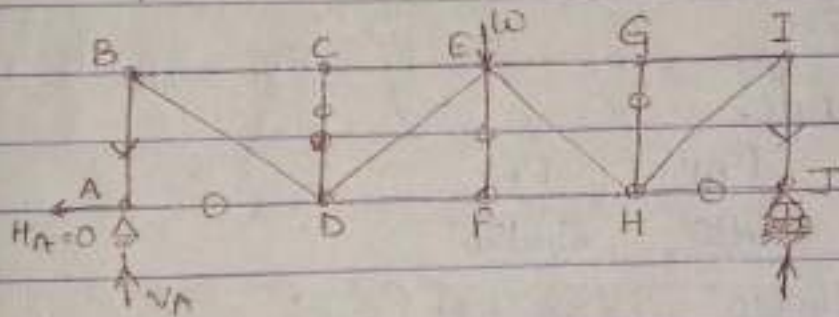
$$\left[\begin{array}{c} \uparrow \\ \downarrow \end{array} \right]$$

+ve -ve

$$+V_A + V_D - 10 \sin 45^\circ = 0$$

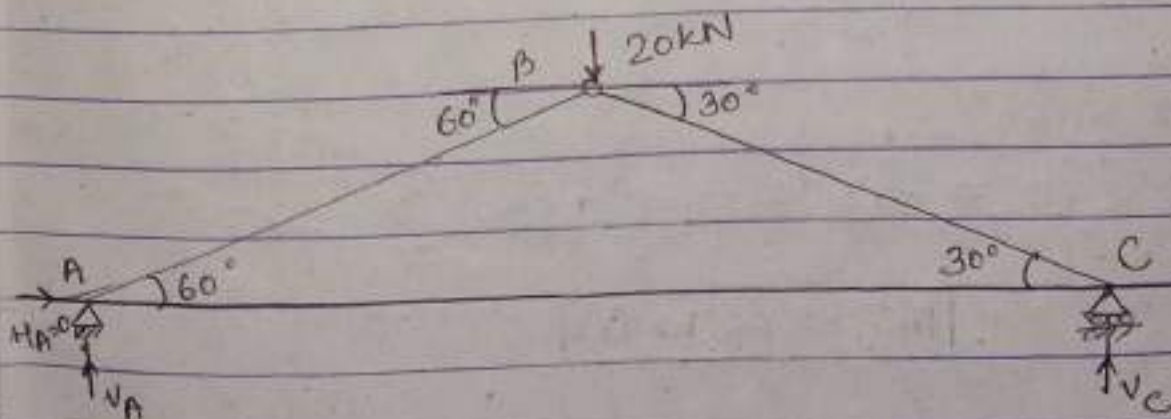
$$V_B = \frac{10}{\sqrt{2}} \text{ kN}$$

Ques 5: The members having zero forces are?



Sol:- Zero force member
AD, CD, EF, GH, HJ

Ques 6:- $F_{AC} = ?$ { 2 Marks }

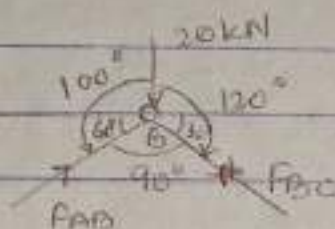


Sol:-

NOTE 1. Since length of the member is not known, we can not find reactions directly, So first take F.B.D of joint 'B' & find AB. Then take F.B.D of

joint 'A' & find AC.

Joint 'B' :-

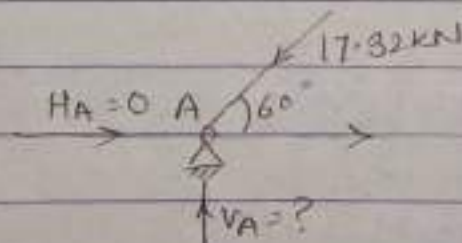


Using Sine rule,

$$\frac{20}{\sin 90^\circ} = \frac{F_{AB}}{\sin 120^\circ} = \frac{F_{BC}}{\sin 150^\circ}$$

$$F_{AB} = 20 \times \sin 120^\circ = 17.32 \text{ kN (C)}$$

Joint 'A'



$$A = 17.32 \sin 60^\circ$$

$$\sum x = 0 \Rightarrow -17.32 \cos 60^\circ + F_{AC} = 0$$

$$\begin{array}{c} \boxed{\rightarrow \leftarrow} \\ + \quad - \end{array}$$

$$\boxed{F_{AC} = 8.66 \text{ kN (T.)}}$$

NOTE :

At joint 'B', if we use eqⁿ of equilibrium, we have to solve two simultaneous equations. It will waste our time. So, Sine rule is used.

2. METHOD OF SECTION :

Concepts ① :-

Equilibrium of a section of a truss is considered in MTD of section.

2. {1 Marks} {Advantage}

The advantage of MTD of section is that force in any Intermediate Member can be found directly without finding forces in any other members.

3. Procedure :-

Step 1 :- Find the support reactions, considering equilibrium of the entire truss.

Step 2 :- Cut the member under consideration by a section ① - ① & consider equilibrium of either L.H.S of ① - ① or R.H.S of ① - ① and use $\sum x = 0$, $\sum y = 0$, $\sum M = 0$, to find the unknown forces in the members.

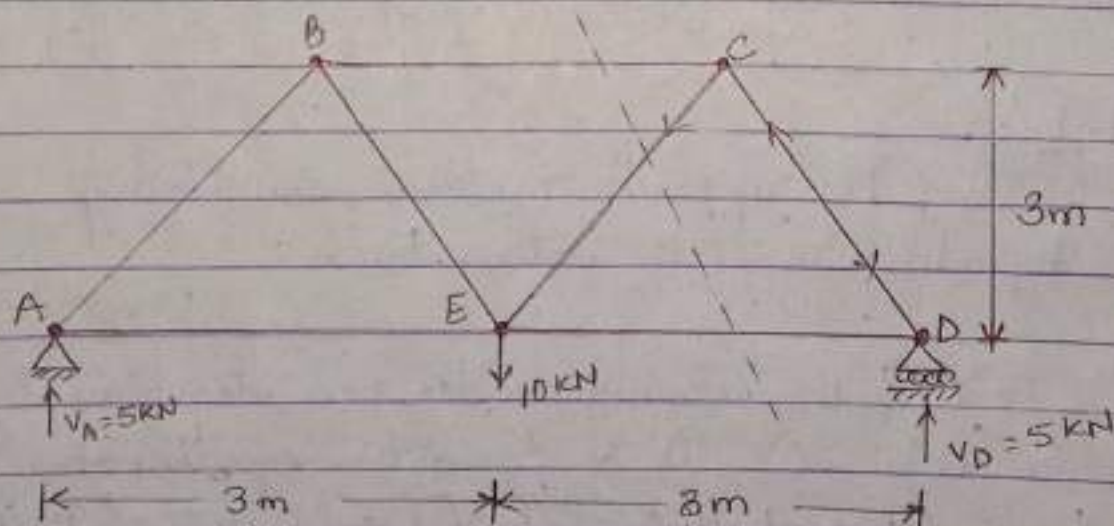
NOTE 1 :- Cut the member such that entire truss divided into two separate parts.

2. Preferably, don't cut more than 3 members in MTD of section we have only 3 eqⁿ of equilibrium, with three eqⁿ of equilibrium we

can only find 3-unknown member forces.

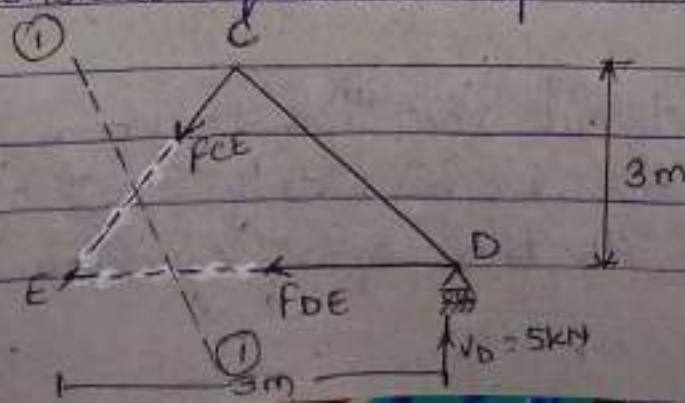
3. Cut the member such that all the cut members do not meet at one joint if they meet at one joint, then $\sum M = 0$ becomes useless eqⁿ. & it becomes a MTD of joints problem.

Ques:- For the truss shown in fig. force in the member BC is.



Sol:- Due to symmetry
 $V_A = V_D = 5 \text{ kN}$

Cut the 'BC' by ① - ①
Consider equilibrium of R.H.S of ① - ①



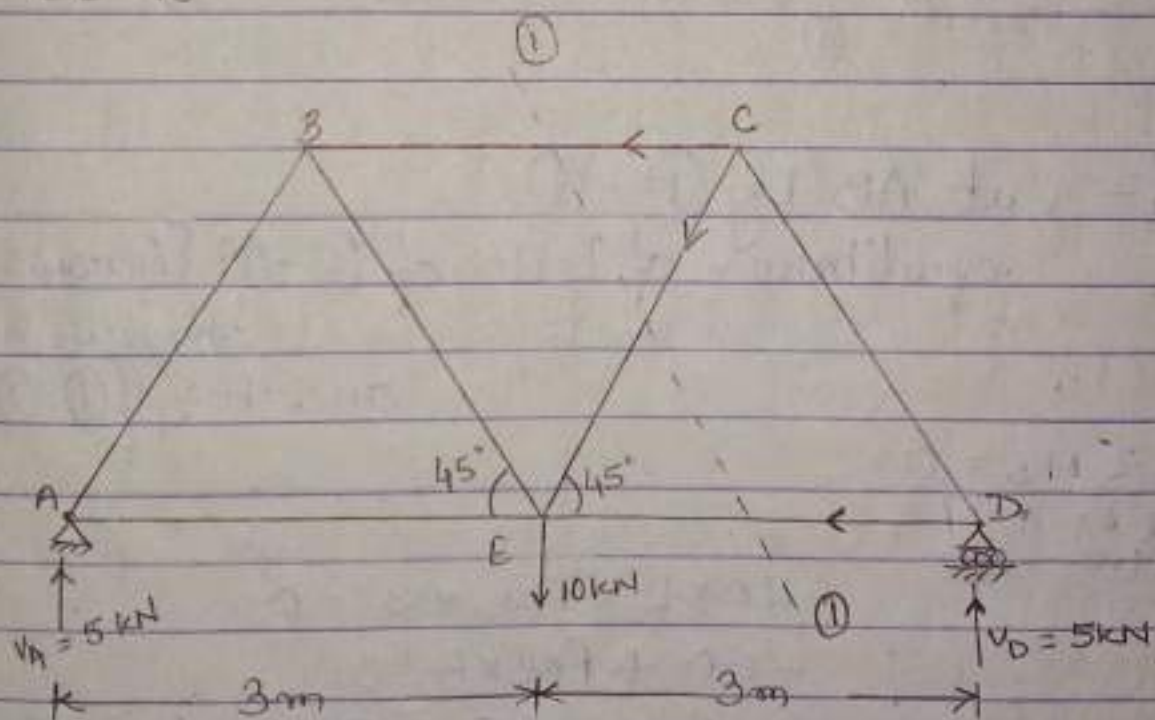
$$\sum M_E = 0 \Rightarrow -F_{BC} \times 3 - 5 \times 3 = 0$$

[$\curvearrowright$ $\curvearrowleft$]
+ve -ve

$F_{BC} = +15 \text{ kN}$ (-ve sign implies our assumed direction for F_{BC} is wrong.
So, arrow must be towards joint.

So, $F_{BC} = 5 \text{ kN (comp)}$ Ans!-

Ques!- For truss shown in fig. force in the member CE is



Sol:- Cut CE by ①-①

Equilibrium of R.H.S of ①-①

{Keep arrows on R.H.S of ①-① only}

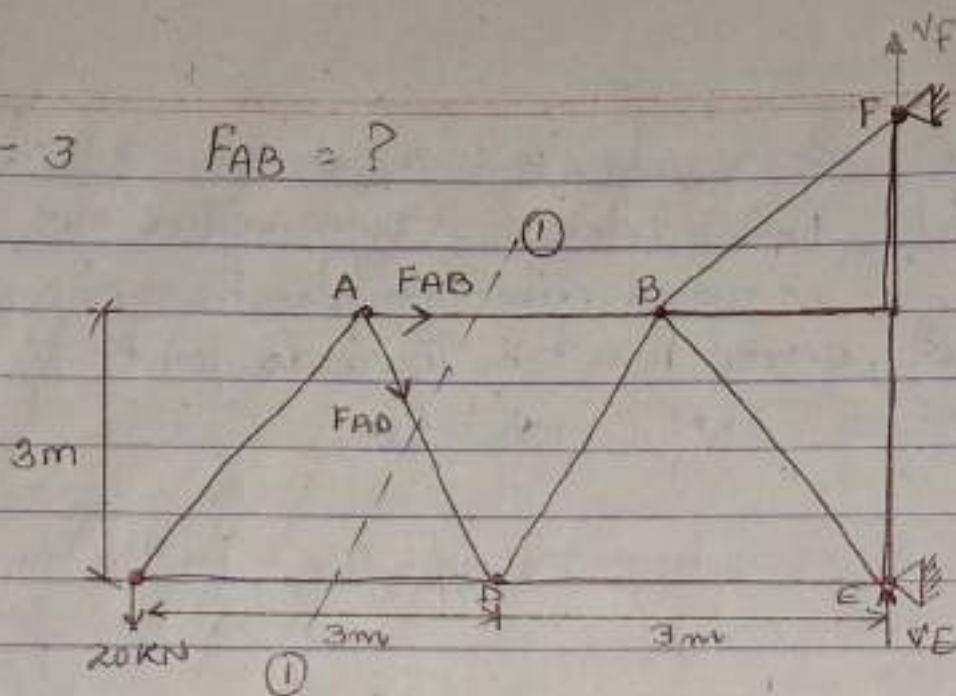
$$\sum F_y = 0$$

[$\uparrow$ $\downarrow$]
+ve -ve

$$+5 - F_{CE} \sin 45^\circ = 0$$

$$F_{CE} = \frac{5}{\sin 45^\circ} = 5\sqrt{2} \text{ kN (T)}$$

Que :- 3 $F_{AB} = ?$



Sol:- Cut AB by (1) - (1)
 equilibrium of L.H.S of (1) - (1) { Arrows must be kept on L.H.S of (1) - (1) }

