K.K. COLLEGE OF ENGINEERING & MANAGEMENT

Branch: Mechanical Engineering

Semester: IV

Applied Thermodynamics ME402

APPLIED THERMODYNAMICS

Course Code-ME 402

Objectives:

- 1) To learn about of 1st law for reacting systems and heating value of fuels.
- 2) To learn about gas and vapor cycles and their first law and second law efficiencies.
- To understand about the properties of dry and wet air and the principles of psychometry.
- 4) To learn about gas dynamics of air flow and steam through nozzles.
- 5) To learn the about reciprocating compressors with and without intercooling
- To analyze the performance of steam turbines.

Module-1

Introduction to solid, liquid and gaseous fuels- Stoichiometry, exhaust gas analysis- First law analysis of combustion reactions- Heat calculations using enthalpy tables- Adiabatic flame temperature- Chemical equilibrium and equilibrium composition calculations using free energy.

Module -II

Vapor power cycles Rankine cycle with superheat, reheat and regeneration, energy analysis Super- critical and ultra super-critical Rankine cycle- Gas power cycles, Air standard Otto, Diesel and Dual cycles- Air standard Brayton cycle, effect of reheat, regeneration and intercooling – Combined gas and vapor power cycles- vapor compression refrigeration cycles, refrigerants and their properties(12hrs)

Module-III

properties of dry and wet air, use of pschyrometric chart, processes involving heating/cooling and humidification/ dehumidification, dew point(4hrs)

Module-IV

Basics of compressible flow. Stagnation properties, Isentropic flow of a perfect gas through a nozzle, chocked flow, subsonic and supersonic flows- normal shocks-use of ideal gas tables for isentropic flow and normal shock flow- Flow of steam and refrigerant through nozzle, super saturation-compressible flow in diffusers, efficiency of nozzle and diffuser. (8hrs)

Module-V

Reciprocating compressors, staging of reciprocating compressors, optimal stage pressure ratio, effect of intercooling, minimum work for multistage reciprocating compressors. (5hrs)

Module-VI

Analysis of steam turbines, velocity and pressure compounding of steam turbine. (3hrs) Outcomes:

- After completing this course the students will get a good understanding of various practical power cycles and heat pump cycles.
- They will be able to analyze energy conversion in various thermal devices such as combustors, air coolers, nozzles, diffusers, steam turbines and reciprocating compressors.
- 3. They will be able to understand phenomena occurring in high speed compressible flows.

MODUE 1 FUEL AND COMBUSTION

Introduction

Fuel is a combustible substance, containing carbon as a main constituent, which on proper burning gives large amount of heat, which can be used economically for domestic and industrial purpose. Example: Wood, charcoal, coal, kerosene, petrol, diesel, producer gas, oil gas etc.

During the process of combustion, carbon, hydrogen, etc., combine with oxygen with a liberation of heat.

The combustion reaction can be explained as

 $C + O_2 \longrightarrow CO_2 + 94 \text{ k cals}$ $2H_2 + O2 \longrightarrow 2H_2O + 68.5 \text{ k cals}$

The calorific value of a fuel depends mainly on the amount of Carbon and Hydrogen.

Requirements of a Good Fuel

A good fuel should have the following characteristics:

- ➤ High calorific value.
- Moderate ignition temperature.
- Low contents of non-combustible matters.
- ➤ Low moisture content.
- Free from objectionable and harmful gases like CO, SOx, H2S.
- Moderate velocity of combustion.
- Combustion should be controllable.
- Easy to transport and readily available at low cost.

Classification of Fuels:

Fuels are classified into

- (i) Primary or Natural fuels These are found in nature.
- (ii) Secondary or Artificial fuels These are derived from primary fuels.

Primary and secondary fuels may also be divided into 3 classes' namely solid, liquid and gaseous fuels.



Fossil fuels

Fossil fuels are those, which have been derived from fossil remains of plant and animal life.

They are found in the earth's crust.

All conventional fossil fuels whether solid, liquid or gaseous (coal, petroleum or Natural gas) contain basically carbon and / or hydrogen. The fuels on combustion in presence of oxygen in the air release heat energy. This heat energy can be utilized for domestic and industrial purposes.

