What is TOM

The subject theory of machine may be defined as that branch of engineering science which deals with the study of relative motion both the various parts of m/c and forces which act on them.

The theory of m/c may be sub divided into the following branches:

1. **Kinemics:** It deals with the relative motion between the various parts of the machine

2. **Dynamics:** It deals with the force and their effects, while acting upon the m/c part in motion.

   Resistance Body: Resistant bodies are those which do not suffer appreciable distortion or change in physical form by the force acting on them e.g., spring, belt.

   Kinematic Link Element: A resistant body which is a part of an m/c and has motion relative to the other connected parts is **term as link.**

   A link may consist of one or more resistant bodies. Thus a link may consist of a number of parts connected in such away that they form one unit and have no relative motion to each other.

- **A link should have the following two characteristics:**
  1. It should have relative motion, and
  2. It must be a resistant body.

Functions of Linkages

The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or *vice versa*. Stated more specifically linkages may be used to convert:

1. Continuous rotation into continuous rotation, with a constant or variable angular velocity ratio.
2. Continuous rotation into oscillation or reciprocation (or the reverse), with a constant or variable velocity ratio.
3. Oscillation into oscillation, or reciprocation into reciprocation, with a constant or variable velocity ratio.

Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

- **Function generation**: the relative motion between the links connected to the frame,
- **Path generation**: the path of a tracer point, or
- **Motion generation**: the motion of the coupler link.

Types

1. **Rigid Link:** It is one which does not undergo any deformation while transmitting motion–C.R, etc.

2. **Flexible Link:** Partly deformed while transmitting motion–**spring, belts.**

3. **Fluid Link:** It formed by having the motion which is transmitted through the fluid by pressure. e. g, hydraulic press, hydraulic brakes.

Kinematic Pair

Two element or links which are connected together in such a way that their relative motion is completely or successfully constrained form a kinematic pair. i.e. The term kinematic pairs actually refer to kinematic constraints between rigid bodies.

The kinematic pairs are divided into **lower pairs** and **higher pairs**, depending on how the two bodies are in contact.

- **Lower Pair:** When two elements have **surface contact** while in motion.
- **Higher Pair:** When two elements have **point or line of contact** while in motion.

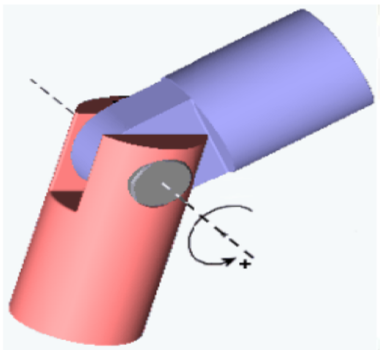
Lower Pairs

A pair is said to be a lower pair when the connection between two elements is through the area of contact. Its 6 types are:

- **Revolute Pair**
- **Prismatic Pair**
- **Screw Pair**
- **Cylindrical Pair**
- **Spherical Pair**
- **Planar Pair.**

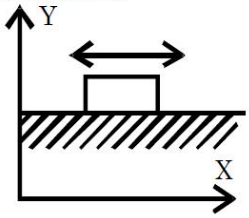
Revolute Pair

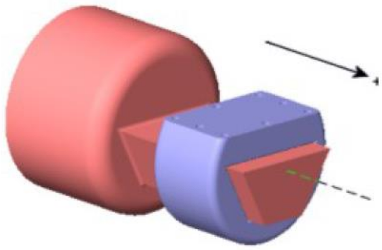
A revolute allows only a relative rotation between elements 1 and 2, which can be expressed by a single coordinate angle 'θ' .Thus a revolute pair has a single degree of freedom.



Prismatic Pair

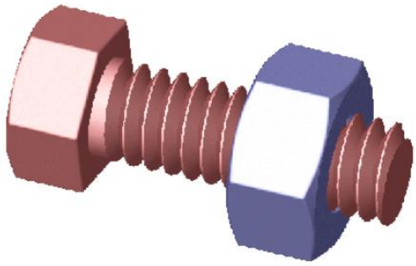
A prismatic pair allows only a relative translation between elements 1 and 2, which can be expressed by a single coordinate 'x'. Thus a prismatic pair has a single degree of freedom.





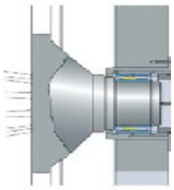
Screw Pair

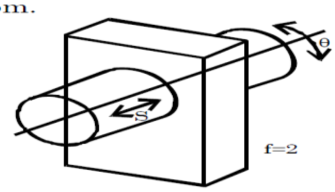
A screw pair allows only a relative movement between elements 1 and 2, which can be expressed by a single coordinate angle 'θ' or 'x'. Thus a screw pair has a single degree of freedom. Example-lead screw and nut of lathe, **screw jack.**



Cylindrical Pair

A cylindrical pair allows both rotation and translation between elements 1 and 2, which can be expressed as two independent coordinate angle 'θ' and 'x'. Thus a cylindrical pair has two degrees of freedom.

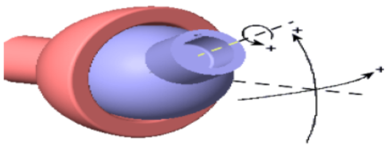


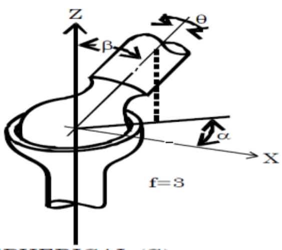


CYLINDRICAL (C)

Spherical Pair

A spherical pair allows three degrees of freedom since the complete description of relative movement between the connected elements needs three independent coordinates. Two of the coordinates 'α' and 'β' are required to specify the position of the axis OA and the third coordinate 'θ' describes the rotation about the axis OA. e.g. – **Mirror attachment of motor cycle.**

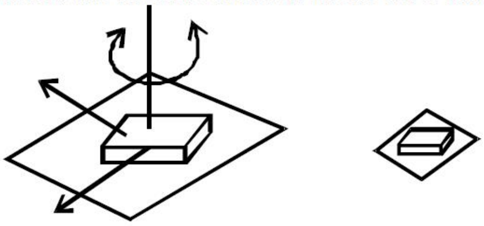


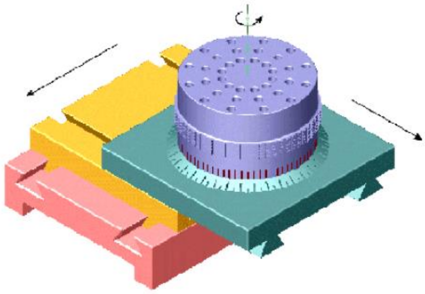


SPHERICAL (G)

Planar Pair

A planar pair allows three degrees of freedom. Two coordinates x and y describe the relative translation in the xy-plane and the third 'θ' describes the relative rotation about the z-axis.





Higher Pairs

A higher pair is defined as one in which the connection between two elements has only **a point or line of contact**. A cylinder and a hole of equal radius and with axis parallel make contact along a surface. Two cylinders with unequal radius and with axis parallel make contact along a line. A point contact takes place when spheres rest on plane or curved surfaces (ball bearings) or between teeth of a skew-helical gears. In roller bearings, between teeth of most of the gears and in cam-follower motion. The degree of freedom of a kinetic pair is given by the number independent coordinates required to completely specify the relative movement.

Wrapping Pairs

Wrapping Pairs comprise belts, chains, and other such devices

Sliding Pair

When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show that a sliding pair has a completely constrained motion.

Turning pair

When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

Rolling pair: When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

According to mechanical constraint between the elements:

1. **Closed Pair:** When two elements of a pair are held together mechanically. e.g., all lower pair and some of higher pair.
2. **Unclosed Pair** (Open Pair): When two elements of a pair are not held together mechanically. e.g., cam and follower.

Kinematic Constraints

Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with **kinematic constraints**. *Kinematic constraints* are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

Types of Constrained Motions

Following are the three types of constrained motions:

1. **Completely constrained motion;** When motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in Fig.5.1

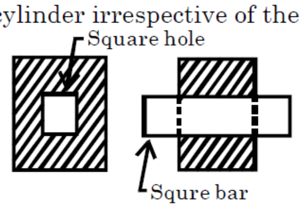


Fig. 5.2 Square bar in a square hole

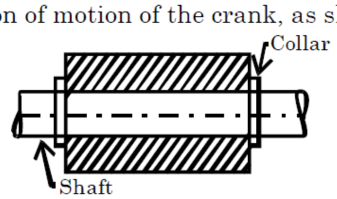


Fig. 5.3 Shaft with collar in a circular hole.