$$\sum M_D = 0$$

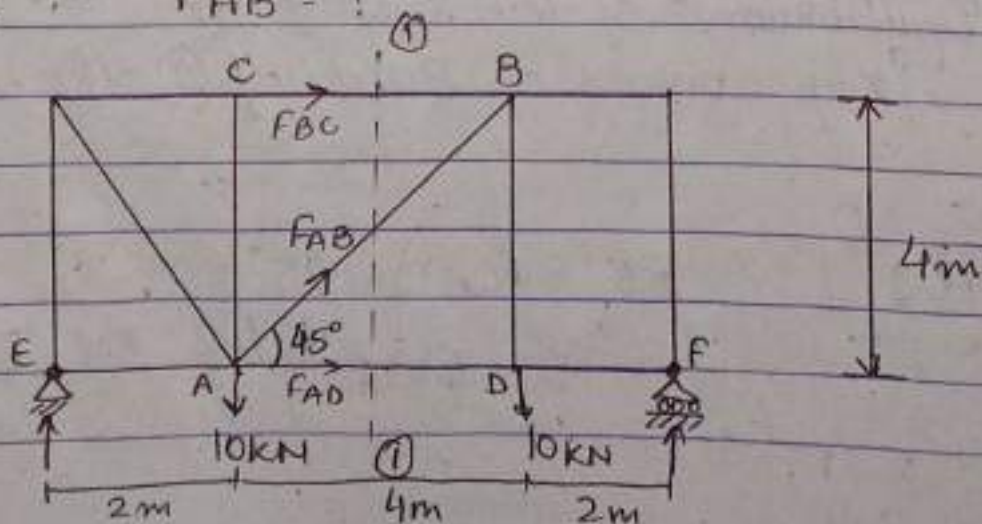
[$\curvearrowright$ +ve $\curvearrowleft$ -ve]

$$-20 \times 3 + F_{AB} \times 3 = 0$$

$$-60 + F_{AB} \times 3 = 0$$

$$F_{AB} = \frac{60}{3} = 20 \text{ kN (T) Ans!-}$$

Que 4 :- $F_{AB} = ?$



Sol:- Cut AB by ①-①

Equilibrium of L.H.S of ①-① (Arrows are kept on L.H.S of ①-①)

$$\Sigma y = 0$$

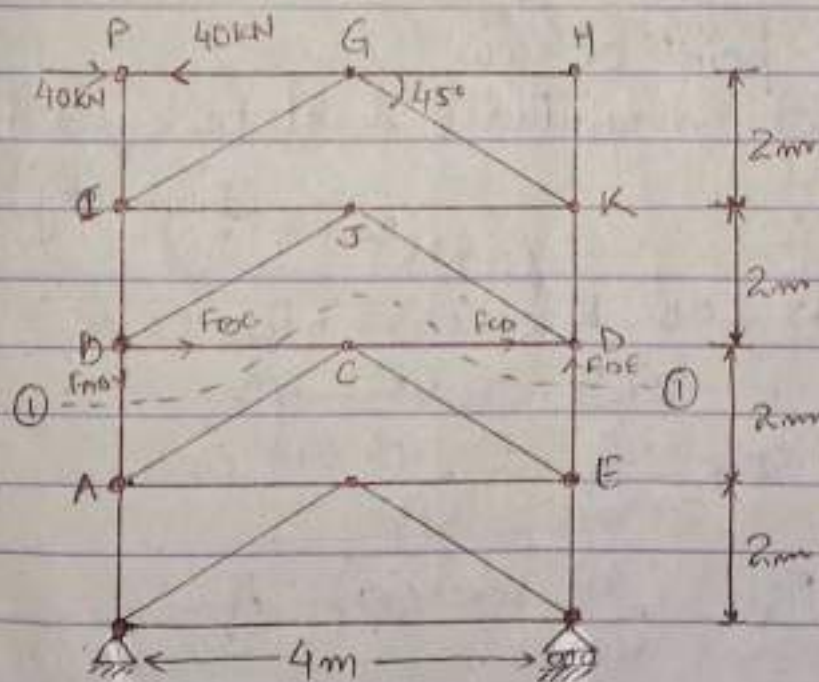
$$\left[\begin{array}{c} \uparrow \\ \downarrow \end{array} \right]$$

+ve -ve

$$+10 - 10 + F_{AB} \sin 45^\circ = 0$$

$$F_{AB} = 0 \text{ Ans:-}$$

Ques: $F_{AB} = ?$



Sol:- Cut AB by ①-①

Equilibrium of upper side of ①-①

{ arrows are kept on upper side of ①-① }

$$\Sigma M_D = 0$$

$$[\curvearrowright]$$

$$+40 \times 4 - F_{AB} \times 4 = 0$$

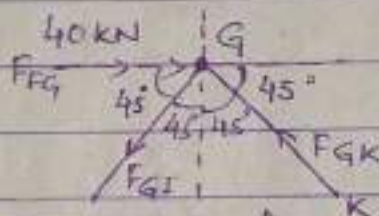
$$F_{AB} = 40 \text{ kN (T)}$$

NOTE: 1. $F_{GH} = 0$

2. $F_{FI} = 0$

$$F_{GH} = 40 \text{ kN (C)} = F_{FG}$$

3. F.B.D of joint 'G'



From $\Sigma y = 0$ point of view

F_{GI} & F_{GK} magnitudes must be equal.

$$\Sigma x = 0$$

$$[\rightarrow \leftarrow]$$

$$\Rightarrow +40 - F_{GK} \cos 45^\circ - F_{GI} \cos 45^\circ = 0$$

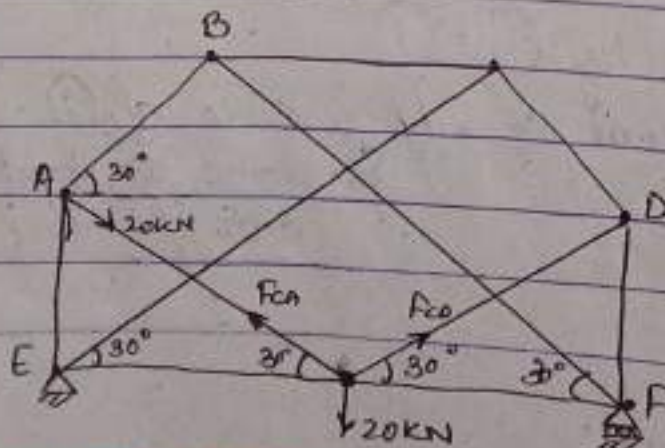
(F_GI) [Because of symmetry]

$$2 F_{GI} \cos 45^\circ = 40$$

$$F_{GI} = \frac{40}{2 \cos 45^\circ} = \frac{40}{\sqrt{2}} \text{ kN (T)}$$

$$F_{GK} = \frac{40}{\sqrt{2}} \text{ kN (C)} \quad \underline{\text{Ans:-}}$$

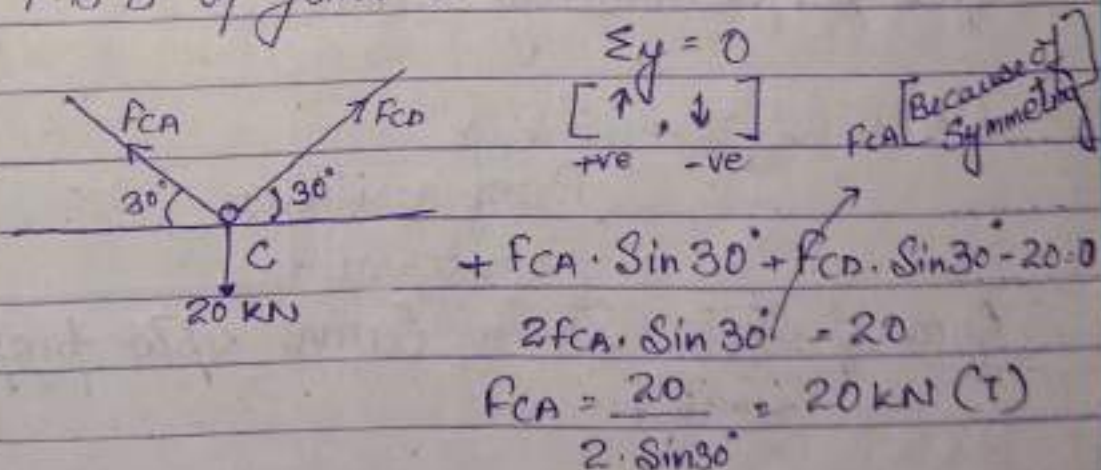
Ques 6:- $F_{AB} = ?$



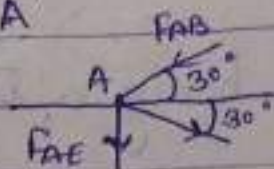
NOTE :-

1. Since all the joints have a minimum of 3-members, method of joints can not be used directly.
2. Since minimum of 4-members are getting cut by any section and the unwanted members are not meeting at a joint, method of sections can not be used directly.
3. From the symmetry of structure & loading, we find that forces in the members CA and CD must be equal. So consider F.B.D of C to find " F_{CA} ", then take F.B.D of joint 'A'. Find F_{AB} .

Sol: - ① F.B.D of joint 'C' :-



② joint 'A'



$$\sum X = 0$$

$$-F_{AB} \cos 30^\circ + 20 \cos 30^\circ = 0$$

$$F_{AB} = 20 \text{ kN (C)} \quad \text{Ans:-}$$

Deflections in determinate trusses :-

→ Strain energy method { or unit load method }

Concept :-

1. Strain energy :-

It is the energy stored due to straining of the body.

2. Resilience :-

The property of the material to store energy within proportionality limit.

3. Proof resilience :-

Max^m strain energy that can be stored upto proportionality limit.

4. Modulus of resilience :-

$$= \frac{\text{Proof resilience}}{\text{Unit volume}}$$

{ Area of Stress - Strain Curve upto proportionality limit }

5. Toughness :-

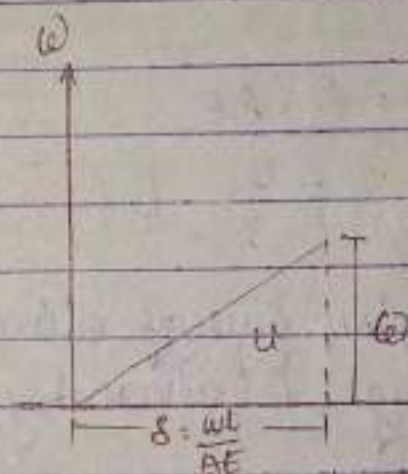
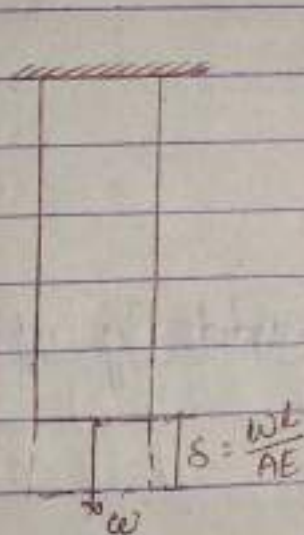
Energy stored in the body upto fracture.

6. Modulus of toughness :-

$$= \frac{\text{Toughness}}{\text{Unit Volume}}$$

7. Strain energy stored due gradually applied loading is given by

$$U = \text{Area of load v/s deflection curve}$$



$\therefore U = \text{Area of load v/s deflection curve}$

$$= \frac{1}{2} \times \delta \times W$$

$$= \frac{1}{2} \times \frac{Wl}{AE} \times W$$

$$U = \frac{W^2 l}{2AE} \quad \text{--- (1)}$$

$$U = \frac{W^2 l}{2AE} \times \frac{A}{A} = \frac{F^2}{2E} \times \text{Vol.}$$

$$U = \frac{F^2}{2E} \times \text{Volume} \quad \text{--- (2)}$$

force δ displacement δ moment δ total diff: δ

when

$$f = \frac{W}{A}$$

$$\text{Volume} = A \times l$$

$$U = \frac{1}{2} \times \delta \times W$$

$$\delta = \frac{WL}{AE} \Rightarrow W = \frac{AE\delta}{L}$$

$$= \frac{1}{2} \times \delta \times \frac{AE\delta}{L}$$

$$U = \frac{\delta^2 \cdot AE}{2L}$$

(3)

8. Strain energy stored due to suddenly applied loading, is given by,

$$U = \text{Area of load v/s deflection}$$

$$= W \times \delta$$

$$= W \times \frac{2Wl}{AE}$$

$$U = \frac{2W^2l}{AE}$$

$$U = \frac{2W^2l}{AE} \times \frac{A}{A} = \frac{f^2}{2E} \times \text{Volume}$$

$$U = \frac{f^2}{2E} \times \text{Volume}$$

$$\text{where } f = \frac{2W}{A}$$

$$\delta = \frac{2Wl}{AE}$$

NOTE 1 :- Stress due to suddenly applied load is twice that of gradually applied load.
 $\left\{ f = \frac{2W}{A} \right\}$

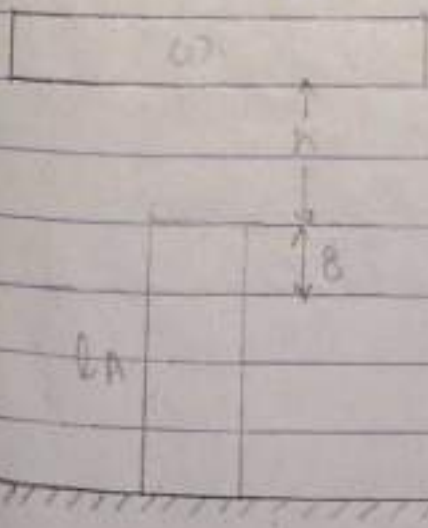
2. Elongation due to suddenly applied load is twice due to gradually applied.