Advantages of Solid fuels

- 1. Handling and transportation of solid fuels are easy.
- 2. Solid fuels are cheap and easily available.
- 3. They have a moderate ignition temperature
- 4. This type of fuel can be stored conveniently without any risk.

Disadvantage of solid fuels:

1. During burning, solid fuels produce a large amount of ash and disposal of ash is a big problem.

2. The calorific value of solid fuel is comparatively low.

3. Since a lot of air is required for complete combustion, its thermal efficiency is not so high.

- 4. A large space is required for storage.
- 5. Combustion is a slow process and it cannot be easily controlled.

Advantages of Liquid fuels

- 1. Liquid fuels do not yield any ash after burning.
- 2. They require comparatively less storage space.
- 3. Calorific value of liquid fuel is higher than that of solid fuels.
- 4. Their combustion is uniform and easily controllable.

Disadvantages of liquid fuels:

- 1. Liquid fuels are comparatively costlier than the solid fuels.
- 2. They give unpleasant odour during incomplete combustion.
- 3. Some amount of liquid fuels will escape due to evaporation during storage.
- 4. Special type of burners are for effective combustion.

Advantages of gaseous fuels:

- 1. Gaseous fuels can be easily transported through the pipes.
- 2. They do not produce any ash or smoke during burning.
- 3. They have high calorific value than the solid fuels.
- 4. They have high thermal efficiency.

Disadvantages of gaseous fuels

- 1. They are highly inflammable and hence the chances for fire hazards are high.
- 2. Since gases occupy a large volume, they require large storage tanks.

Combustion:- Combustion is an exothermic chemical reaction, which is accompanied by development of heat and light at a rapid rate, temperature rises considerably.

Factors affecting the rate of combustion

The rate of combustion depends on the following factors:

- 1. The concentration of the fuel and air.
- 2. The nature of the combustible substance
- 3. The temperature

4. With increase in pressure or surface area of the fuel the rate of combustion can be increased.

5. It increases within increase in pressure of air.

6. It Increases with preheating of fuel and air.

Combustion stoichiometry

When molecules undergo chemical reaction, the reactant atoms are rearranged to form new combinations. For example, hydrogen and oxygen react to form water:

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

two atoms of hydrogen and one atom of oxygen form one molecule of water - number of atoms of H and O must be the same on both sides Such reaction equation represents initial and final states and does not indicate actual path of reaction, which may involve many intermediate steps and intermediate species.

Relative masses of molecules are obtained by multiplying number of moles of each species by the molecular weights (kg/kg-mol). For hydrogen-oxygen reaction,

 $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$

$$(1 \operatorname{mol} H_2) \left(\frac{2 \operatorname{kg}}{\operatorname{kg-mol} H_2} \right) + \left(\frac{1}{2} \operatorname{mol} O_2 \right) \left(\frac{32 \operatorname{kg}}{\operatorname{kg-mol} O_2} \right)$$
$$= (1 \operatorname{mol} H_2 O) \left(\frac{18 \operatorname{kg}}{\operatorname{kg-mol} H_2 O} \right)$$

mass of reactants equals mass of products, although moles of reactants do not equal moles of products - fixed p and T, ideal gas,

1 volume $H_2 + 1/2$ volume $O_2 = 1$ volume H_2O

Stoichiometric calculations are done by performing atom balance for each of the elements in mixture. The theoretical amount of air required to burn a fuel completely to products with no dissociation is defined as stoichiometric air.

In most combustion calculations dry air is assumed as a mixture of 79% (vol) N_2 and 21% (vol) O_2 or 3.764 moles of N_2 per mole O_2 Molecular weight of pure air is 28.96, as it also contains small amounts of argon, carbon dioxide, hydrogen etc.