The motion of a square bar in a square hole, as shown in Fig.5.2, and the motion of a shaft with collars at each in a circular hole, as shown in Fig. 5.3, are also examples of completely constrained motion.

2. Incompletely constrained motion: When the motion between a pair can take place in more than one direction, then the motion is called an incomplete constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 5.4., is an incompletely constrained motion as it may either rotate or slide in a hole . These both motions have no relationship with the other.

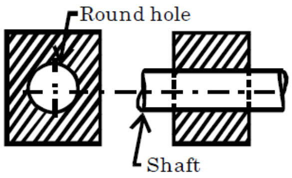


Fig. 5.4 Shaft in a circular hole.

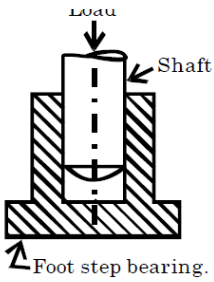


Fig. 5.4 Shaft in a foot step bearing.

3. Successfully constrained motion: When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a **foot-step bearing** as shown in Fig 5.5. The shaft may rotate in a bearing or it may rotate in a bearing or it may upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion. But if the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Kinematic chain

A kinematic chain is a **series of links** connected by kinematic pairs. The chain is said to be closed chain if every link is connected to at least two other links, otherwise it is called an open chain. A link which is connected to only one other link is known as singular link. If it is connected to two other links, it is called binary link. If it is connected to three other links, it is called ternary link, and so on. A chain which consists of only binary links is called simple chain.

If each link is assumed to form two pairs with two adjacent links, then relation between the No. of pairs (p) formatting a kinematic chain and the number of links (l) may be expressed in the from of an equation:

$$\mathbf{l = 2p - 4}$$

Since in a kinematic chain each link forms a part of two pairs, therefore there will be as many links as the number of pairs.

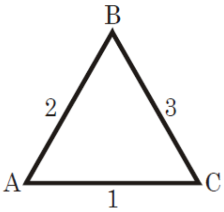
Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression:

$$j = \frac{3}{2} l - 2$$

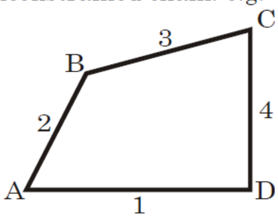
Where,
l = no. of links

p = no. of pairs
j = no. of binary joints.

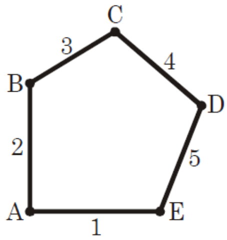
If L.H.S. > R.H.S. ⇒ Structure
L.H.S. = R.H.S. ⇒ Constrained chain
L.H.S. < R.H.S. ⇒ Unconstrained chain. e.g.



Structure



Constrained chain

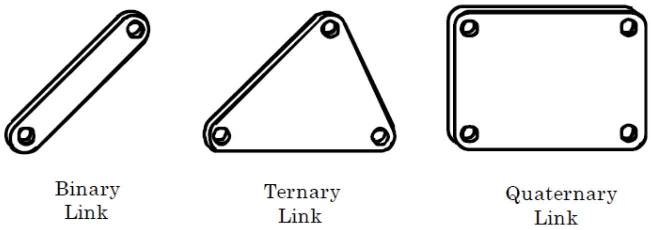


Unconstrained chain

Types of Joins

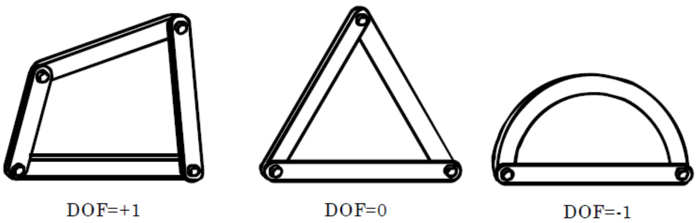
1. Binary Joint
2. Ternary Joint
3. Quaternary Joint.

Links, Joints and Kinematic Chains



Every link must have nodes. The number of nodes defines the type of link.

- **Binary link** - One link with two nodes
- **Ternary link** - One link with three nodes
- **Quaternary link** - One link with four nodes



Now that we have defined degrees of freedom, We can look at the illustration above and determine the degrees of freedom of each.

	Joint Type	DOF	Description
A	First order pin joint	1	two binary links joined at a common point
B	Second order pin joint	2	three binary links joined at a common point
C	Half Joint	1 *or* 2	Rolling or sliding *or* both

Mechanism

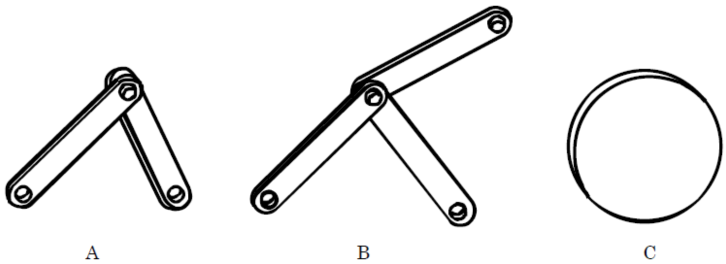
Mechanism: When one of the link of a kinematic chain is fixed, it will be a mechanism. If the different link of the same kinematic chain is fixed, the result is a different mechanism. The primary function of a mechanism is to transmit or modify motion.

Machine: When a mechanism is required to transmit power or to do some particular kind of work it is known as a machine.

Structure: An assemblage of resistant bodies having no relative motion between them and meant for carrying load having straining action called structure.

Inversions: Mechanism is one in which one of the link of kinematic chain is fixed. Different mechanism are formed by fixing different link of the same kinematic chain are known as inversions of each other.

Mechanisms and Structures



- A **mechanism** is defined by the number of **positive** degrees of freedom. If the assembly has zero or negative degrees of freedom it is a structure.
- A **structure** is an assembly that has **zero** degrees of freedom. An assembly with **negative** degrees of freedom is a structure with residual stresses.

Degrees of freedom

Degree of Freedom: It is the **number of independent variables** that must be specified to define completely the condition of the system.
A kinematic chain is said to be movable when its d.o.f. ≥ 1 otherwise it will be locked. If the d.o.f. is 1 the chain is said to be constrained.

Figure4-1 shows a rigid body in a plane. To determine the DOF this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be translated along the x axis, translated along the y axis, and rotated about its central.

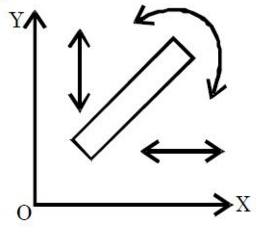


Figure 4-1 Degrees of freedom of a rigid body in a plane

Consider a pencil on a table. If the corner of the table was used as a reference point, two independent variables will be required to fully define its position. Either an X-Y coordinate of an endpoint and an angle or two X coordinates *or* two Y coordinates. No single variable by itself can never fully define its position. Therefore the system has two degrees of freedom.

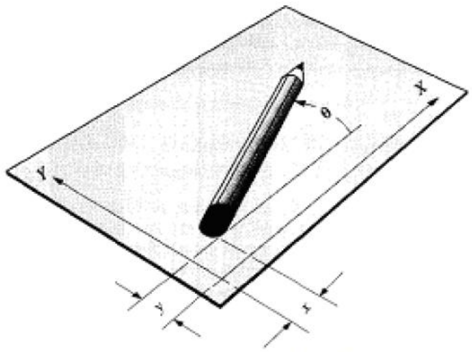


Fig. Degree of freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions Along the x, y and z axes and three rotary motions around the x, y and z axes respectively.

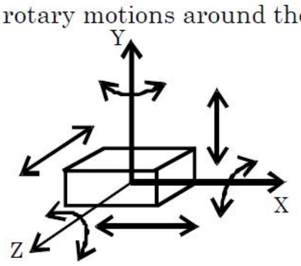


Figure 4-2 Degrees of freedom of a rigid body in space

Unconstrained rigid body in space possesses 6 d.o.f.

Joint/Pair	D.O.F.	Variable
Pin Joint	1	θ
Sliding Joint	1	S
Screw Pair	1	θ or S
Cost. Pair	2	θ, S
Spherical Pair	3	θ, j, ψ
Planar Pair	3	xy θ

→ Revolute Pair

Kutzbach criterion

The number of degrees of freedom of a mechanism is also called the mobility of the device. The **mobility** is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The **Kutzbach criterion** calculates the mobility.

In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism.

For example, Figure shows several cases of a rigid body constrained by different kinds of pairs.