3. Strain energy due to suddenly applied load is 4 times more than gradually applied load.

9. Impact:

Dynamic action of a moving load is called Impact.

10. Strain energy due to impact loading is given by:



$$U = W(h + g) = \frac{AE\delta^2}{2l}$$

If,

$$\delta_{st} = \frac{Wl}{AE}$$

Solving the eqⁿ, we get

$$\delta = \delta_{st} + \sqrt{(\delta_{st})^2 + 2\delta_{st} \cdot h}$$

If, $h = 0$

$$\delta = \delta_{st} + \delta_{st} = 2\delta_{st}$$

If $h=0$, Impact load becomes suddenly applied load.

$$U = \frac{f^2}{2E} \times \text{Volume}$$

$$\text{where, } f = \frac{w}{A} \left[1 + \sqrt{1 + \frac{2AEh}{w \cdot l}} \right]$$

$$\rightarrow \text{If } [h=0, f=2w]$$

11. Castigliano's theorem II :-

It states that the first partial derivative of total strain energy wrt to a load in a structure at any point gives deflection at that point.

$$\delta = \frac{\partial u}{\partial w}$$

$$\theta = \frac{\partial u}{\partial M}$$

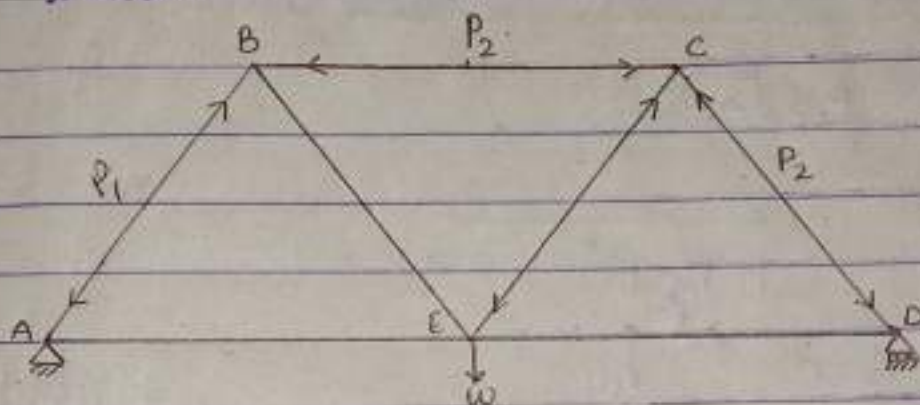
12. Castigliano's theorem - I :-

$$p = \frac{\partial u}{\partial \delta}$$

13. Application of Castigliano's theorem II :-

i) Application :

To find deflection at any joint in a truss.



U = total strain energy stored in all axially loaded members.

$$= \frac{\sum P^2 \cdot l}{2AE}$$

Where, $P = P_1, P_2, \text{ etc. } \dots$ = forces in all members due to applied load 'W' etc.

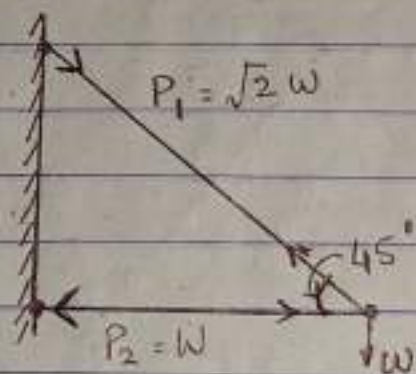
δ_{ve} = Vertical deflection of joint 'E'

$$= \frac{\partial U}{\partial W} = \sum \frac{2P \cdot \frac{\partial P}{\partial W} \cdot l}{2AE} = \sum \frac{P \cdot k \cdot l}{AE}$$

$$\boxed{\delta_v = \sum \frac{P \cdot k \cdot l}{AE}}$$

where :- $K = \frac{\partial P}{\partial w}$ - forces in all members due to unit vertical load at E { where we want to find deflection }

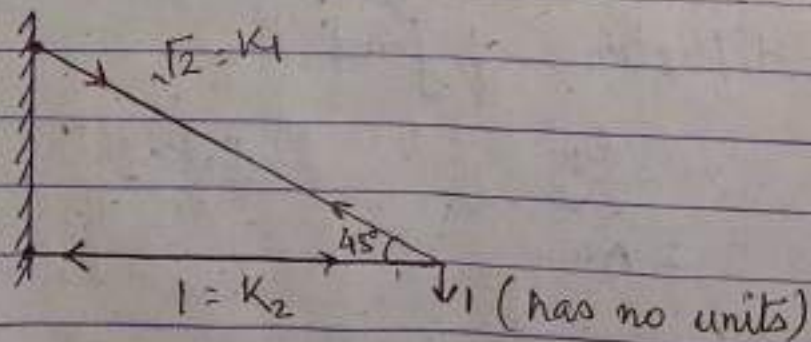
Explⁿ :-



$$\frac{\partial P_1}{\partial w} = \frac{\partial}{\partial w} (\sqrt{2}w) = \sqrt{2} \text{ (has no units) } = \text{force in the members due to unit load.}$$

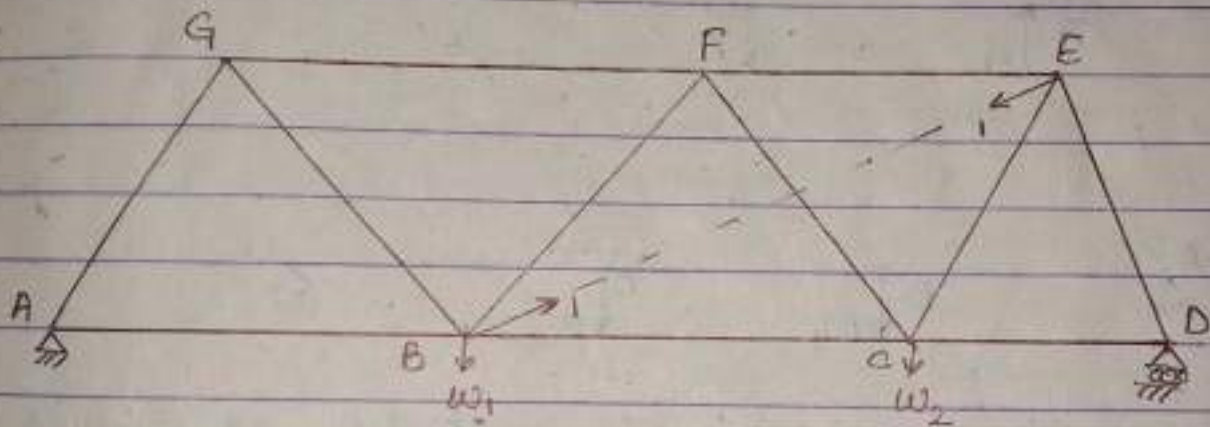
$$\frac{\partial P_2}{\partial w} = \frac{\partial}{\partial w} (w) = 1 \text{ { has no units } }$$

if apply unit load,



ii) 2nd application of Castigliano's theorem IInd:

→ to find relative joint displacement



→ If we want to find Relative displacement of two joints B & E, apply unit load at B & E in the direⁿ of BE as shown in fig & find forces in all members due to these two unit loads {K values} then relative displacement of two joints B & E will be

$$\delta_{BE} = \sum \frac{P \cdot K \cdot L}{AE}$$

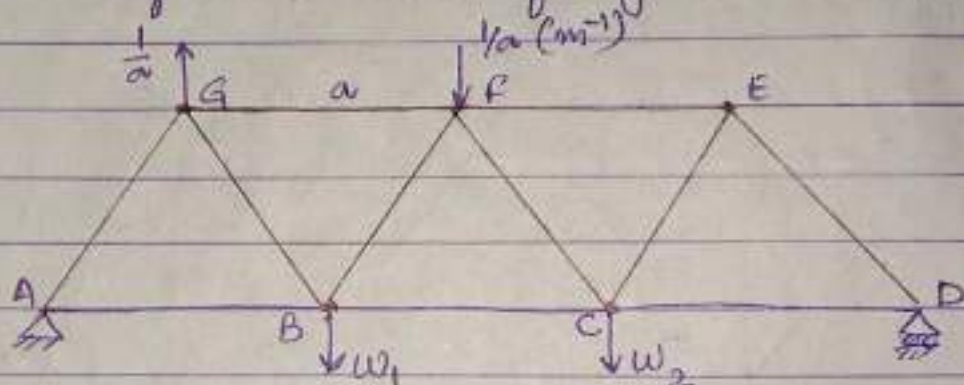
where,

$P = P_1, P_2, \text{etc}$ = forces in all members due to applied loads.

$K = K_1, K_2, \text{etc}$ = forces in all members due to two unit loads at B & E.

(iii) 3rd application of Castigliano's theorem - II :-

→ to find rotation of any member GF



$\frac{1}{a} \times a = 1$ { has no units } { unit couple }

→ If we want to find rotation of any member 'GF', Apply Unit Couple at G & F. as shown in fig. (the two forces $\frac{1}{a}$ produce unit moment.)

$$\frac{1}{a} \times a = 1$$

Now,

→ find forces in all members due to this unit couple { K value }

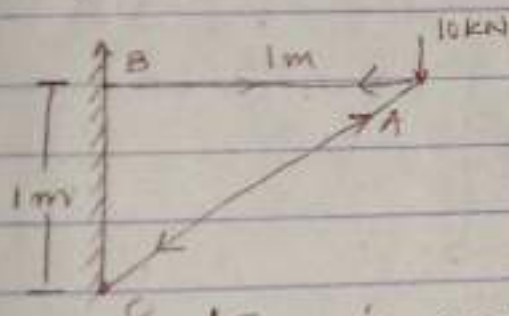
→ then the rotation of the member GF is given by

$$\theta_{GF} = \sum \frac{P \cdot K \cdot L}{AE}$$

where,
K = forces in all members due to unit couple at G and F.

P = Forces in all members due to applied load.

Q. For truss shown in fig. find vertical deflection of Joint 'A'.



AE - is const for all member

NOTE :

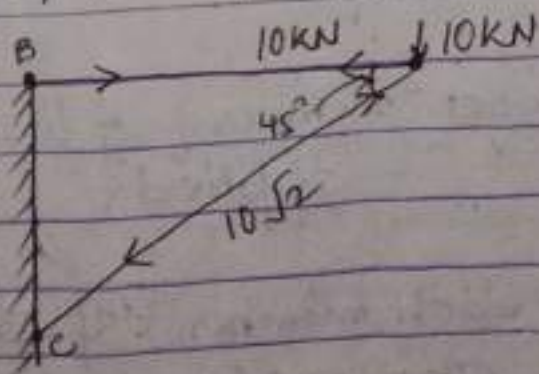
$$\delta_{VA} = \sum \frac{P \cdot K \cdot L}{AE}$$

where,

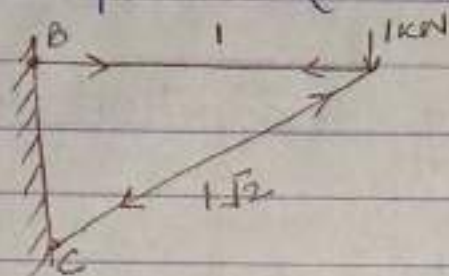
P = Forces in all members due to applied load.

K = Forces in all member due to unit vertical load at A only.