	% (vol)	% (mass)
N ₂	78.09	75.02
O ₂	20.95	23.15
Ar	0.93	1.28
CO ₂	0.03	0.05
Ne , neon He , helium Kr , kripton Xe , xenon H ₂ , hydrogen		

Fuel composition, for 1 kg of fuel,

С	kg of carbon
h	kg of hydrogen
0	kg of oxygen
n	kg of nitrogen
s	kg of sulphur
w	kg of water
а	kg of ash, etc

c + h + o + n + s + w + a = 1 kg

For complete combustion of the fuel the following reactions take place carbon :

$$C + O_2 \rightarrow CO_2$$

1 mol C + 1 mol O ₂	1 mol CO ₂
12 kg C + 32 kg O ₂	44 kg CO ₂
1 kg C + (32/12) kg O ₂	(44/12) kg CO ₂

hydrogen :

$$\begin{array}{rrrr} H_2 \ + \ \frac{1}{2}O_2 \ \rightarrow \ H_2O \\ 2 \ \mathrm{kg} \ \mathrm{H}_2 \ + \ 16 \ \mathrm{kg} \ \mathrm{O}_2 & 18 \ \mathrm{kg} \ \mathrm{H}_2\mathrm{O} \\ 1 \ \mathrm{kg} \ \mathrm{H}_2 \ + \ 8 \ \mathrm{kg} \ \mathrm{O}_2 & 9 \ \mathrm{kg} \ \mathrm{H}_2\mathrm{O} \end{array}$$

Theoretical Calculation of Minimum Air required for Combustion

In order of achieve efficient combustion of fuel, it is essential that the fuel is brought into intimate contact with sufficient quantity of air to burn all the combustible matter under appropriate conditions.

The correct conditions are

- i). Intimate mixing of air with combustible matter and
- ii). Sufficient time to allow the combustion process to be completed.

If these factors are inappropriate, inefficient combustion occurs.

The elements usually present in common fuels which enter into the process of combustion are mainly **C**, **H**, **S** and **O**.

Nitrogen, ash and CO2 (if any) present in the fuel are incombustible matters and hence they do not take any oxygen during combustion.

Air contains 21% oxygen by volume and 23% of oxygen by weight.

Hence from the amount of oxygen required by the fuel, the amount of air can be calculated.

From the combustion reaction equations, we can calculate the quantity of oxygen by weight or volume and from this, the weight or volume of air required can be calculated.

For example,

i) Combustion of Carbon

C12 + O322 CO 44 (by weight) 2

12 parts by weight of carbon require 32 parts by weight of oxygen for complete combustion.

(or)

1 part by volume of carbon requires 1 part by volume of oxygen for complete combustion.

C parts by weight of carbon require = parts by weight of O_2 12 32C

ii) Combustion of Hydrogen

Oxygen when present in the fuel is always in combination with hydrogen. So, the quantity of hydrogen in combination with oxygen, which is present in the fuel, will not take part in the combustion reaction. Therefore, the quantity of hydrogen in combination with oxygen is deduced from the total hydrogen in the fuel.

Now, the quantity of hydrogen available for combustion reaction will be,

$$H-\frac{O}{8}$$

where H is the total quantity of hydrogen and O is the total quantity of oxygen in the fuel. (In water the quantity of hydrogen in combination with oxygen is one-eighth of the weight of oxygen).

 $\begin{array}{ccc} 2H_2 &+O_2 \longrightarrow 2H_2O\\ 2 \times 2 & 32 & 36 \text{ (by weight)} \end{array}$

4 parts by weight of hydrogen require 32 parts by weight of oxygen for complete combustion.

(or)

2 parts by volume of hydrogen require 1 part by volume of oxygen for complete combustion.

: H parts by weight of hydrogen requires

$$=\frac{32H}{4}$$
 parts by weight of O₂

But, some of the hydrogen is present in the combined form with oxygen (i.e, as H2O). This combined hydrogen does not take part in the combustion reaction. Therefore, the quantity of combined hydrogen must be deduced from the total hydrogen in the fuel.

$$\therefore H - \frac{O}{8}$$
 part by weight of hydrogen requires

$$\frac{[H-\frac{O}{8}] \times 32}{4} = 8[H-\frac{O}{8}] \text{ parts by weight of } O_2$$

iii) Combustion of Sulphur

$$\begin{array}{ccc} S & + & O_2 & \longrightarrow & SO_2 \\ 32 & & 32 & & 64 \text{ (by weight)} \end{array}$$

32 parts by weight of sulphur requires 32 parts by weight of oxygen for complete combustion.

(or)

1 part by volume of sulphur requires 1 part by volume of oxygen for complete combustion.

: S parts by weight of sulphur requires

$$= \frac{32 \times S}{32} = S \text{ parts by weight of } O_2$$

Consequently, theoretical amount of oxygen required for the complete combustion of 1kg of solid or liquid fuel.