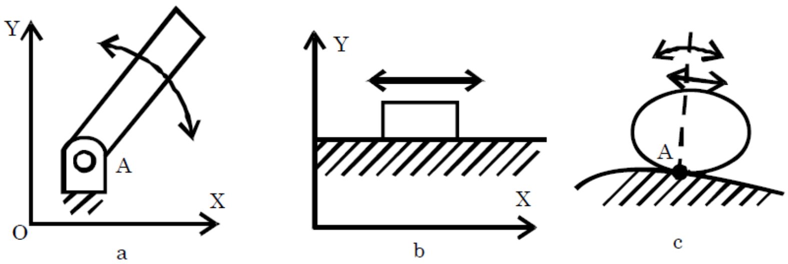


Figure. Rigid bodies constrained by different kinds of planar pairs

In Figure-a, a rigid body is constrained by a **revolute pair** which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the x and y axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure -b, a rigid body is constrained by a **prismatic pair** which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the x axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the y axis.

In Figure-c, a rigid body is constrained by a **higher pair.** It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

Now let us consider a plane mechanism with / number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be (l-1) and thus the total number of degrees of freedom will be 3 (l-1) before they are connected to any other link. In general, a mechanism with l number of links connected by j number of binary joints or lower pairs (i.e. single degree of freedom pairs) and h number of higher pairs (i.e. two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$$n = 3(l - 1) - 2j - h$$

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.
If there are no two degree of freedom pairs (i.e. higher pairs), then h = 0. Substituting h= 0 in equation (i), we have

$$n = 3(l - 1) - 2j$$

Where,

n = degree of freedom
l = no. of link
j = no. of joints/no. of lower pair
h = no. of higher pair.

In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a, b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.

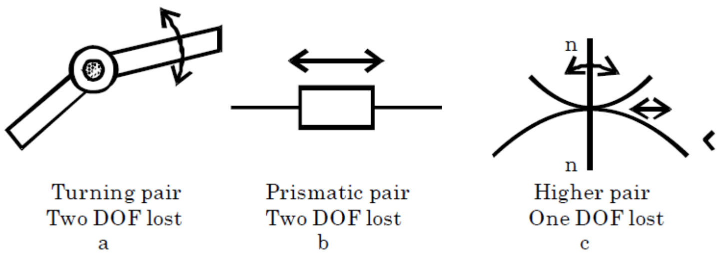


Figure. Kinematic Pairs in Planar Mechanisms

Grubler Criterion

Grubler's Criterion for Plane Mechanisms

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting n = 1 and h = 0 in Kutzbach equation, we have

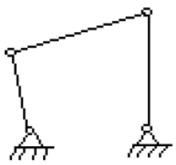
$$1 = 3(l-1)-2j \quad \text{or} \quad 3l-2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints can not have odd number of links. The simplest possible mechanisms of this type are a four bar mechanism and a slider – crank mechanism in which l = 4 and j = 4.

Grashof 's law

In the range of planar mechanisms, the simplest groups of lower pair mechanisms are four bar linkages. A **four bar linkage** comprises four bar-shaped links and four turning pairs as shown in Figure 5-8.



The link opposite the frame is called the **coupler link**, and the links which are hinged to the frame are called **side links**. A link which is free to rotate through 360 degree with respect to a second link will be said to **revolve** relative to the second link (not necessarily a frame). If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.

Some important concepts in link mechanisms are:

1. **Crank**: A side link which revolves relative to the frame is called a *crank*.
2. **Rocker**: Any link which does not revolve is called a *rocker*.
3. **Crank-rocker mechanism**: In a four bar linkage, if the shorter side link revolves and the other one rocks (*i.e.*, oscillates), it is called a *crank-rocker mechanism*.
4. **Double-crank mechanism**: In a four bar linkage, if both of the side links revolve, it is called a *double-crank mechanism*.
5. **Double-rocker mechanism**: In a four bar linkage, if both of the side links rock, it is called a *double-rocker mechanism*.

Classification

Before classifying four-bar linkages, we need to introduce some basic nomenclature.

In a four-bar linkage, we refer to the *line segment between hinges* on a given link as a **bar** where:

- s = length of shortest bar
- l = length of longest bar
- p, q = lengths of intermediate bar

Grashof's theorem states that a four-bar mechanism has *at least* one revolving link if

$$\mathbf{s + l \leq p + q}$$

and all three mobile links will rock if

$$s + l > p + q$$

The inequality 5-1 is **Grashof's criterion**.

All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

Case	l + s vers. p + q	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Double rocker
4	=	Any	Change point

5	>	Any	Double-rocker

Table: Classification of Four-Bar Mechanisms

From Table we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links. However, this condition is necessary but not sufficient. Mechanisms satisfying this condition fall into the following three categories:

1. When the shortest link is a side link, the mechanism is a crank-rocker mechanism. The shortest link is the crank in the mechanism.
2. When the shortest link is the frame of the mechanism, the mechanism is a double-crank mechanism.
3. When the shortest link is the coupler link, the mechanism is a double-rocker mechanism.

Inversion of Mechanism

Method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of the mechanism.

Inversion is a term used in kinematics for a reversal or interchanges of form or function as applied to kinematic chains and mechanisms.

Types of Kinematic Chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

1. Four bar chain or quadric cyclic chain,

2. Single slider crank chain, and

3. Double slider crank chain.

For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure 5-14a can be inverted into the mechanisms shown in Figure 5-14b, c, and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in Figure 5-15 is the same as that in Figure 5-14b.

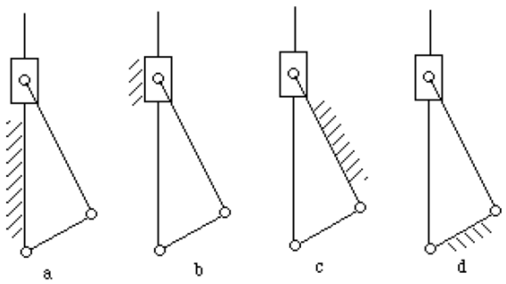


Figure: Inversions of the crank-slide mechanism

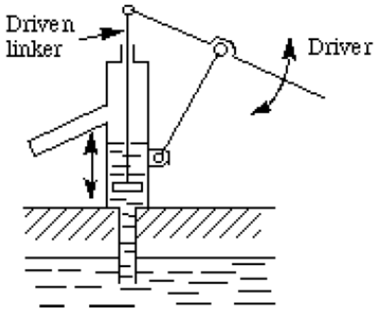


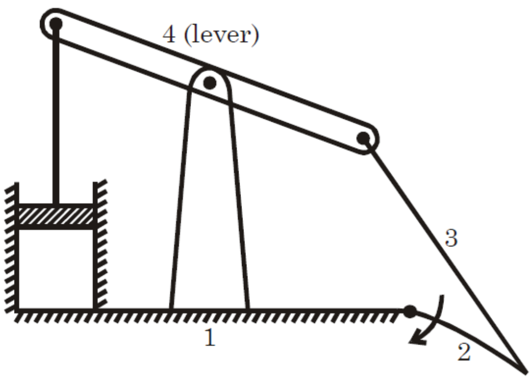
Figure. A pump device

- Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

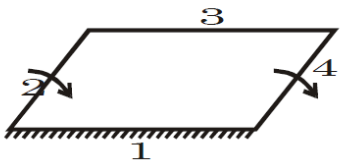
- Inversion of a kinematic chain has no effect on the relative motion of its links.
- The motion of links in a kinematic chain relative to some other links is a property of the chain and is not that of the mechanism.
- For **L** number of links in a mechanism, the number of possible inversions is equal to L.

1. Inversion of four bar chain

(a) Crank and lever mechanism/Beam engine (1st inversion).



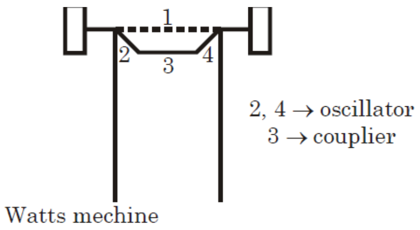
(b) Double crank mechanism (Locomotive mechanism) 2nd inversion.



(c) Double lever mechanism (Ackermann steering) 3rd inversion.

2, 4 → Oscillator

3 → Coupler



2. Inversion of the Slider-Crank Mechanism

Inversion is a term used in kinematics for a reversal or interchanges of form or function as applied to kinematic chains and mechanisms. For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure (a) can be inverted into the mechanisms shown in Figure (b), (c), and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in Figure is the same as that in Figure (b).