Sol. - 1st step :- 'P' values :-



IInd Step :- 'K' values



$$l_{AC} = 1\sqrt{2} \text{ m}$$

IIIrd Step :-

$$\delta_{VA} = \sum \frac{P \cdot k \cdot l}{AE}$$

Members	P	K	l	$\delta_{VA} = \sum \frac{P \cdot k \cdot l}{AE}$
AB	+10	1	1	$\frac{10}{AE}$
AC	-10√2	-1/√2	√2 m	$\frac{-10\sqrt{2}(-1/\sqrt{2})\sqrt{2}}{AE} = \frac{20\sqrt{2}}{AE}$

$$\delta_{VA} = \frac{38.28}{AE} \text{ Ans.}$$

NOTE :-

Deflection of a truss at any joint
 $= \delta_v = \sum \frac{P \cdot K \cdot L}{AE} = \sum K \delta$

where,

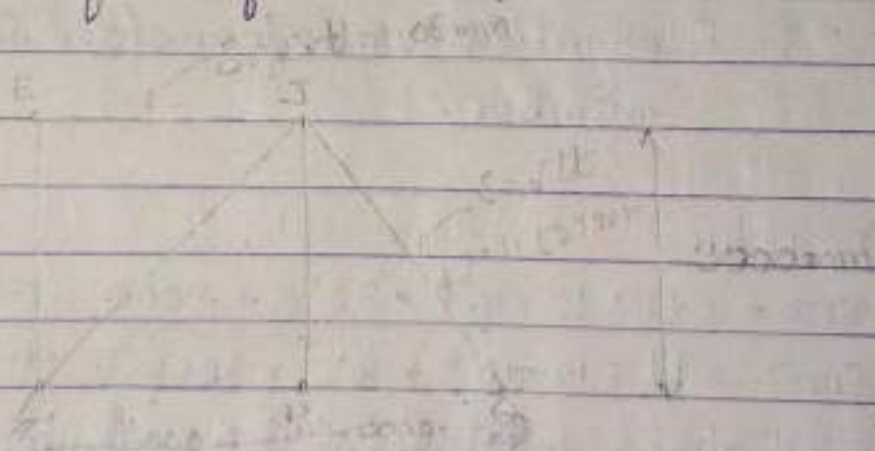
$\delta = \frac{PL}{AE}$ for each member { if loads are applied }

$\delta = \alpha t l$ for each member { if temp. changes are given }

Ques:

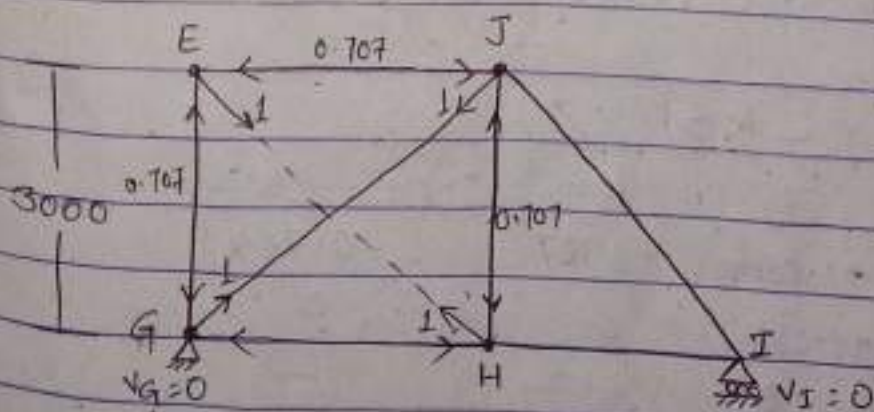
(2 Marks)

The members EJ & IJ of a steel truss shown in fig. are subjected to a temp. rise of 30°C . If $\alpha = 12 \times 10^{-6} \text{ mm/}^\circ\text{C}$, the displacement of joint E relative to joint H along the direction of HE of the truss is?



Sol:-

1. To find relative joint displacement of E & H in the dirⁿ of EH apply unit load at each joint along HE as shown in fig.



then,

$$\delta_{EH} = \frac{\sum PKL}{AE} = \sum K\delta$$

where,

K = forces in all members due to two unit loads at H & E.

δ = Elongation or expansion in each member.

2. δ values

$$\delta_{EJ} = \alpha \Delta L = 12 \times 10^{-6} \times 30^\circ C \times 3000 = 1.08 \text{ mm}$$

$$\delta_{IJ} = \alpha \Delta L = 12 \times 10^{-6} \times 30^\circ C \times 3000\sqrt{2} = 1.527 \text{ mm}$$

δ for all other members = 0 { No. temp. change }

3. K values { K values are found as shown in fig. }

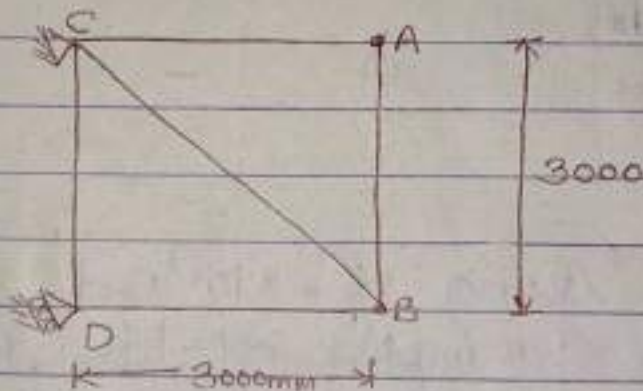
4. Tension, elongation, +ve
Compⁿ, contraction, -ve

$$\delta_{EH} = \sum K\delta$$

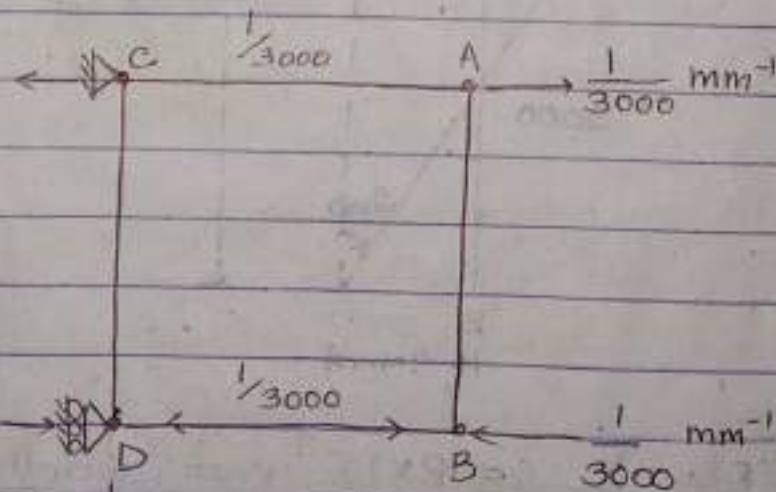
Members	δ	K	$\delta_{EH} = \sum K\delta$
EJ	+1.08mm	-0.707	-0.764
IJ	+1.527	0	0
for all other members	0	—	—

$\delta_{EH} = -0.764 \text{ mm}$ } -ve sign implies that δ_{EH} is opposite to the direction of unit loads, i.e., E & H are move away from each other by 0.764 mm .

Ques:- For the truss given in the fig. the rotation of the member AB is



Sol:- i) find the rotation of the member AB, apply unit couple at A & B as shown in fig.



$$\text{then } \theta_{AB} = \frac{\sum PKL}{AE} = \sum K\delta$$

② 'K' values they are found as shown in fig.

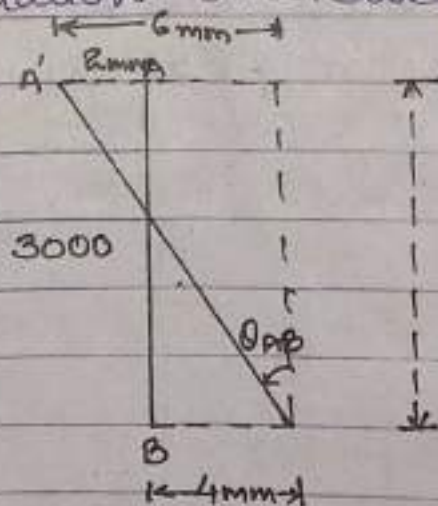
③ $\theta_{AB} = \sum K \cdot \delta$

Member	K	δ	$\theta_{AB} = \sum K \delta$
AC	$+1/3000$	-2	$-2/3000$
BD	$-1/3000$	+4	$-4/3000$
for all other members	0	-	0
			$-6/3000$

$$\theta_{AB} = -6/3000 = -2 \times 10^{-3} \text{ rad}$$

{ -ve sign implies rotation θ_{AB} in opposite to applied couple. So, AB rotation anticlockwise. }

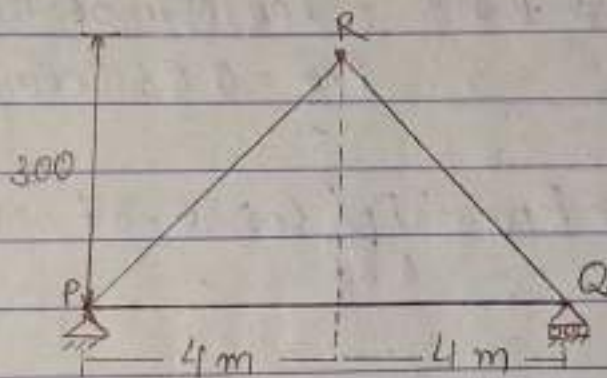
IInd Method



$$\tan \theta \approx \theta = \frac{6}{3000} = 2 \times 10^{-3} \text{ rad (anticlockwise)}$$

Ques: { 2 Marks }

For the truss shown in fig PQ is short by 3mm, magnitude of vertical displacement of joint 'R' is



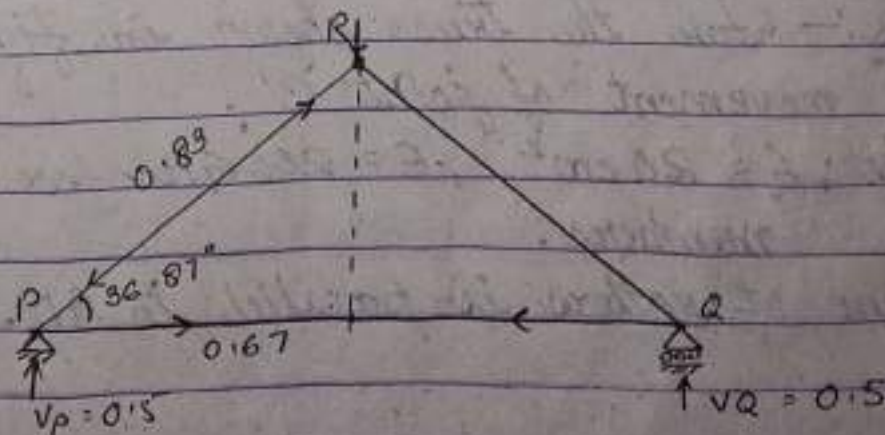
Sol:- 1. $\delta_{VR} = \sum \frac{PKL}{AE} = \sum K \cdot \delta$

where, δ = elongation or contraction for each member

$\delta_{PQ} = -3\text{mm}$ { -ve because it is short or contracted by 3mm }

$\delta = 0$, for all other member.

2. 'K' values:



$$\tan \theta = \frac{3}{4} \Rightarrow \theta = 36.87^\circ$$

Joint 'P' :-

$$\sum y = 0 \Rightarrow +0.5 - F_{PR} \sin 36.87 = 0$$

$$F_{PR} = 0.83 \text{ (comp)}$$

$$\sum x = 0 \Rightarrow F_{PQ} - \frac{F_{PR}}{0.83} \cos 36.87 = 0.67 \text{ (T)}$$

$$(3) \delta_{VR} = \sum K \cdot \delta$$

Member	δ	K	$\delta_{VR} = \sum K \delta$
PQ	-3	+0.67	-2 mm
For all other member	0	-	0

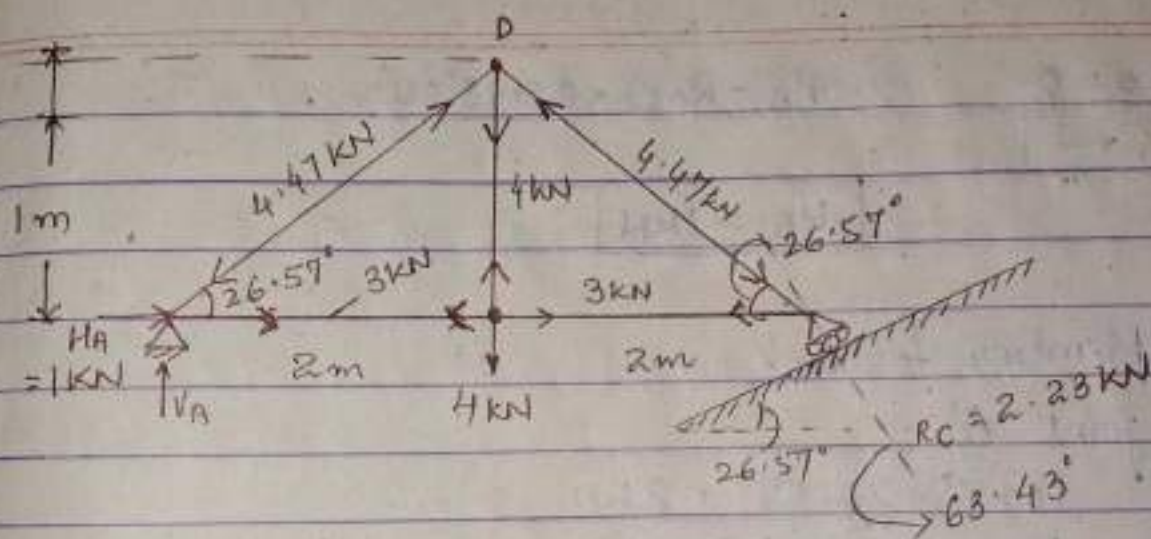
$$\delta_{VR} = -2 \text{ mm} \text{ } \{-ve \text{ Sign implies}\}$$

deflection is opposite to unit load i.e., joint 'B' moves upward by '2 mm'.

Ques:- For the truss shown in fig. find movement of joint 'C'.

Take : $A = 20 \text{ cm}^2$, $E = 200 \text{ GPa}$ for all members.

Plane of rollers is parallel to AD.



Sol:- $\tan \theta = \frac{1}{2} \Rightarrow \theta = 26.57^\circ$

1. Roller's always move parallel to the plane of the roller base. So unit load must be applied parallel to the plane of the roller base to find movement of the roller.

$$\delta_c = \sum \frac{PKL}{AE}$$

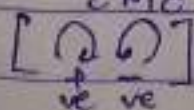
where, P = forces in all members due to the applied loads.

K = forces in all members due to single unit load at 'C' parallel to base of rollers.