Theoretical minimum
$$O_2 = \left[\frac{32}{12} \times C + 8(H - \frac{O}{8}) + S\right]$$
 kg

Since mass % of O2 in air is 23, the amount of air required theoretically for combustion of 1 kg of the fuel is,

Air (theoretical) =
$$\frac{100}{23} \left[\frac{32}{12} \times C + 8(H - \frac{O}{8}) + S \right]$$
 kg

Volume of Air Required for Complete Combustion of Gaseous Combustible Matters

i).

 $\begin{array}{ll} H_2(g) & + 1/2O_2 \longrightarrow H_2O \\ 1 \text{ vol.} & 0.5 \text{ vol.} \\ 1 \text{ volume of } H_2(g) \text{ requires } 0.5 \text{ vol. of oxygen} \end{array}$

ii).

 $CH_4 + 2 O_2 \longrightarrow CO_2 + 2 H_2O$ 1 vol 2 vol 1 volume of CH₄ requires 2.0 volume of oxygen

iii).

 $\begin{array}{c} CO(g) + \frac{1}{2}O_2 & \longrightarrow & CO_2 \\ 1 \text{ vol} & 0.5 \text{ vol} \\ 1 \text{ volume of CO requires 0.5 volume of oxygen} \end{array}$

iv).

$$C_{2}H_{4(g)} + 3O_{2} \longrightarrow 2CO_{2} + 2H_{2}O$$
1 vol 3 vol
1 volume of $C_{2}H_{4(g)}$ requires 3.0 vol. of oxygen

Amount of O2 required by the fuel will be given by subtracting the amount of O2 already present in the fuel from the total or theoretical amount of O2 required by the fuel.

∴ Net amount of O₂ required=Total amount of O₂ required – O₂ already present in the fuel.

Air contains 21 % of O2 by volume and 23 % of O2 by weight. Hence from the amount of

O2 required by the fuel, the amount of air required can be calculated.

- Minimum weight of air required = x minimum O2
- > Minimum volume of air required = x minimum O2
- Molecular mass of air is taken as 28.94 g/mol
- > Density of air at NTP = 1.29 kg/cm^2
- 22.4 litres (or 22,400 ml) of any gas at NTP (i.e 0°C and 760 mm of Hg) has a mass equal to its 1 mol (gram molecular weight)
- Thus 22.4 litres of CO2 at NTP will have a mass of 44 g (44 is the molecular weight of CO2)

Excess air for combustion

- It is necessary to supply excess air for complete combustion of the fuel. It is found out from the theoretical amount of air as follows.
- > The amount of air required if excess air is supplied.

Theoretical amount of air 100 x [100 + % of excess air]

STOICHIOMETRIC AIR FUEL (A/F) RATIO

sufficient oxygen for complete combustion of the fuel.

A weak mixture is one which has an excess of air.

A rich mixture is one which has a deficiency of air.

The percentage of excess air is given as under :

%age excess air = $\frac{\text{Actural A/F ratio} - \text{Stoichiometric A/F ratio}}{\text{Stoichiometric A/F ratio}}$

The ratios are expressed as follows :

- For gaseous fuels By volume
- ➢ For solid and liquid fuels By mass

For *boiler* plant the mixture is usually greater than 20% weak ; for *gas turbines* it can be as much as 300% weak. *Petrol engines* have to meet various conditions of load and speed, and operate over a wide range of mixture strength. The following definition is used: The working value range between 80% (weak) and 120% (rich).

Note. The reciprocal of the air fuel ratio is called the *fuel-air* (F/A) ratio.

AIR-FUEL RATIO FROM ANALYSIS OF PRODUCTS

When analysis of combustion products is known air-fuel ratio can be calculated by the following methods :

1. Fuel composition known

(i) Carbon balance method (ii) Hydrogen balance method

(*iii*) Carbon-hydrogen balance method.

2. Fuel composition unknown

(*i*) Carbon-hydrogen balance method.

1. Fuel composition known

(*i*) *Carbon balance method*. When the fuel composition is known, the carbon balance method is quite accurate if *combustion takes place with excess air and when free (solid) carbon is not present in the products*. It may be noted that the Orsat analysis will not determine the quantity of solid carbon in the products.