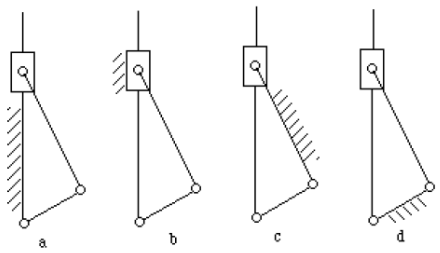


Figure. Inversions of the crank-slide mechanism

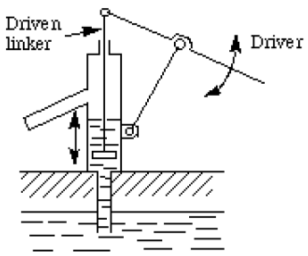
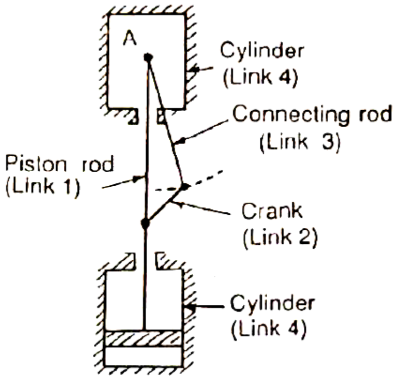


Figure. A pump device

Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

1. Pendulum pump or Bull engine: In **this** mechanism the inversion is obtained by fixing cylinder **or link4 (i.e.** sliding **pair), as** shown in figure below.



The **duplex** pump which is used to supply feed water to boilers uses this mechanism.

2. Oscillating cylinder engine: It is used to convert reciprocating motion into rotary motion.

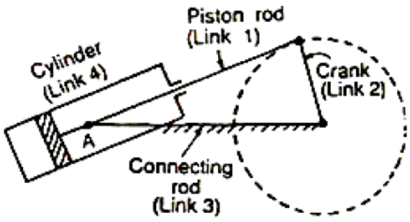


Fig. Oscillating cylinder engine

3. Rotary internal combustion engine or Gnome engine:

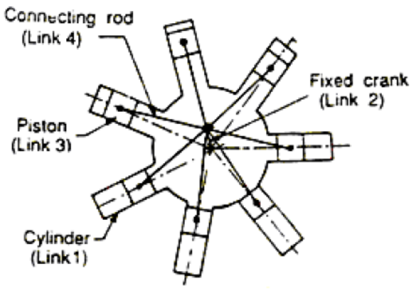
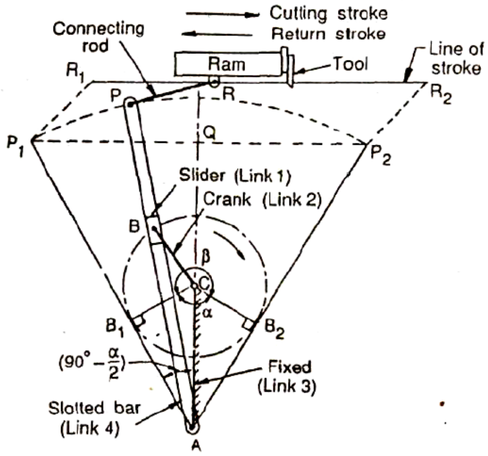


Fig. Rotary internal combustion engine

Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place.

Quick return motion mechanism

4. Crank and slotted lever quick return motion mechanism: This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \text{ or } \frac{360° - \alpha}{\alpha}$$

Length of stroke

$$= 2\ AP \times \frac{CB}{AC}$$

Note: We see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

5. Whitworth quick return motion mechanism:
This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) formatting the turning pair is fixed, as shown in Figure. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar *PA* (link 1) which oscillates at a pivoted point *D*. The connecting rod *PR* carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line *RD* produced, i.e. along a line passing through *D* perpendicular to *CD*.

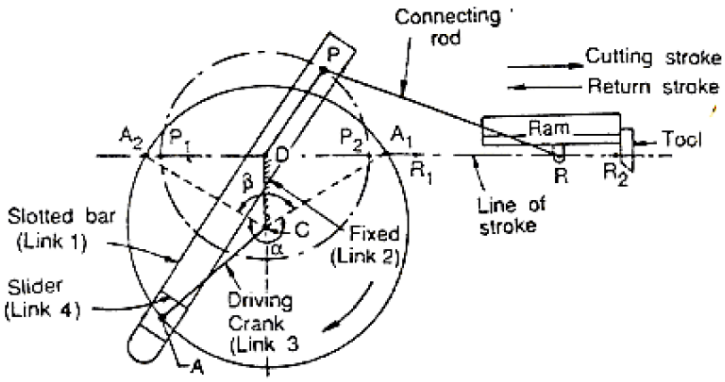


Fig. Whitworth quick return motion mechanism

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \text{ or } \frac{360° - \beta}{\beta}$$

Note: In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 P_2 = PR$. The length of effective stroke is also equal to 2 PD

Inversion of Double slider crank chain

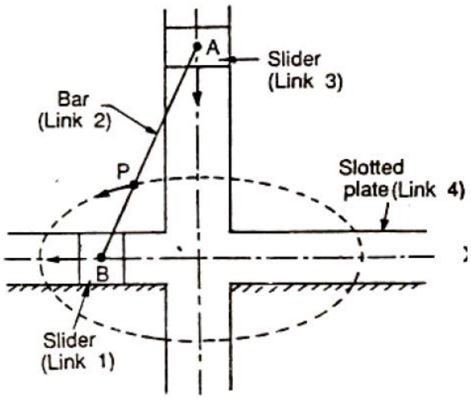
It has four binary links, two revolute pairs, two sliding pairs.
Its various types are:

- Elliptical Trammel
- Scotch Yoke mechanism
- Oldham's coupling.

Elliptical trammels

It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Figure. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link *AB* (link 2) is a bar which forms turning pair with links 1 and 3.

When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as *P* traces out an ellipse on the surface of link 4, as shown in Figure (*a*). A little consideration will show that *AP* and *BP* are the semi-major axis and semi-major axis of the ellipse respectively. This can be proved as follows:



Note: If *P* is the mid-point of link *BA*, then *AP* = *BP*.' Hence if P is the midpoint of link *BA*, it will trace a circle.

Scotch yoke mechanism

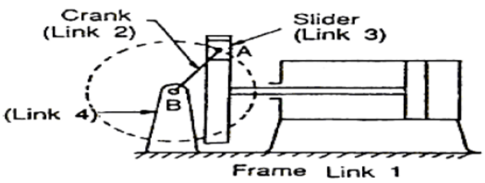
Here the constant rotation of the crank produces harmonic translation of the yoke. Its four binary links are:

1. Fixed Link
2. Crank
3. Sliding Block
4. Yoke

The four kinematic pairs are:

1. revolute pair (between 1 & 2)
2. revolute pair (between 2 & 3)
3. prismatic pair (between 3 & 4)
4. prismatic pair (between 4 & 1)

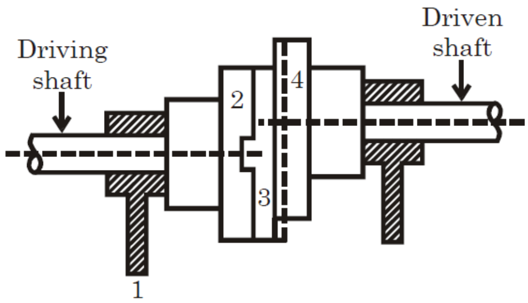
This mechanism is used for converting rotary motion into a reciprocating motion.

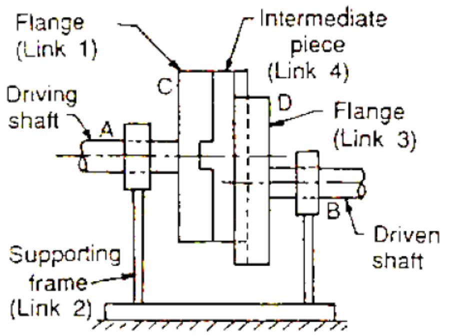


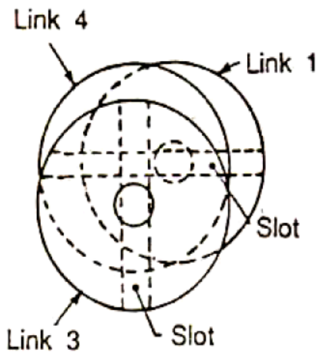
(e)

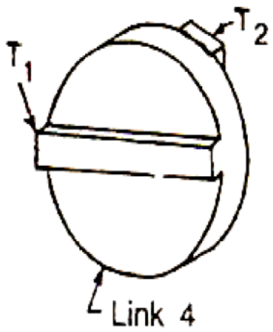
Oldham's coupling

- It is used for transmitting angular velocity between two parallel but eccentric shafts.
- An Oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart.
- Oldham's coupling is the inversion of double slider crank mechanism.
- The shafts are coupled in such a way that if one shaft rotates the other shaft also rotates at the same speed.