'P' values

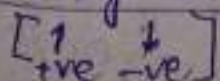
i) Support reactions

$$\sum M_C = 0 \Rightarrow +V_A \times 4 - 4 \times 2 = 0$$



$$V_A = 2 \text{ kN}$$

$$\sum Y = 0$$



$$\Rightarrow V_A + R_C \sin 63.43^\circ - 4 = 0$$

$$R_C = \frac{2}{\sin 63.43^\circ} = 2.23 \text{ kN}$$

$$\Sigma x = 0 \Rightarrow +H_A - R_C \cos 68.43^\circ = 0$$

$\left[\begin{array}{c} \rightarrow \\ \leftarrow \end{array} \right]$
 $\begin{array}{c} +ve \\ -ve \end{array}$

$H_A = 1 \text{ kN}$

(ii) Member forces:
joint 'A'

$$F_{AD} \sin 26.57^\circ = 2 \text{ kN}$$

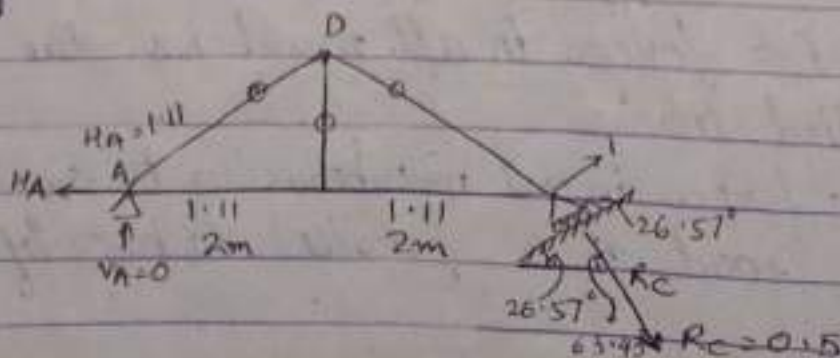
$$F_{AD} = 4.47 \text{ kN (Comp)}$$

$$\Sigma x = 0 \Rightarrow +1 - 4.47 \cos 26.57^\circ + F_{AB} = 0$$

$$F_{AB} = 3 \text{ kN (T)}$$

Because of symmetry find other forces as shown in fig.

III - Step 'K' values



Support reactions :-

$$\Sigma M_C = 0 \Rightarrow V_A \times 4 = 0$$

$$\left[\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} \right]$$

$\begin{array}{c} +ve \\ -ve \end{array}$

$$V_A = 0$$

$$\Sigma y = 0 \Rightarrow$$

$$\left[\begin{array}{c} \uparrow \\ \downarrow \end{array} \right]$$

$\begin{array}{c} +ve \\ -ve \end{array}$

$$1 \times \sin 26.57 + R_C \times \sin 68.43 = 0$$

$$R_C = \frac{\sin 26.57}{\sin 68.43} = -0.5$$

[-ve sign implies towards]

$$\sum x = 0 \Rightarrow -H_A + 1 \cdot \cos 26.57 + 0.5 \cos 63.43 = 0$$

[→ ←]
+ve -ve

$$H_A = 1.11 \text{ kN}$$

(iv) Step :-

$$\delta_c = \frac{\sum PkL}{AE}$$

Member	P	K	L	$\delta_c = \sum \frac{PKL}{AE}$
AB	+3	+1.11	2m	6.66/AE
BC	+3kN	1.11	2m	6.66/AE
for all other members	-	0	-	0

$$\delta_c = \frac{2 \times 6.66}{AE} = \frac{2.66}{20 \times 10^{-4} \times 200 \times 10^6}$$

$$= 0.033 \text{ mm Ans!-}$$

$$A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2$$

$$= 200 \times 10^6 \text{ kN/m}^2 \text{ Ans!-}$$

IV. Application of Castigliano's Theorem II :-

Forces in members of redundant trusses :

Concepts ①

External Indeterminacy (D_{sc}) or External Redundancy
 { it deals with support reactions }

→ It is the no. of reaction in excess of equilibrium eqⁿ

$$D_{sc} = r - s$$

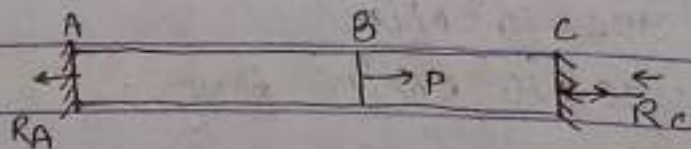
$r \rightarrow$ no. of unknown reaction
 $s \rightarrow$ no. of eqⁿ of equilibrium available.

(2) Internal Indeterminacy (D_{si})
 { it deals with internal member forces only? }

It is no. of members in excess of $(2j - 3)$
 i.e.,

$$D_{si} = m - (2j - 3)$$

In SOM,



$$D_s = r - s = 2 - 1 = 1 \text{ degree}$$

$r =$ No. of unknowns $= (R_A, R_C) = 2$

$s =$ No. of eqⁿ of equilibrium available $= 1$

$$R_A + R_C = D \quad \text{--- (1)}$$

$$\Sigma x = 0$$

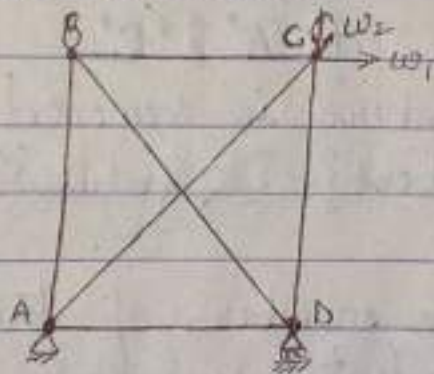
So, one add compatibility condition is ready.

→ elongation in AB = Contraction in BC - (2)

Concept (3):-

Additional eqⁿ, called compatibility conditions are developed to find forces in redundant trusses.

ex:-

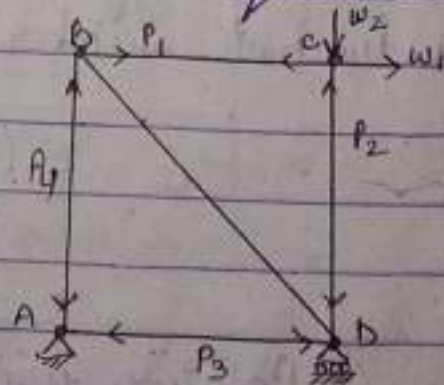
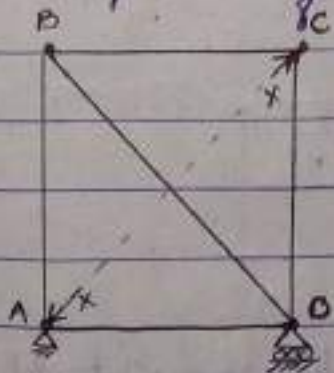


Actual Truss:-

$$m = 6, J = 4$$

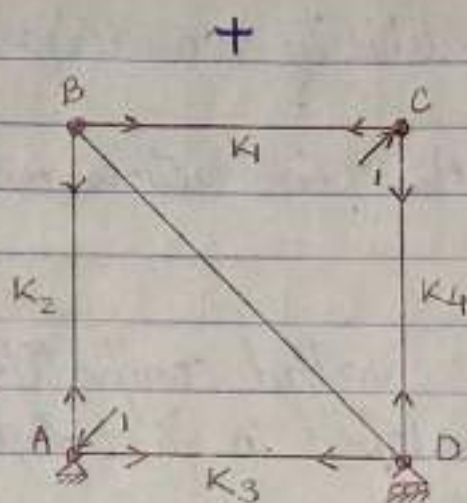
$$Dsi = m - (2j - 3) = 1 \text{ degree}$$

So, one compatibility conditions is required.



Remove any redundant member AC, & replace it by its force 'x' as shown in fig.

Remove redundant & make the truss determinate truss and find forces in all members {P values}



Apply unit loads at 'A' & 'C'

Where redundant members is removed. { find forces in all members } { K values }

→ Final forces in the members of truss { Excluding redundant members }.

$$= P + Kx$$

→ Strain energy stored in all members {excluding redundant member}.

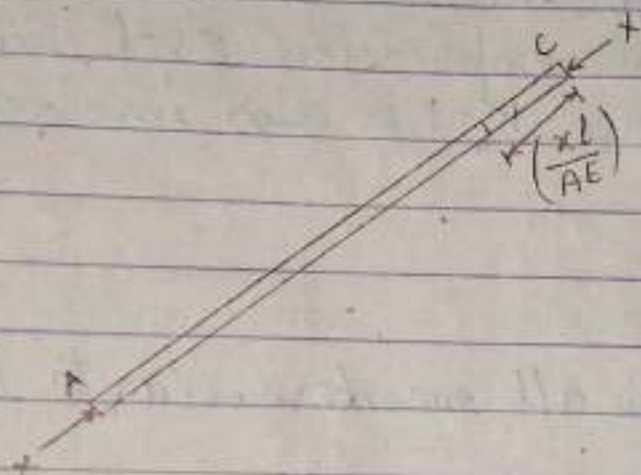
$$U = \frac{\sum (P + Kx)^2 \cdot l}{2AE}$$

→ δ_{AC} = Relative joint displacement of 'A' & 'C'.

$$= \frac{\partial U}{\partial x} = - \frac{x \cdot l}{AE}$$

← Compatibility conditions

{ -ve because unit load dirⁿ & deformations in the members are always opposite to each other }



$$\delta_{AC} = \sum \frac{2(P + Kx)K \cdot L}{2AE} = -\frac{xL}{AE}$$

$$\delta_{AC} = \sum \frac{PKL}{AE} + \sum \frac{K^2L}{AE} x = -\frac{xL}{AE}$$

5 members 5 members

$$\sum \frac{PKL}{AE} + \sum \frac{K^2L}{AE} x = -\frac{xK^2L}{AE} \quad (K=1) \text{ for redundant members}$$

$$\sum \frac{PKL}{AE} + \sum \frac{K^2L}{AE} x = 0$$

5 members 6 members

$$x = -\frac{\sum \frac{PKL}{AE}}{\sum \frac{K^2L}{AE}}$$

** Compatibility Condition

If 'AE' is constant for all members

$$x = -\frac{\sum PKL}{\sum K^2L}$$

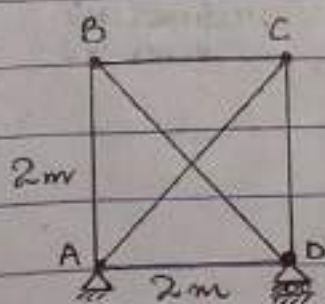
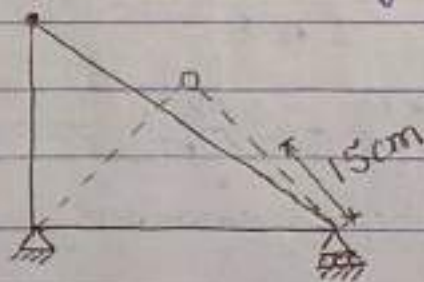
Note :-

In the express, $\sum PKL$, redundant force. 'x' is not involved because it is removed, but in the expression $\sum K^2L$, redundant member force $K=1$ is also involved.

Concept (4) :-

Forces in all members, due to load fit alone :-

{ No external loads are acting on the truss } :



$$L_{AC} = 2\sqrt{2} = 2.828m$$

If any member 'AC' is too long or too short by an amount 's' then then member 'AC' pulled or pushed to keep it in position in the truss. Then forces are developed in all other members of the ~~truss~~ truss.

{ In a statically determinate truss, forces will not be developed due to lack of fit }

$$U = \frac{\sum (P + kx)^2 \cdot l}{2AE}$$

$$U = \frac{\sum (kx) \cdot l}{2AE}$$

δ_{AC} : Relation joint displacement : $\delta = \frac{\partial U}{\partial x}$

$$= \frac{\sum 2(kx) \cdot kl}{2AE}$$

$$\delta = \frac{\sum k^2 l}{AE} \times x$$

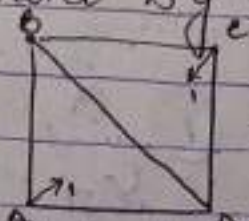
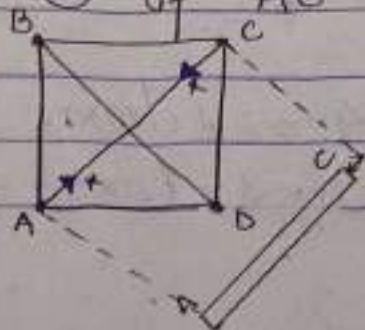
So, x = forces in redundant member (due to lack of fit)

$$x = \frac{\delta}{\frac{\sum k^2 l}{AE}}$$

If 'AE' is in all members

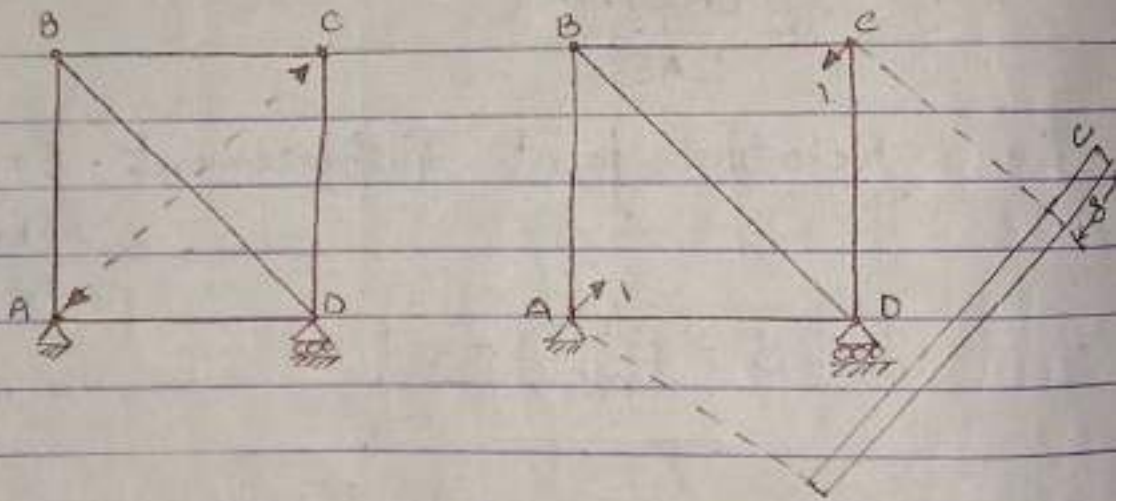
$$x = \frac{\delta(AE)}{\sum k^2 l}$$

Note :- ① If 'AC' is too short by 's'