(ii) Hydrogen balance method. This method is used when solid carbon is suspected to be present.

(*iii*) Carbon-hydrogen balance method. This method may be employed when there is some uncertainty about the nitrogen percentage reported by the Orsat analysis.

2. Fuel composition unknown

When the fuel composition is not known the carbon-hydrogen balance method has to be employed.

HOW TO CONVERT VOLUMETRIC ANALYSIS TO WEIGHT ANALYSIS ?

The conversion of volumetric analysis to weight analysis involves the following step :

1. Multiply the volume of each constituent by its molecular weight.

2. Add all these weights and then divide each weight by the total of all and express it as percentage.

HOW TO CONVERT WEIGHT ANALYSIS TO VOLUMETRIC ANALYSIS ?

1. Divide the weight of each constituent by its molecular weight.

2. Add up these volumes and divide each volume by the total of all and express it as a percentage.

11.13. WEIGHT OF CARBON IN FLUE GASES

The weight of carbon contained in one kg of flue or exhaust gas can be calculated from the amounts of CO2 and CO contained in it.

In eqn. [11.4 (*b*)], it was shown that 1 kg of carbon produces 11/3 kg of CO2 when completely burnt. Hence 1 kg of CO2 will contain 3/11 kg of carbon.

In eqn. [11.6 (*a*)], it can be seen that 1 kg of carbon produces 7/3 kg of CO, hence 1 kg CO contains 3/7 kg of carbon.

Therefore, weight of carbon per kg of fuel

$$=\left(\frac{3}{11}CO_2+\frac{3}{7}CO\right)$$

WEIGHT OF FLUE GASES PER KG OF FUEL BURNT

Due to supply of air, the weight of flue gas or exhaust gas is always more than that of fuel burnt. The actual weight of dry flue gases can be obtained by comparing the weight of carbon present in the flue gases with the weight of carbon in the fuel, since there is no loss of carbon during the combustion process. As the analysis of the exhaust gases is volumetric, so this *must first be reduced to weight analysis*.

Also, total weight of carbon in one kg of flue gas is

$$= \left(\frac{3}{11}CO_2 + \frac{3}{7}CO\right)$$

: The weight of flue gas/kg of fuel burnt

= Weight of carbon in one kg of fuel Weight of carbon in one kg of flue gas.

Flue Gas Analysis [Orsat's Method]

The mixture of gases (like CO2, O2, CO, etc.,) coming out from the combustion chamber is called flue gases. The analysis of a flue gas would give an idea about the complete or incomplete combustion process. The analysis of flue gases is carried out using orsat's apparatus.

Description of orsat's apparatus

It consists of a horizontal tube. At one end of this tube, an U-tube containing fused CaCl2 is connected through a 3-way stop cock. The other end of the tube is connected with a graduated burette. The burette is surrounded by a water jacket to keep the temperature of gas as a constant. The lower end of the burette is connected to a water

reservoir by means of a rubber tube. The level of water in the burette can be raised or lowered by raising or lowering the reservoir.

The horizontal tube is also connected with three different absorption bulbs I, II and III for absorbing CO2, O2 and CO.

I - bulb : It consists of 'potassium hydroxide' solution and it absorbs only CO2.

II - bulb : It consists of 'alkaline pyrogallol' solution and it absorbs only CO2 and O2.

III - bulb : It consists of 'ammoniacal cuprous chloride' solution and it absorbs CO2, O2 and CO.



Working

The 3-way stop cock is opened to the atmosphere and the reservoir is raised, till the burette is completely filled with water and air is excluded from the burette. The 3-way stop cock is now connected to the flue gas supply and the flue gas is sucked into the burette and the volume of flue gas is adjusted to 100 cc by

raising and lowering the reservoir. Then the 3-way stop cock is closed.

a) Absorption of CO2

The stopper of the absorption bulb-I, containing KOH solution, is opened and all the gases is passed into the bulb-I by raising the level of water in the burette. The gas enters into the bulb-I, where CO2 present in the flue gas is absorbed by KOH.