- The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces.
- The intermediate piece (link 4) which is a circular disc, have two tongues (i.e. diametrical projections) T_1 and T_2 on each face at right angles to each other.
- The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3).
- The link 4 can slide or reciprocate in the slots in the flanges.

Let the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Then the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

:. **Maximum** sliding' speed of each tongue (in m/s),

$$v = \omega \cdot r$$

Where ω = Angular velocity of each shaft in rad/s, and
r = Distance between the axes of the shafts in metres.

Hooke's Joint (Universal Coupling)

This joint is used to connect two non-parallel intersecting shafts. It also used for shafts with angular misalignment where flexible coupling does not serve the purpose. Thus Hooke's Joint connecting two rotating shafts whose axes lies in one plane.

Velocity

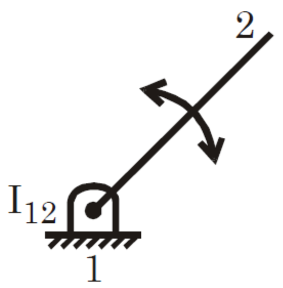
The concept of velocity and acceleration images is used extensively in the kinematic analysis of mechanisms having ternary, quaternary, and higher- order links. If the velocities and accelerations of any two points on a link are known, then, with the help of images the velocity and acceleration of any other point on the link can be easily determined. An example is

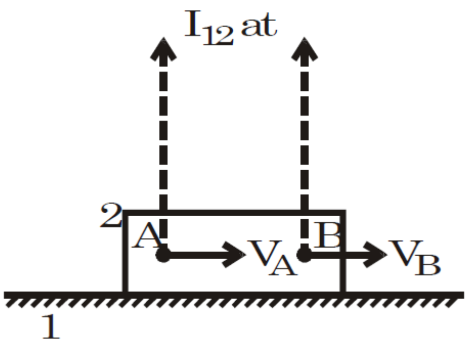
1. Instantaneous Centre Method
2. Relative Velocity Method

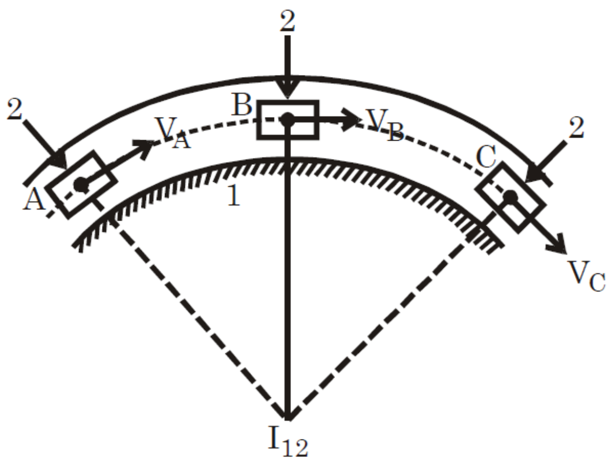
Velocity by Instantaneous Centre Method

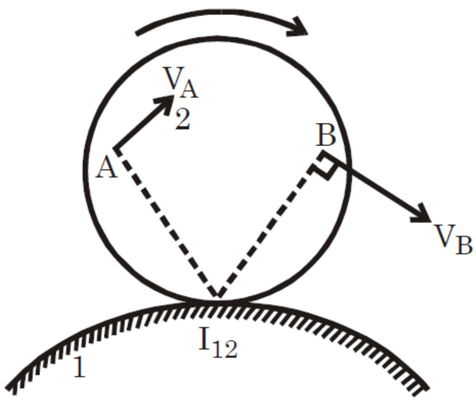
Instantaneous centre is one point about which the body has pure rotation. Hence for the body which having straight line motion, the radius of curvature of it is at infinity and hence instantaneous centre of this ties at infinite.

Special cases of ICR

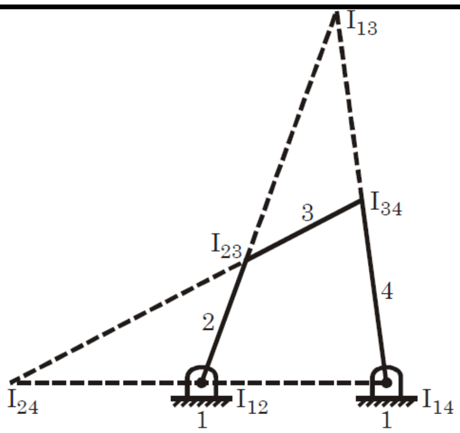








Types of ICR:



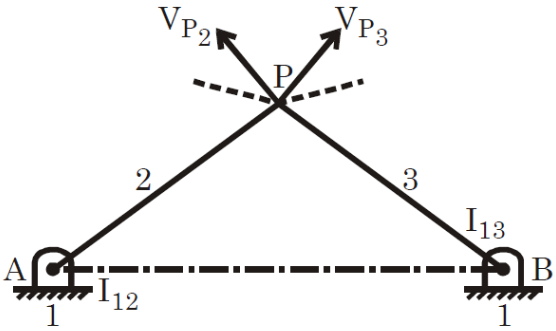
(i) Fixed ICR: I_{12}, I_{14}

(ii) Permanent ICR: I_{23}, I_{34}

(iii) Neither Fixed nor Permanent I.C: I_{13}, I_{24}

Three-Centre-in-line Theorem (Kennedy's Theorem)

Kennedy Theorem states that "If three links have relative motion with respect to each other, their relative instantaneous centre lies on straight line".



The Theorem can be proved by contradiction.
The Kennedy Theorem states that the three IC I_{12}, I_{13}, I_{23} must all lie on the same straight line on the line connecting two pins.

Let us suppose this is not true and I_{23} is located at the point P. Then the velocity of P as a point on link 2 must have the direction V_{P_2}, $\perp$ to AP. Also the velocity of P as a point on link 3 must have the direction V_{P_3}, $\perp$ to BP. The direction is inconsistent with the definition that an instantaneous centre must have equal absolute velocity as a part of either link. The point P chosen therefore, cannot be the IC I_{23}.
This same contradiction in the direction of V_{P_2} and V_{P_3} occurs for any location chosen for point P, except the position of P chosen on the straight line passing through I_{12} and I_{13}. This justify the Kennedy Theorem.

Properties of the IC:

1. A rigid link rotates instantaneously relative to another link at the instantaneously centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. In other words, the velocity of the IC relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link.

Number of I.C in a mechanism:

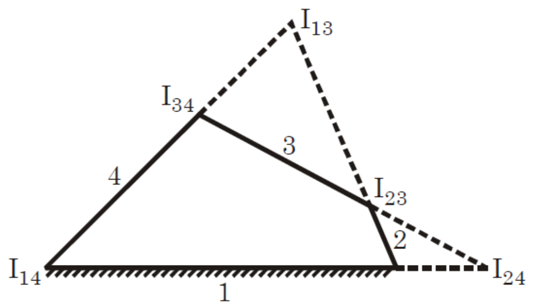
$$N = \frac{n(n-1)}{2}$$

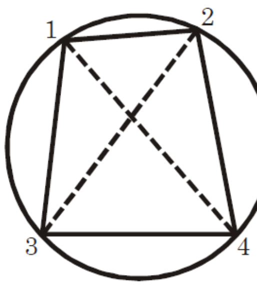
N = no. of I.C.
n = no. of links.

1. Each configuration of the link has one centre.
   The instantaneous centre changes with alteration of configuration of mechanism.

Method of locating instantaneous centre in mechanism

Consider a pin jointed four bar mechanism as shown in fig. The following procedure is adopted for locating instantaneous centre.





1. First of all, determine the no. of IC.

$$N = \frac{n(4-1)}{3} = \frac{4(4-1)}{2} = 6$$

2. Make a list at all the instantaneous centre in a mechanism.

Links	1	2	3	4
–	12	23	34	–
IC	13	24		
	14			

3. Locate the fixed and permanent instantaneous centre by inspection. In fig I_{12} and I_{14} are fixed I.Cs and I_{23} and I_{34} are permanent instantaneous centre locate the remaining neither fixed nor permanent IC by Kennedy's Theorem. This is done by circle diagram

as shown mark the points on a circle equal to the no. of links in mechanism. In present case 4 links.

4. Join the points by solid line to show these centres are already found. In the circle diagram these lines are 12, 23, 34, and 14 to indicate the ICs I_{12}, I_{23}, I_{34} and I_{14},

5. In order to find the other two IC, join two such points that the line joining them forms two adjacent triangles in the circle diagram. The line which is responsible for completing two triangles should be a common side to the two triangles. In fig join 1 and 3 to form triangle 123 and 341 and the instantaneous centre I_{13} will lie on the intersection of I_{12}, I_{23} and $I_{14}\, I_{34}$. similarly IC I_{24} is located.