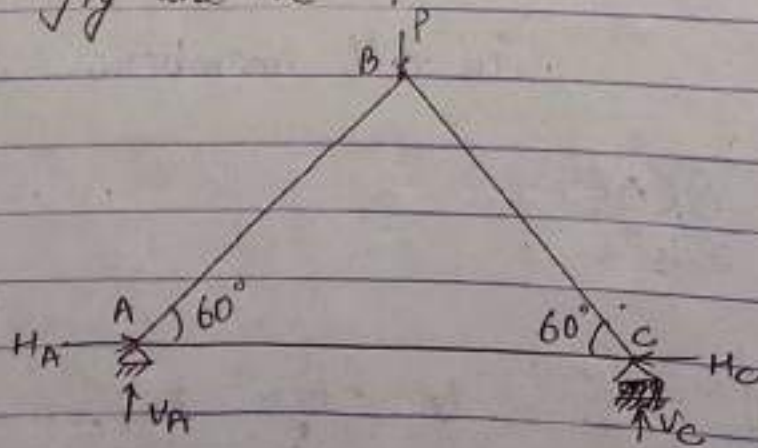
Unit load and δ are in same direction. So take δ as '+ve'

- ② If members 'AC' is too long by '8'
 { It should be compressed and kept in position }
 { force in AC is compressive }



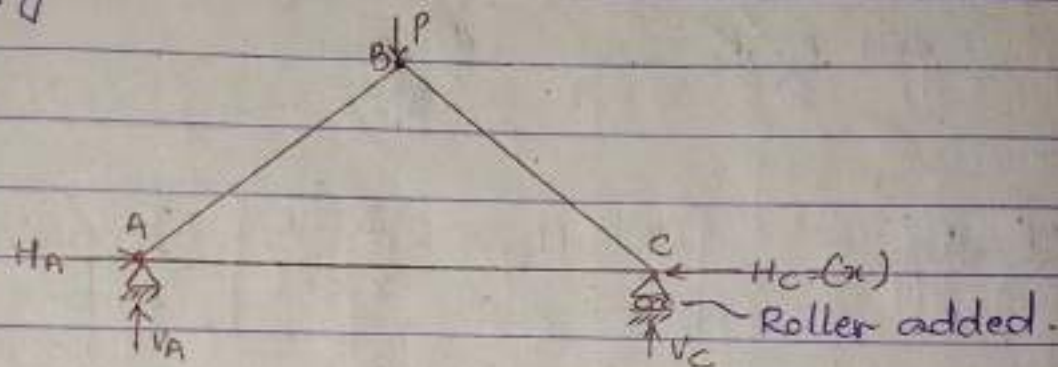
Since unit load '8' are in same dir take δ as '+ve'.

Ques:- {2 Marks} For the truss shown in fig the $H_c = ?$



Solⁿ:- ① $D_{sc} = r - s = 4 - 3 = 1$ degree
 $D_{si} = m - (2j - 3) = 3 - (2 \times 3 - 3) = 0$

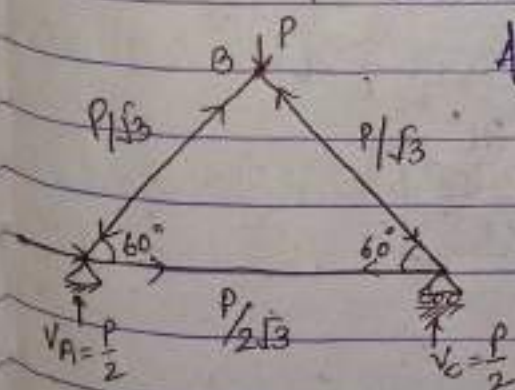
Since the truss is externally indeterminate to one degree, remove the redundant reaction H_c and replace it by its force 'x' as shown in fig:-



Since $D_{SO} = 1$, we req^d on compatibility condition $\Rightarrow \delta_{H_C} = \frac{\partial u}{\partial x} = 0$

$$\Rightarrow x = - \frac{\sum P k L}{\sum k^2 L} \quad \text{Compatibility conditions}$$

1 Step: 'P' values { forces in all members after redundant is removed }



At joint A:-

$$\sum y = 0$$

$$= \frac{P}{2} - F_{AB} \sin 60^\circ = 0$$

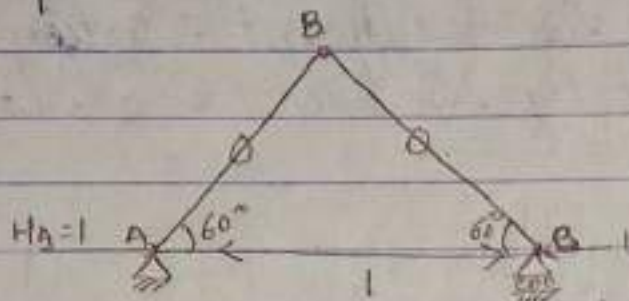
$$F_{AB} = \frac{P}{2 \sin 60^\circ} = \frac{P}{\sqrt{3}} \text{ (comp)}$$

$$\sum x = 0$$

$$-\frac{P}{\sqrt{3}} \cos 60^\circ + F_{AC} = 0$$

$$F_{AC} = \frac{P}{2\sqrt{3}} \text{ (T)}$$

II Step :- 'K' values :



III Step :-

$$X = H_C = - \frac{\sum P k l}{\sum K^2 l}$$

Tension +ve
Comp. -ve

Member	P	K	l	Pkl	K ² l	Final forces F = P + Kx
AC	$+P/2\sqrt{3}$	-1	l	$-\frac{Pl}{2\sqrt{3}}$	l	$\frac{+P}{2\sqrt{3}} + (-1) \times \frac{P}{2\sqrt{3}} = 0$
AB	$-P/\sqrt{3}$	0	l	0	0	$-P/\sqrt{3}$
BC	$-P/\sqrt{3}$	0	l	0	0	$-P/\sqrt{3}$

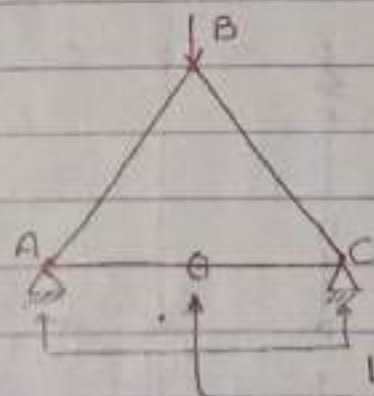
$$x = H_C = - \frac{\sum P k l}{\sum K^2 l} = - \frac{\left[\frac{-Pl}{2\sqrt{3}} \right]}{[l]}$$

$$= \frac{P}{2\sqrt{3}}$$

$$H_C = \frac{P}{2\sqrt{3}} \quad \text{Ans.}$$

Ques:- In the above Pb the force in the member AC is 'zero' { becz the member AC can not elongate or contract because of two hinged supports? }

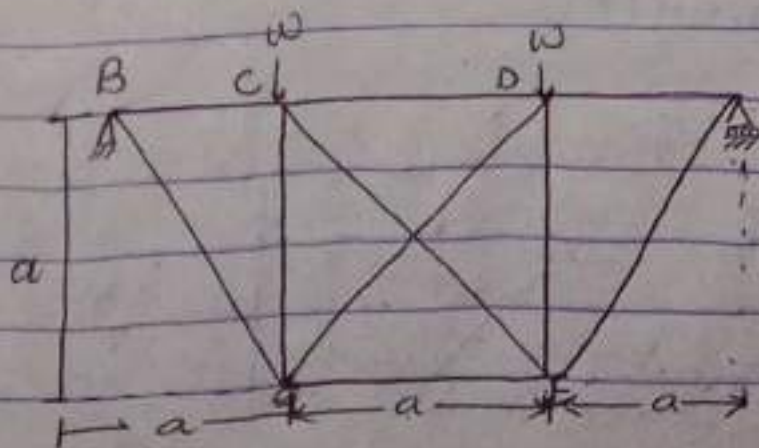
So $f_{AC} = 0$



because both supports is fixed & it cannot fixed move

Ques:- a) For the truss shown in fig, find axial force in the member GF, AE is same to all members.

(b) Evaluate the forces in the same members if it is fabricated 0.1% too short & the structure carries no external loads.



Sol:- 1st step :- Find Dsc & Dsi

$$D_{sc} = r - s = 3 - 3 = 0 \text{ } \{0 \text{ implies determinate}\}$$

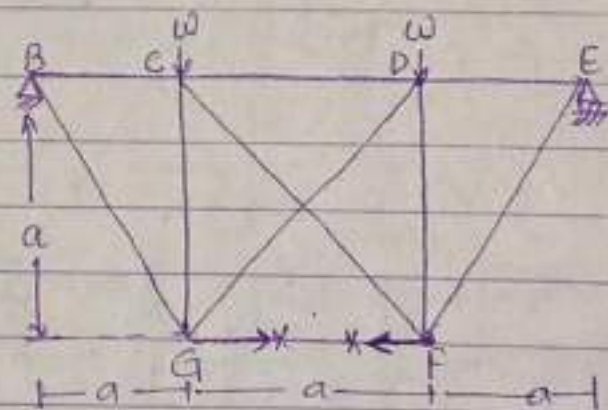
$$D_{si} = m - (2j - 3) = 10 - (2 \times 6 - 3)$$

$$= 10 - 9 = 1 \text{ degree}$$

Since the truss is internally redundant.

Remove any redundant

member 'GF' & replace it by its force 'x' as shown in fig.



We should not remove EC or EB bcz

structure will collapse, we can remove

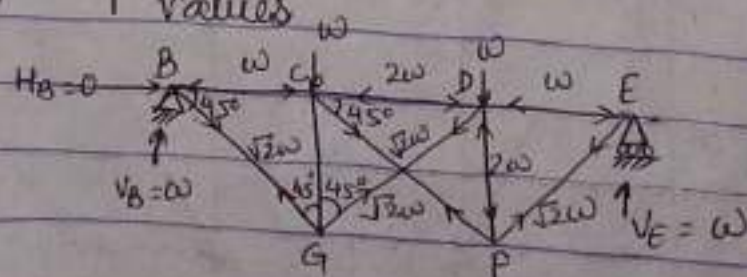
only one of the six members in the central panel.

$$\text{Compatibility Condition} = x = - \frac{\sum PKL}{\sum K^2 L} = f_{GF}$$

where P = forces in all members due to applied load } after redundant member is removed

K = forces in all members due to unit loads at G and F.

2nd Step :- 'P' values



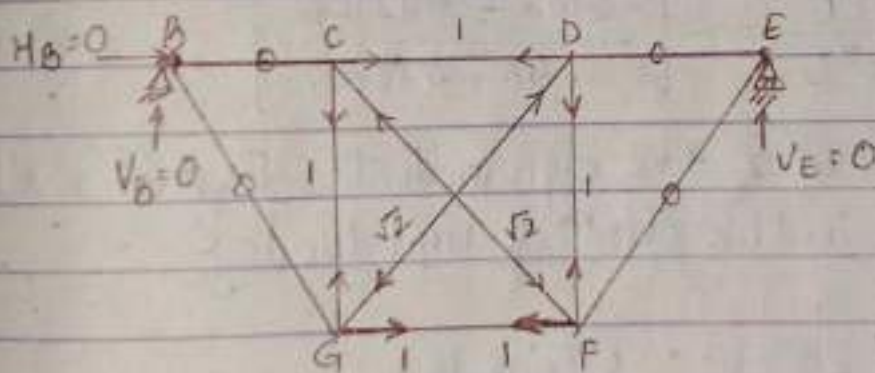
Joint 'C' :-

$$\sum x = 0$$

$$\Rightarrow w + \sqrt{2}w \cos 45^\circ - F_{CD} = 0$$

$$F_{CD} = 2w$$

IIIrd Step :- 'K' Values :-



IVth Step :-

$$X = F_{FG} = -\frac{EPKL}{2k^2l}$$

Tension +ve
compression -ve.

Member	P	K	L	Pkl	k ² l	Final forces F = P + kx
BC	-w	0	a	0	0	-w + (0) \times 1 \cdot 20w = -w
CD	-2w	1	a	-2wa	a	
DE	-w	0	a	0	0	
EF	\sqrt{2}w	0	\sqrt{2}a	0	0	
GB	\sqrt{2}w	0	\sqrt{2}a	0	0	
GC	-2w	1	a	-2wa	a	

GD	$\sqrt{2}w$	$-\sqrt{2}$	$\sqrt{2}a$	$-2\sqrt{2}wa$	$2\sqrt{2}a$
FD	$-2w$	1	a	$-2wa$	a
FC	$\sqrt{2}w$	$-\sqrt{2}$	$\sqrt{2}a$	$-2\sqrt{2}wa$	$2\sqrt{2}a$
GF	0	+1	a	0	a
			$-6wa - 4\sqrt{2}wa$	$4a + 4\sqrt{2}a$	
			$\Sigma P_k L$	$\Sigma k^2 L$	

$$x = - \frac{\Sigma P_k L}{\Sigma k^2 L} = - \left[\frac{-6wa - 4\sqrt{2}wa}{4a + 4\sqrt{2}a} \right]$$

$= 1.207 w$ $\frac{a}{L}$ +ve sign indicates F_{GF} (i.e. 'x') acts in the dirⁿ of unit loads }

$$F_{GF} = 1.207 w \quad (\text{Tensile})$$

$$F_{GD} = P + kx = \sqrt{2}w + (-\sqrt{2})(1.207w) = -0.293 w \quad (\text{comp}).$$

Part (2) :- Force in GF when it is fabricated 0.1% too short.

(No external loads are acting on the truss)

GF is 0.1% too short

$$\text{So, } \delta = \frac{0.1}{100} \times a = 0.001 a$$

NOTE :- Since the member 'GF' is too short, it must be pulled & kept in position.