The gas is again sent to the burette. This process is repeated several times to ensure complete absorption of CO2. The decrease in volume of the flue gas in the burette indicates the volume of CO2 in 100 cc of the flue gas.

b) Absorption of O2

Stop cock of bulb-I is closed and stop cock of bulb-II is opened. The gas is again sent into the absorption bulb-II, where O2 present in the flue gas is absorbed by alkaline pyrogallol. The decrease in volume of the flue gas in the burette indicates the volume of O2.

c) Absorption of CO

Now the stop cock of bulb-II is closed and stop cock of bulb-III is opened. The remaining gas is sent into the absorption bulb-III, where CO present in the flue gas is absorbed by ammoniacal cuprous chloride. The decrease in volume of the flue gas in the burette indicates the volume of CO. The remaining gas in the burette after the absorption of CO2, O2 & CO is taken as nitrogen.

Significance (or) uses of flue gas analysis

1. Flue gas analysis gives an idea about the complete or incomplete combustion process.

2. If the flue gases contain considerable amount of CO, it indicates that incomplete combustion is occurring and it also indicates that the short supply of O2.

3. If the flue gases contain considerable amount of O2, it indicates that complete combustion is occurring and also it indicates that the excess of O2 is supplied.

INTERNAL ENERGY AND ENTHALPY OF FORMATION

The first law of thermodynamics can be applied to any system. Non-flow and steadyflow energy equations deducted from this law must be applicable to systems undergoing combustion processes. It has been proved experimentally that the energy released, when a unit mass of a fuel undergoes complete combustion, depends on the *temperature at which the process is carried out*.

Thus such quantities quoted are related to temperature. Now it will be shown that if the energy released by a fuel at one temperature is known then it can be calculated at other temperatures. The process of combustion is defined as taking place from reactants at a state identified by the reference temperature T0 and another property, either pressure or volume, to products at the same state.

Let UR0 = Internal energy of the *reactants* (which is a mixture of fuel and air) at T0,

UP0 = Internal energy of *products* of combustion at *T*0,

UR1 = Internal energy of *reactants* at temperature T1,

UP1 = Internal energy of *products* at temperature *T*1,

UR2 = Internal energy of *reactants* at temperature T2,

UP2 = Internal energy of *products* at temperature *T*2,

 $\Delta U0$ = Constant volume heat of combustion,

Q = Heat transferred to the surroundings during the process, and

W = Work obtained during combustion process.

Analysis for a non-flow process involving combustion at 'constant volume' :

When the combustion process is carried out at *constant volume* then the non-flow energy

equation, Q = (U2 - U1) + W, can be applied to give

 $Q = (U_{P_0} - U_{R_0})$ where, W = 0 for constant volume combustion, $U_1 = U_{R_0}$, and $U_2 = U_{P_0}$.

The internal energy change is independent of the path between the two states and dependsonly on the initial and final values and is given by the quantity Q. The heat so transferred is called the *internal energy of combustion* at T0 (or *constant volume heat of combustion*), and is denoted by $\Delta U0$. Thus,

$$\Delta U_0 = U_{P_0} - U_{R_0}$$

 $\Delta U0$ is a *negative quantity* since the internal energy of the reactants includes the potential chemical energy and heat is transferred *from* the system.

It may be noted that in case of real constant volume combustion processes the initial and final temperatures will not be same as T0 (reference temperature). The change in internal energy, for analytical purposes, between reactants at state 1 to products at state 2 can be considered in the following *three steps* (stages) :

(*i*) The change for the reactants from state 1 to *T*0.

(*ii*) The constant volume combustion process from reactants to products at T0.

(*iii*) The change for the products from T0 to state 2.



The entire process can be thought of as taking place in piston-cylinder device as shown in



Thus U2 - U1, the change in internal energy between states 1 and 2, can be written as (UP2 - UR1) to show the chemical changes involved and this can be further expanded for analytical

purposes as follows :

$$U_{P_2} - U_{R_1} = (U_{P_2} - U_{P_0}) + (U_{P_0} - U_{R_0}) + (U_{R_0} - U_{R_1})$$

i.e.,

The values of $(U_{R_0} - U_{R_1})$ and $(U_{P_2} - U_{P_0})$ can be calculated from the following relations :

$$U_{R_0} - U_{R_1} = \sum_{R} n_i (u_{i_0} - u_{i_1}) \qquad \dots (11.15)$$

where, u_i = Tabulated value of the internal energy for any constituent at the required temperature T_0 or T_1 in heat unit per mole

 n_i = Number of moles of the constituent, and

 \sum_{R} = Summation for all the constituents of the reactants denoted by *i*.