Angular Velocity Ratio Theorem

According to this Theorem "the ratio of angular velocity of any two links moving in a constrained system is inversely proportional to the ratio of distance of their common instantaneous centre from their centre of rotation".

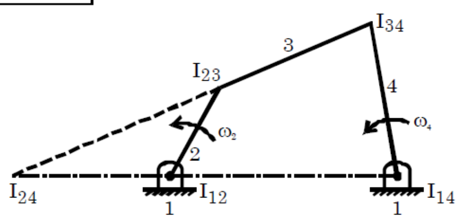
$$\frac{\omega_2}{\omega_3} = \frac{I_{13}\, I_{23}}{I_{12}\, I_{23}}$$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14}\, I_{24}}{I_{12}\, I_{24}}$$

Indices of Merit (Mechanical Advantage)

From previous concept are know that

$$\boxed{\frac{\omega_2}{\omega_4} = \frac{I_{14}\, I_{24}}{I_{12}\, I_{24}}}$$ as per angular velocity ratio Theorem.



Let T_2 represent the input torque T_4 represent the output torque. Also consider that there is no friction or inertia force.

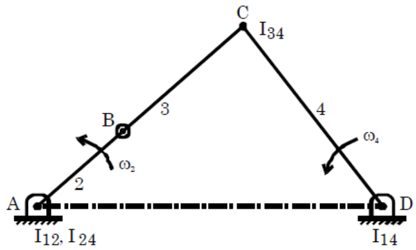
Then $T_2\omega_2 = T_4\omega_4$

−ve sign indicates that power is applied to link 2 which is negative of the power applied to link 4 by load.

$$\therefore \qquad \boxed{\frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \frac{I_{14}\, I_{24}}{I_{12}\, I_{24}}}$$

The mechanical advantage of a mechanism is the instantaneous ratio of the output force (torque) to the input force (torque). From above equation we know that mechanical advantage is the reciprocal of the velocity ratio.

Fig shows a typical position of four bar linkage in toggle, where link 2 and 3 are on the same straight line.



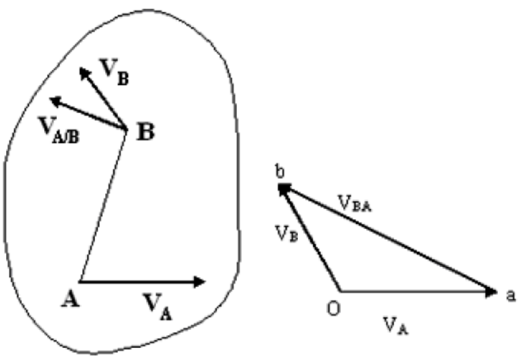
At this position, I_{12} and I_{24} is coincident at A and hence, the distance $I_{24}\,I_{24}$ is zero,

$$\therefore \quad \frac{\omega_4}{\omega_2} = \frac{I_{12}\,I_{24}}{I_{14},I_{24}} = \frac{0}{I_{14}\,I_{24}} = 0$$

$$\therefore \quad \boxed{\omega_4 = 0}$$

$$\therefore \text{ Mechanical advantage } \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \infty$$

Hence the mechanical advantage for the toggle position is infinity

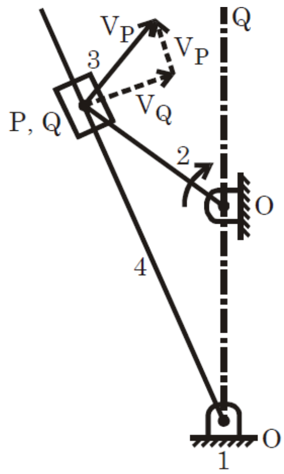


The relative velocity method is based upon the velocity of the various points of the link. Consider two points A and B on a link. Let the absolute velocity of the point A i.e. VA is known in magnitude and direction and the absolute velocity of the point B i.e. VB is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown.

1. Take some convenient point o, Known as the pole.
2. Through o, draw oa parallel and equal to VA, to some convenient scale.
3. Through a, draw a line perpendicular to AB. This line will represent the velocity of B with respect to A, i.e.
4. Through o, draw a line parallel to VB intersecting the line of VBA at b.
5. Measure ob, which gives the required velocity of point B to the scale.

1. Relative Velocity and Acceleration:

Relative velocity of coincident points in two kinematic links:

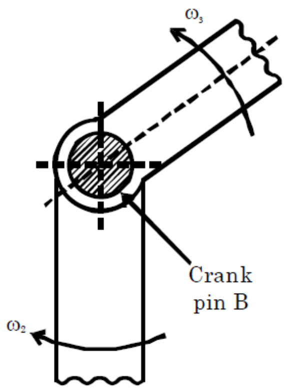


P on link 2 and 3
Q on link 4

Rubbing Velocity:

Let r_b = radius of pin B.

$\omega_{2/3}$ = relative angular velocity between link 2 and 3.



$$\boxed{\upsilon_{rub} = r_b.\omega_{2/3}}$$

$\omega_{2/3} = \omega_2 \pm \omega_3$, + for opposite rotation.

Relative Acceleration Method:

$$\boxed{f = f_c + f_t}$$

f = total acceleration

f_c = Centripetal acceleration

f_t = Tangential acceleration

$f_c = r\omega^2 = \frac{V^2}{r}$ $\qquad$ $f_t = r\alpha$

Where,
r = radius of rotation of a point on link

ω = Angular velocity of rotation

V = linear velocity of a point on link

α = Angular acceleration

Direction of f_c is along radius of rotation and towards centre.

Direction of f_t is perpendicular to radius of rotation.

Corioli's Component of Acceleration:

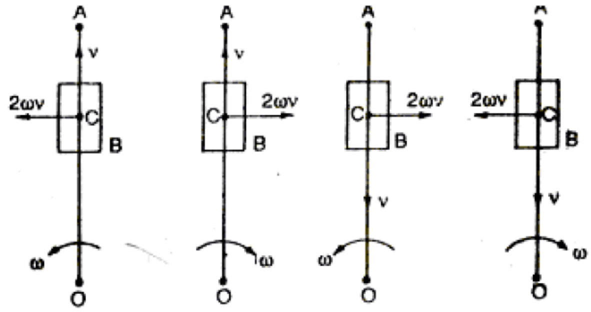


Fig. Direction of coriolis component of acceleration

This tangential component of acceleration of the slider *B* with respect to the coincident point *C* on link is known as coriolis component of acceleration and is always perpendicular to the link.

∴ Coriolis component of the acceleration of *B* with respect to *C*,

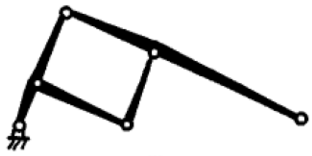
$$a^c_{BC} = a^t_{BC} = 2\omega.v$$

Where ω = Angular velocity of the link *OA*, and

V = Velocity of slider *B* with respect to coincident point *C*.

Pantograph

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.



(i)

A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc., on enlarged or reduced scales. It is, sometimes, used as an indicator rig in order to reproduce to a small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine. It is also used to guide cutting tools.

Module 4

1. ***According to the position of axes of the shafts*****.** The axes of the two shafts between which the motion is to be transmitted, may be

(*a*) Parallel, (*b*) Intersecting, and (*c*) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 12.1. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing***. These gears have teeth parallel to the axis of the wheel as shown in Fig. 12.1. Another name given to the spur gearing is ***helical gearing***, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 12.2 (*a*) and (*b*) respectively. The double helical gears are known as ***herringbone gears***. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 12.2 (*c*). These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing***. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears***.

The two non-intersecting and non-parallel *i.e.* non-coplanar shaft connected by gears is shown in Fig. 12.2 (*d*). These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing***. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids***.

Notes : (*a*) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as ***mitres***.

(*b*) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(*c*) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.

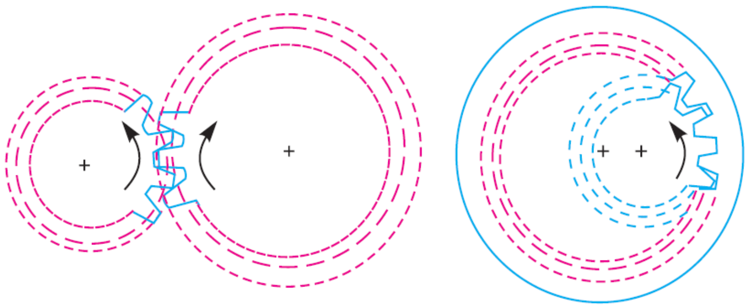
2. ***According to the peripheral velocity of the gears*****.** The gears, according to the peripheral velocity of the gears may be classified as :

(*a*) Low velocity, (*b*) Medium velocity, and (*c*) High velocity.