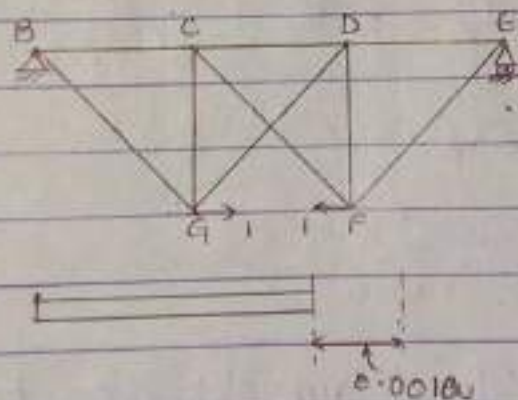
So, Force in GF is tensile due to this force, forces are developed in other members also.

$$U = \sum \frac{(P + kx)^2 \cdot L}{2AE} = \sum \frac{(kx)^2 \cdot L}{2AE}$$

$$\delta_{GF} = \delta = \frac{\partial U}{\partial x} = \sum \frac{2(kx) \cdot k \cdot L}{2AE}$$

$$= \sum \frac{k^2 L}{AE} x$$

$$x = \frac{(\delta) AE}{\sum k^2 L}$$



$$A = 20 \text{ cm}^2 = 20 \times 10^{-6} \text{ m}^2$$

$$E = 200 \text{ GPa}$$

$$= 200 \times 10^9 \text{ N/m}^2$$

$$= 200 \times 10^6 \text{ kN/m}^2$$

Since unit load direction & deformation (0.001a) are in same dirⁿ, take δ as '+ve'.

$$\rightarrow \text{Substituting, } x = F_{GF} = \frac{\delta(AE)}{\sum k^2 L} = \frac{(0.001a) \cdot AE}{(4a + 4\sqrt{2}a)}$$

$$F_{GF} = 1.03 \times 10^{-4} AE = 1.03 \times 10^{-4} \times (20 \times 10^{-6} \times 200 \times 10^6)$$

$$F_{GF} = 41.2 \text{ kN} \left\{ \begin{array}{l} \text{+ve sign indicates } F_{GF} \text{ acts in} \\ \text{the dir}^n \text{ of unit loads} \end{array} \right\}$$

$$\text{So, } F_{GF} = 41.2 \text{ kN (T)}$$

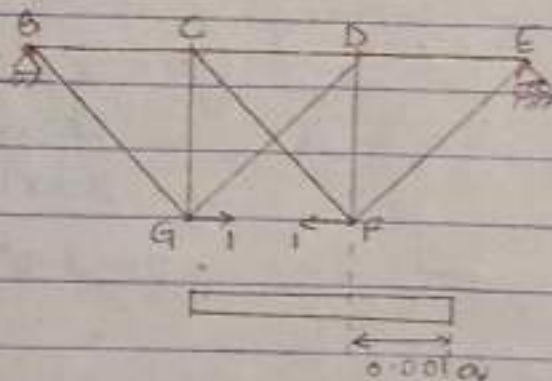
$\therefore P = 0$ [because no external load acting on the truss]

Due to fabrication error,

$$F_{GD} = Kx = \left(\frac{-\sqrt{2}}{2} \right) \times 41.2 = -58.26 \text{ kN (comp.)}$$

from
table

NOTE: In the above problem if the member GF is 0.1% too long, force in the member GF is



→ Since 'S' & Unit load are opposite to each other, take 'S' as '-ve'.

$$X = F_{GF} \cdot \frac{(-S) \cdot AE}{\sum K^2 L}$$

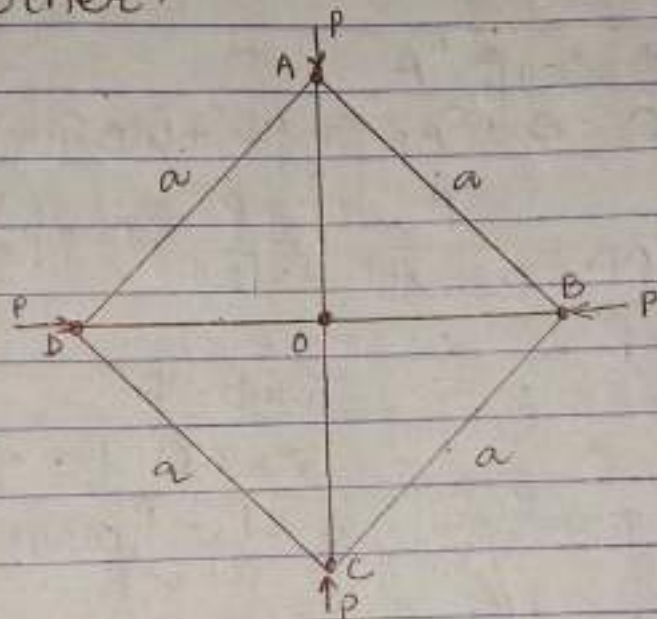
$$= -41.2 \text{ kN}$$

-ve sign indicates F_{GF}
acts opposite to dirⁿ of
unit load

So, $F_{GF} = 41.2 \text{ kN (comp.)}$

Ques:- Find the forces in members of the truss shown in fig. AE is constant for all members, also find the distance by which the points A & C moves towards

each other.

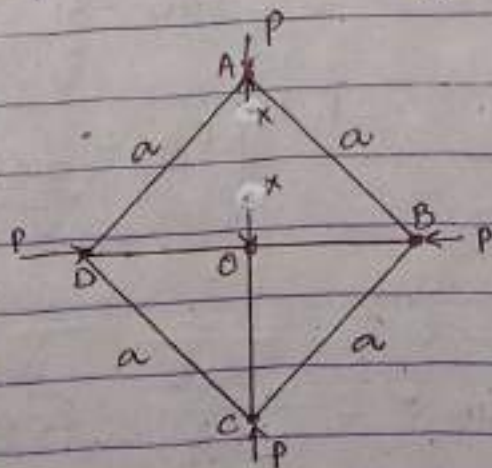


Sol:- 1st Step :-

No support. No external indeterminacy

$$D_{si} = m - (2j - 3) = 8 - (2 \times 5 - 3) = 1 \text{ degree}$$

Since, the truss is internally indeterminate to one degree, remove any redundant member 'OA' & replace it by force 'x' as shown in fig.



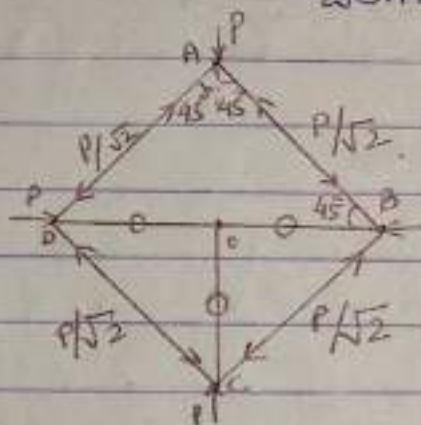
Then compatibility condition $\Rightarrow x = -\frac{\sum P_k l}{\sum k^2 l} = \frac{FOA}{\sum k^2 l}$

2nd Step :-

At joint 'A'

$$\Sigma y = 0 \Rightarrow +F_{AD} \sin 45^\circ + F_{AB} \sin 45^\circ - P = 0$$

$$F_{AD} = \frac{P}{2 \sin 45^\circ} = \frac{P}{\sqrt{2}} \text{ (comp.)}$$



Joint 'D'

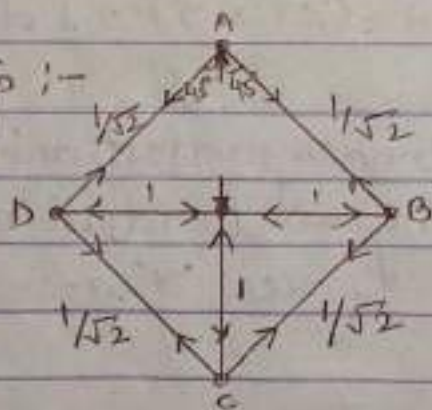
$$\Sigma x = 0 \quad \left[\begin{array}{c} \rightarrow \text{ve} \\ \leftarrow \text{ve} \end{array} \right]$$

$$\Rightarrow +P - \frac{P}{\sqrt{2}} \cos 45^\circ - \frac{P}{\sqrt{2}} \cos 45^\circ$$

$$+ F_{CD} = 0$$

$$F_{CD} = 0$$

3rd Step :-



Joint 'A'

$$\Sigma y = 0$$

$$\Rightarrow -F_{AD} \cos 45^\circ - F_{AB} \cos 45^\circ + 1 = 0$$

$$F_{AD} = \frac{1}{2 \cos 45^\circ}$$

$$F_{AD} = \frac{1}{\sqrt{2}}$$

4th Step :-

$$x = F_{OA} = - \frac{\Sigma P_k l}{\Sigma k^2 l}$$

Tension +ve
Comp. -ve

Member	P	K	L	PKL	K ² L	Final forces F = P + Kx
AB	$-P/\sqrt{2}$	$1/\sqrt{2}$	a	$-Pa/2$	$a/2$	$-\frac{P}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right) \times 414P = -0.414P$
BC	$-P/\sqrt{2}$	$1/\sqrt{2}$	a	$-Pa/2$	$a/2$	$-0.414P$
CD	$-P/\sqrt{2}$	$1/\sqrt{2}$	a	$-Pa/2$	$a/2$	"
DA	$-P/\sqrt{2}$	$1/\sqrt{2}$	a	$-Pa/2$	$a/2$	"
OB	0	-1	$a/\sqrt{2}$	0	$a/\sqrt{2}$	$-0.414P$ (comp)
OC	0	-1	$a/\sqrt{2}$	0	$a/\sqrt{2}$	"
OD	0	-1	$a/\sqrt{2}$	0	$a/\sqrt{2}$	"
OA	0	-1	$a/\sqrt{2}$	0	$a/\sqrt{2}$	$-0.414P$ (comp)

$$\begin{aligned} & -2Pa \quad 2a + \frac{4a}{\sqrt{2}} \\ & \Sigma PKL \quad \Sigma K^2L \end{aligned}$$

$$X = F_{OA} = - \left[\frac{\Sigma PKL}{\Sigma K^2L} \right] = - \left[\frac{-2Pa}{2a + \frac{4a}{\sqrt{2}}} \right]$$

$= +0.414P$ { +ve sign indicates F_{OA} acts in the dirⁿ of unit loads. So, $F_{OA} = 0.414P$ (comp)

Part 2 :- Relative joint displacement of A & C :

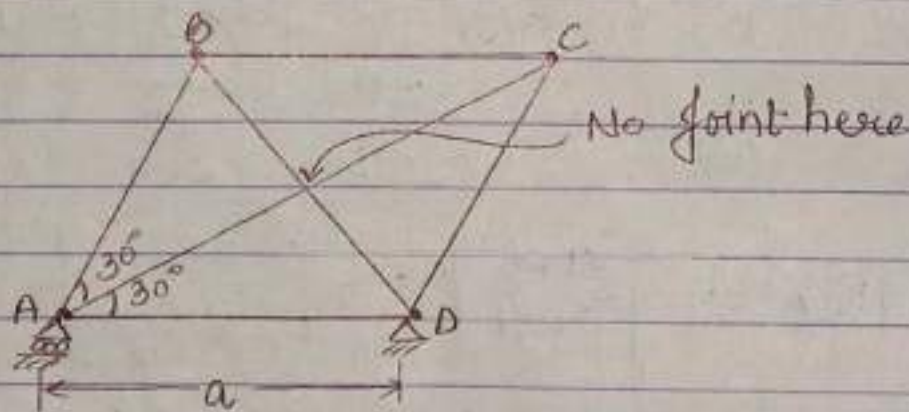
→ Since members OA & OC are present, the relative joints displacement of A & C will be the total contraction of the members OA & OC.

$$\delta_{AC} = \left(\frac{PL}{AE} \right)_{OA} + \left(\frac{PL}{AE} \right)_{OC} = \left[\frac{(0.414P) \times \frac{a}{\sqrt{2}}}{AE} \right] \times 2$$

$$\boxed{\delta_{AC} = \frac{0.585 Pa}{AE}}^{**}$$

Ques:- {20 Marks}

When fabricating the pinned jointed truss shown in fig. it is found that the member AC is fabricated too long. Determine the forces in the members after assembly. AE is const. for all members.



Sol:- 1st Step :-

$$D_{sc} = r - s = 3 - 3 = 0$$

$$D_{si} = m - (2j - 3) = 6 - (2 \times 4 - 3) = 1 \text{ degree}$$

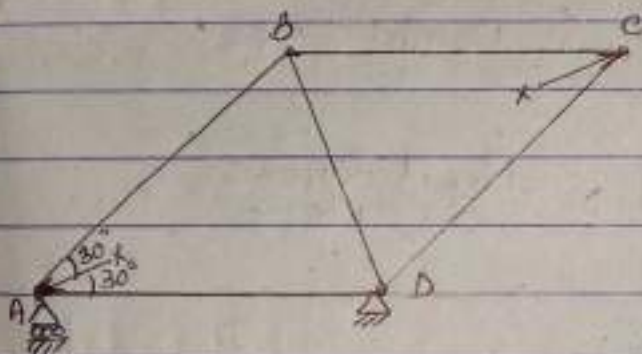
NOTE :- 1. If the truss is statically determinate truss, force will not be developed in any member due to lack of fit.

→ Since the truss is indeterminate truss, forces will be developed in all members due to lack of fit.

2. Since the member is too long it must be compressed & kept in position,

So compressive force is developed in 'AC'.

Remove the redundant member AC & replace it by its compressive force 'x' as shown in fig.



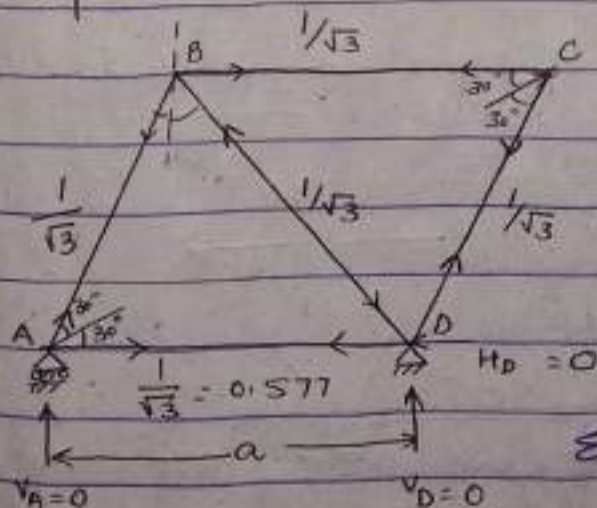
$$U = \frac{\sum (P + kx)^2 l}{2AE}$$

Compatibility Condition

$$S_{AC} = \delta = \frac{\partial U}{\partial X} = \sum \frac{2(Kx)k \cdot l}{2AE} = \sum \frac{k^2 l}{AE} \times$$

So,	$x = f_{AC} = \frac{\delta (AE)}{\sum K^2 L}$
-----	---

2nd Step: 'K' values :-



Joint A'

$$\sum y = 0$$

$$-1 \sin 30^\circ + F_{AB} \sin 60^\circ = 0$$

$$F_{AB} = \frac{\sin 30^\circ}{\sin 60^\circ} = 0.577 = \frac{1}{\sqrt{3}} \text{ (T)}$$

$$\sum x = 0 \Rightarrow +\frac{1}{3} \cos 60^\circ - \cos 30^\circ + \cos 0^\circ$$

$$F_{AD} = \frac{1}{\sqrt{3}} (T) (0.577)$$

At joint 'B'

From $\sum y = 0$ Consideration,

$$F_{BD} = F_{BA} = \frac{1}{\sqrt{3}}$$

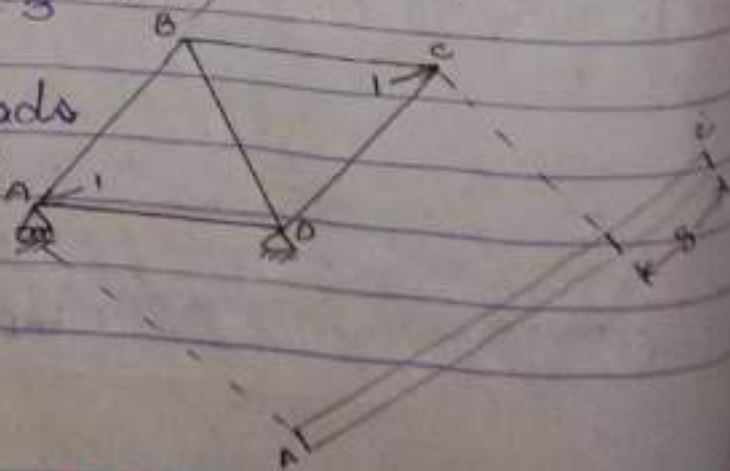
3rd Step :-

$$F_{AC} = \frac{S(AE)}{\sum k^2 l} \quad \begin{array}{l} \text{Tension +ve} \\ \text{Comp. -ve} \end{array}$$

Member	K	$K^2 l$	Final Forces = 0 $F = P + Kx$
AB	$1/\sqrt{3}$	$a/3$	
BC	$+1/\sqrt{3}$	$a/3$	
CD	$+1/\sqrt{3}$	$a/3$	
DA	$+1/\sqrt{3}$	$a/3$	
BD	$-1/\sqrt{3}$	$a/3$	
AC	-1	$\sqrt{3} \cdot a$	
		$\frac{5a}{3} + \sqrt{3}a$	
		$\sum k^2 l$	

$$F_{AC} = x = \frac{S(AE)}{\sum k^2 l} = \frac{S \cdot (A \cdot E)}{\left(\frac{5a}{3} + \sqrt{3}a\right)} = \frac{0.294 AE S}{a}$$

[Since 'S' and unit loads are in same dirⁿ, S_A is taken as +ve]



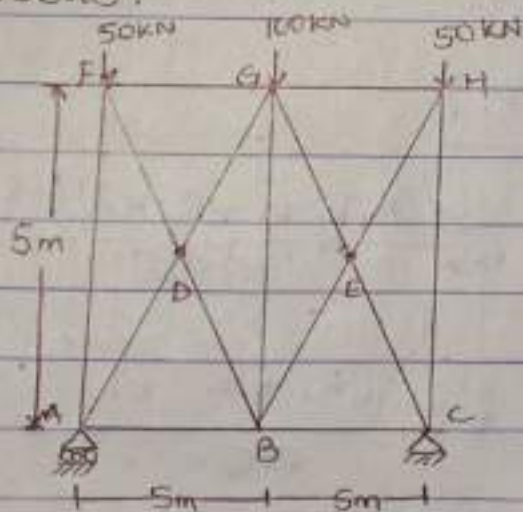
+ve sign indicates FAC acts in the dirⁿ of unit loads.

So,

$$F_{AC} = \frac{0.294 \text{ AES}}{a} \text{ (comp)}$$

Ques:- {20 Marks}

Find forces in members of truss as shown in fig. Assume AE is constant for all members.

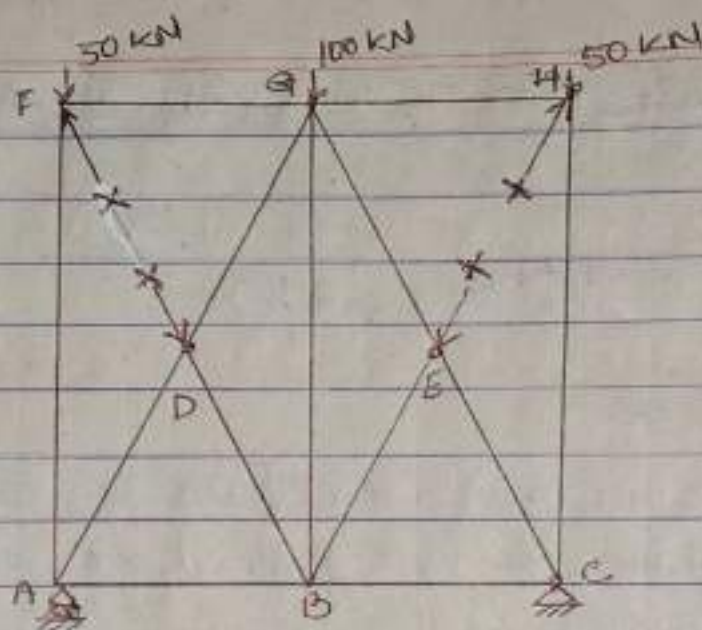


Sol:- 1st Step :-

$$D_{se} = r - s = 3 - 3 = 0$$

$$D_{si} = m - (2j - 3) = 15 - (2 \times 8 - 3) = 2 \text{ degree.}$$

→ Since the truss is internally indeterminate to two degrees, Remove 2-members FD & HE symmetrically & replace them by their forces 'x' as shown in fig { because of symmetry of structure & loading F_{FD} & F_{HE} are equal }



$$U = \frac{\sum (P + k_1 x + k_2 x)}{2AE}$$

$$\delta_{FD} = \frac{\partial U}{\partial x} = 0 \Rightarrow \sum \frac{\partial (P + k_1 x + k_2 x) (k_1 + k_2) \cdot l}{2AE}$$

$$= \sum \frac{P(k_1 + k_2)l}{AE} + \sum \frac{(k_1 + k_2)^2 x l}{AE} = 0$$

$$x = - \frac{\sum P(k_1 + k_2) \cdot l}{\sum (k_1 + k_2)^2 l} \quad \text{Compatibility Condition}$$

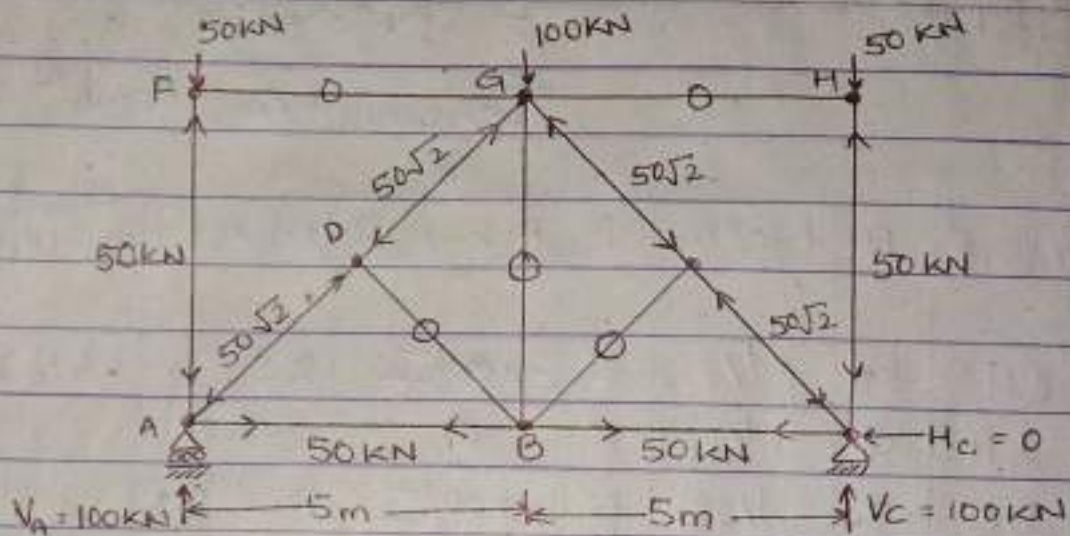
where, k_1 = Forces in all members due to unit loads at F and D

k_2 = Forces in all members due to unit loads at H and E.

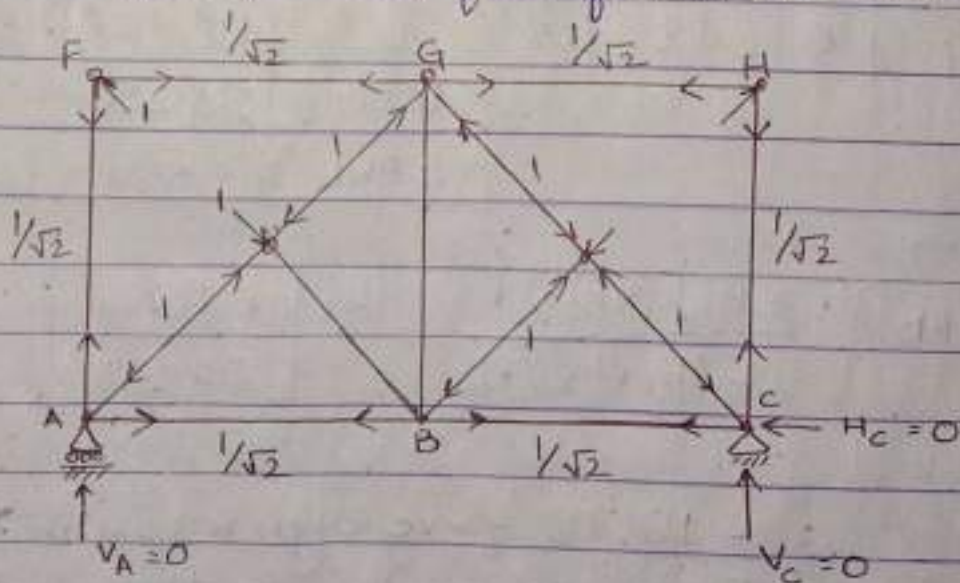
2nd step:-

P values { forces in all members after 2 redundant members are

Removed }



3rd Step: $\{ (K_1 + K_2) \text{ values} \}$ Apply unit loads at F, D, H, E and find forces in all members.



Joint 'B':-

$$\sum F_y = 0 \Rightarrow -1 \cos 45^\circ - 1 \cos 45^\circ + F_{BG} = 0$$

$$F_{BG} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ (T)}$$

4th Step :- $x = F_{FD} = \frac{-\sum P(k_1 + k_2) \cdot l}{\sum (k_1 + k_2)^2 \cdot l}$

Tension +ve, Compⁿ -ve

Members	P	$k_1 + k_2$	l	$P(k_1 + k_2)l$	$(k_1 + k_2)^2 \cdot l$	Final forces $F = P + (k_1 + k_2)x$
AF, CH	-50	$+1/\sqrt{2}$	5	$-250/\sqrt{2}$	2.5	-63.27 kN
FG, GH	0	$1/\sqrt{2}$	5	0	2.5	
AB, BC	+50	$1/\sqrt{2}$	5	$+200/\sqrt{2}$	2.5	
AG, GC	$-50\sqrt{2}$	-1	$5\sqrt{2}$	+500	$5\sqrt{2}$	+18.78 kN(T)
BD, BE	0	-1	$2.5\sqrt{2}$	0	$2.5\sqrt{2}$	
FB, HE	0	-1	$2.5\sqrt{2}$	0	$2.5\sqrt{2}$	
BG	0	$\sqrt{2}$	5	0	10	.
				1000	53.25	
				$\sum PkL$	$\sum k^2L$	

$$x = F_{FD} = - \frac{\sum P(k_1 + k_2) l}{\sum (k_1 + k_2)^2 \cdot l} = - \left[\frac{+1000}{53.25} \right]$$

= -18.78 { -ve sign implies that F_{FD} is opposite to the dirⁿ of unit load }.

$$F_{FD} = 18.78 \text{ kN (T)}$$

All other members :-

$$F_{AF} = P + (K_1 + K_2) x$$

$$= -50 + \frac{1}{\sqrt{2}} \times (-18.78) = -63.27 \text{ kN.}$$