3. ***According to the type of gearing.*** The gears, according to the type of gearing may be classified as :

(*a*) External gearing, **(*b*)** Internal gearing, and **(*c*)** Rack and pinion.

In ***external gearing***, the gears of the two shafts mesh externally with each other as shown in Fig. 12.3 (*a*). The larger of these two wheels is called ***spur wheel*** and the smaller wheel is called **pinion.** In an external gearing, the motion of the two wheels is always ***unlike***, as shown in Fig. 12.3 (*a*).



(*a*) External gearing. (*b*) Internal gearing.

Fig. 12.3

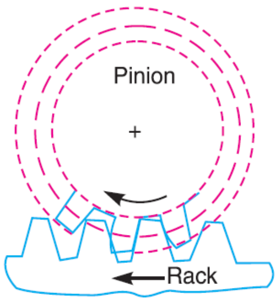


Fig. 12.4. Rack and pinion.

In ***internal gearing***, the gears of the two shafts mesh ***internally*** with each other as shown in Fig. 12.3 (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the two wheels is always ***like***, as shown in Fig. 12.3 (*b*).

4. ***According to position of teeth on the gear surface.*** The teeth on the gear surface may be **(*a*)** straight, **(*b*)** inclined, and **(*c*)** curved.

Terms Used in Gears

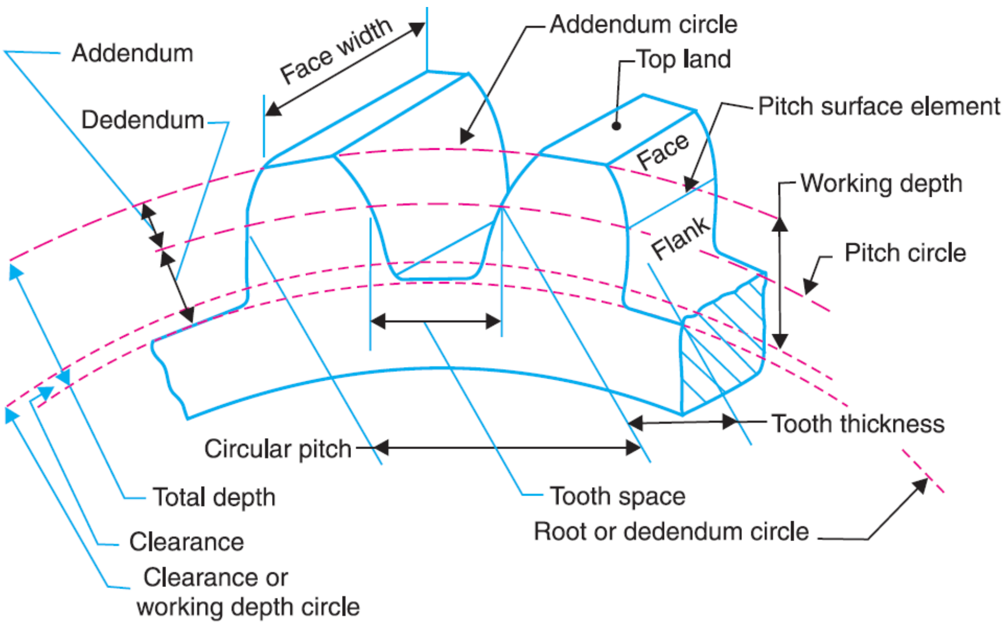


Fig. 12.5. Terms used in gears.

1. ***Pitch circle.*** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. *Pitch circle diameter*. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. *Pitch point*. It is a common point of contact between two pitch circles.

4. *Pitch surface*. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. *Diametral pitch*. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d . Mathematically,

Diametral pitch, $$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module*. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

Module, $m = D/T$

13. ***Clearance.*** It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. ***Total depth.*** It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

25. ***Path of contact.*** It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

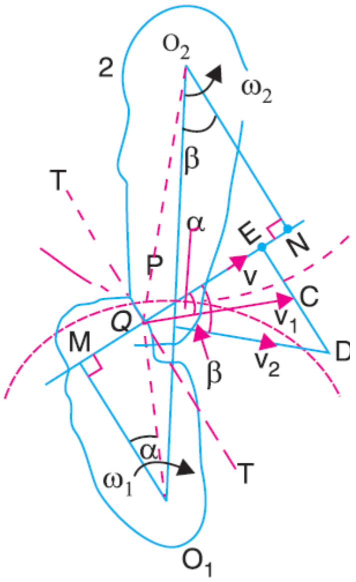
26. ****Length of the path of contact.*** It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. *****Arc of contact.*** It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(*a*) ***Arc of approach.*** It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) ***Arc of recess.*** It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing



Let T T be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

$$\therefore \qquad v_1 \cos\alpha = v_2 \cos\beta$$

Fig. 12.6. Law of gear

$$(\omega_1 \times O_1 Q) \cos\alpha = (\omega_2 \times O_2 Q) \cos\beta$$

$$(\omega_1 \times O_1 Q) \frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q) \frac{O_2 N}{O_2 Q} \quad \text{or} \quad \omega_1 \times O_1 M = \omega_2 \times O_2 N$$

$$\therefore \qquad \frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

Combining equations ***(i)*** and ***(ii)***, we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point *P* must be the fixed point (called pitch point) for the two wheels. In other words, ***the common normal at the point of contact between a pair of teeth must always pass through the pitch point.*** This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as *law of gearing.*

Velocity of Sliding of Teeth $v_S = (\omega_1 + \omega_2) QP$

Notes : 1. We see from equation *(ii)*, that the **velocity of sliding is proportional to the distance of the point of contact from the pitch point.**

12.10. Cycloidal Teeth

A ***cycloid*** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid***.

In Fig. 12.7 (*a*), the fixed line or pitch line of a rack is shown. When the circle *C* rolls without slipping above the pitch line in the direction as indicated in Fig. 12.7 (*a*), then the point *P* on the circle traces epi-cycloid *PA*. This represents the face of the cycloidal tooth profile. When the circle *D* rolls without slipping below the pitch line, then the point *P* on the circle *D* traces hypo-cycloid *PB*, which represents the flank of the cycloidal tooth. The profile *BPA* is one side of the cycloidal rack tooth. Similarly, the two curves *P′A′* and *P′B′* forming the opposite side of the tooth profile are traced by the point *P′* when the circles *C* and *D* roll in the opposite directions.

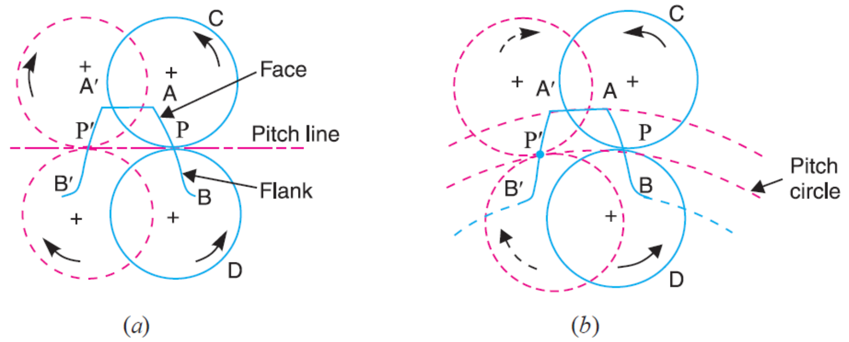


Fig. 12.7. Construction of cycloidal teeth of a gear.

12.11. Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

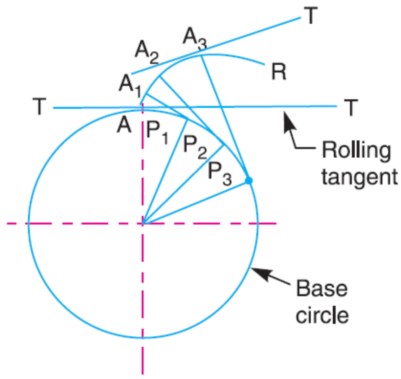


Fig. 12.9. Construction of involute.

Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, ***normal at any point of an involute is a tangent to the circle.***

Length of Path of Contact

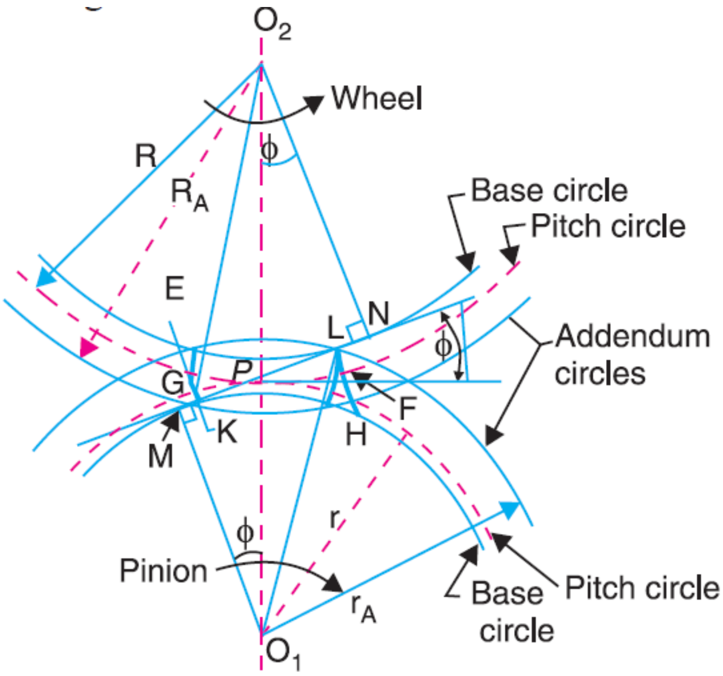


Fig. 12.11. Length of path of contact.

We have discussed in Art. 12.4 that the length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as ***path of approach*** and the part of the path of contact PL is known as ***path of recess***.

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.

From Fig. 12.11, we find that radius of the base circle of pinion,

$$O_1M = O_1P\cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and

$$PN = O_2P\sin\phi = R\sin\phi$$

$\therefore$ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

and

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

$\therefore$ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

Length of Arc of Contact

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

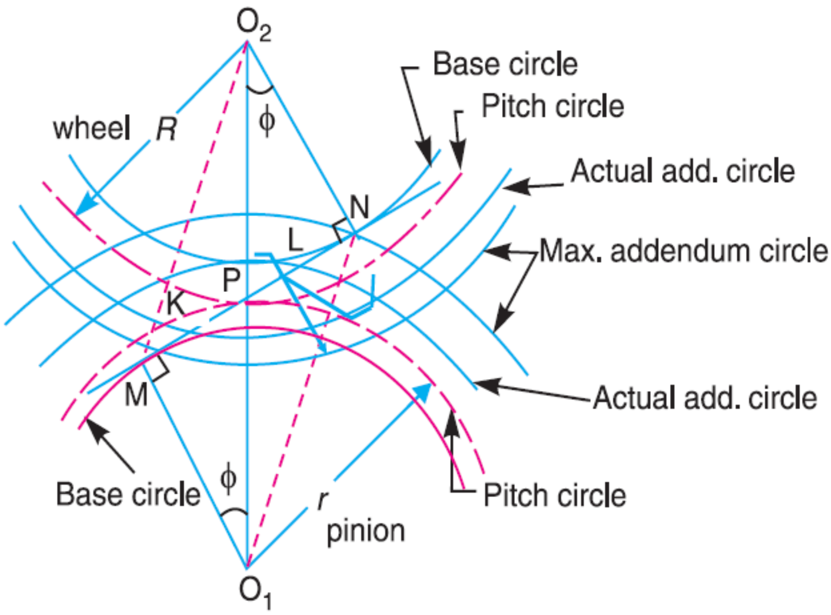
Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

$$p_c = \text{Circular pitch} = \pi m, \text{ and}$$

$$m = \text{Module}.$$

Interference in Involute Gears



words, ***interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.***

When interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig. 12.13. In such a case,

Maximum length of path of approach,

$$MP = r \sin \phi$$

and maximum length of path of recess,

$$PN = R \sin \phi$$

∴ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

Note : In case the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then

Path of approach, $KP = \frac{1}{2} MP$

or $\sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi = \dfrac{r \sin \phi}{2}$

and path of recess, $PL = \frac{1}{2} PN$

or $\sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi = \dfrac{R \sin \phi}{2}$

∴ Length of the path of contact

$$= KP + PL = \frac{1}{2} MP + \frac{1}{2} PN = \frac{(r + R) \sin \phi}{2}$$

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

$$t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

Minimum Number of Teeth on the Wheel in Order to Avoid Interference

$$T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

13.2. Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train, **2.** Compound gear train, **3.** Reverted gear train, and **4.** Epicyclic gear train.

13.3. Simple Gear Train

When there is only one gear on each shaft, as shown in Fig. 13.1, it is known as ***simple gear train.*** The gears are represented by their pitch circles.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that when the number of intermediate gears are ***odd,*** the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig. 13.1 (*b*).

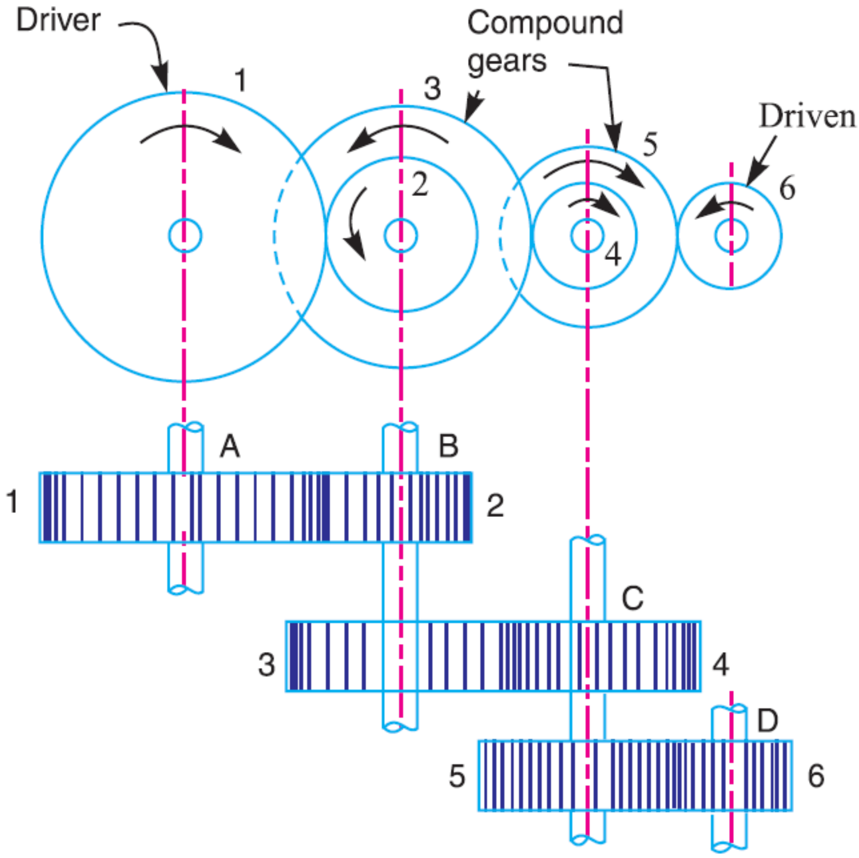
But if the number of intermediate gears are ***even,*** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. 13.1 (*c*).

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Compound Gear Train



$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

13.7. Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. 13.6, where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or ***vice- versa***, but if gear A is fixed and the arm is rotated about the axis of gear A (*i.e.* O_1), then the gear B is forced to rotate ***upon*** and ***around*** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi.*** means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound***.

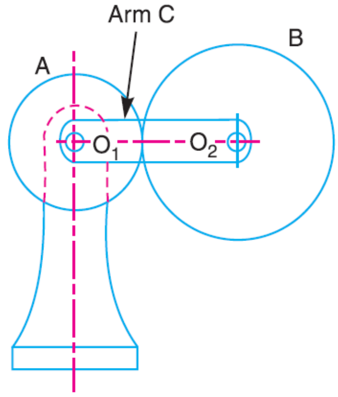


Fig. 13.6. Epicyclic gear train.

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x \times \frac{T_A}{T_B}$

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. When the gear *A* makes one revolution anticlockwise, the gear *B* will make *T_A / T_B revolutions, clockwise. Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear *A* makes + 1 revolution, then the gear *B* will make ($-T_A / T_B$) revolutions. This statement of relative motion is entered in the first row of the table (see Table 13.1).

Secondly, if the gear *A* makes + *x* revolutions, then the gear *B* will make $-x \times T_A / T_B$ revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by *x*.

Thirdly, each element of an epicyclic train is given + *y* revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

2. *Algebraic method.* In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train *viz*. some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations ; and hence to determine the motion of any element in the epicyclic gear train.

Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C

$$= N_A - N_C$$

and speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in ***opposite*** directions.

$$\therefore \qquad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \qquad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.