If mass base is used for tabulated values or calculation, then

$$U_{R_0} - U_{R_1} = \sum_{R} m_i (u_{i_0} - u_{i_1}) \qquad \dots (11.16)$$

where, u_i = Internal energy per unit mass.

The above expression in terms of the *specific heats* (average values for the required temperature range) may be written as

$$U_{R_0} - U_{R_1} = \sum_R m_i c_{vi} (T_0 - T_1) = (T_0 - T_1) \sum_R m_i c_{vi} \qquad \dots (11.17)$$

For products, similar expressions may be written as :

It may be noted that $n_i C_{vi} = m_i c_{vi}$

Analysis for a steady flow or 'constant pressure' combustion process :

In such an analysis the *changes in enthalpy* (H) are important. An analysis carried out as above will give the following expressions :

$$H_{P_2} - H_{R_1} = \underbrace{(H_{P_2} - H_{P_0})}_{\text{Products}} + \Delta H_0 + \underbrace{(H_{R_0} - H_{R_1})}_{\text{Reactants}} \qquad \dots (11.18)$$

where, $\Delta H_0 = H_{P_0} - H_{R_0}$, and is always negative

 $[\Delta H_0 = Enthalpy of combustion at T_0 or the constant pressure heat of combustion at T_0] Expressions for change of enthalpy of reactants and products :$ Reactants :

$$H_{R_0} - H_{R_1} = \sum_R n_i (h_{i_0} - h_{i_1})$$
 ... on mole basis ...(11.20)

...(11.19)

Products :
$$H_{P_2} - H_{P_0} = \sum_{p} n_i (h_{i_2} - h_{i_0})$$
 ... on mole basis ...(11.22)

$$\begin{split} H_{P_2} - H_{P_0} &= \sum_{p} m_i (h_{i_2} - h_{i_0}) & \dots \text{ on mass basis} & \dots (11.23) \\ H_{P_2} - H_{P_0} &= \sum_{p} m_i c_{pi} (T_2 - T_0) \end{split}$$

$$= (T_2 - T_0) \sum_{p} m_i c_{pi} \dots \text{ in terms of mean specific heats } \dots [11.23 (a)]$$

It may be noted that $n_i C_{pi} = m_i c_{pi}$ From the definition of the enthalpy of a perfect gas

$$H = U + pV = U + nR_0T$$

So if we are concerned only with gaseous mixtures in the reaction then for products and reactants $% \left({{{\mathbf{x}}_{i}}} \right)$

$$H_{P_0} = U_{P_0} + n_p R_0 T_0$$
$$H_{R_0} = U_{R_0} + n_R R_0 T_0$$

and

where n_p and n_R are the moles of products and reactants respectively and the temperature is the reference temperature T_0 .

Thus, using eqns. (11.13) and (11.19), we have

$$\Delta H_0 = \Delta U_0 + (n_P - n_R) R_0 T_0 \qquad ...(11.24)$$

If there is no change in the number of moles during the reaction or if the reference temperature is absolute zero, then ΔH_0 and ΔU_0 will be equal.

ENTHALPY OF FORMATION $(\nabla H f)$

A combustion reaction is a particular kind of chemical reaction in which products are formed from reactants with the release or absorption of energy as heat is transferred to as from the surroundings. In some substances like hydrocarbon fuels which are many in number and complex in structure the heat of reaction or combustion may be calculated on the basis of known values of the enthalpy of formation, ΔHf of the constituent of the reactants and products at the temperature T0 (reference temperature). The enthalpy of formation (ΔHf) is the increase in enthalpy when a compound is formed from its constituent elements in their natural form and in a standard state. The standard state is 25°C, and 1 atm. pressure, but it must be borne in mind that not all substances can exist in natural form, e.g. H2O cannot be a vapour at 1 atm. and 25°C. The expression of a particular reaction, for calculation purposes, may be given as :

$$\Delta H_0 = \sum_{P} n_i \Delta H_{f_i} - \sum_{R} n_i \Delta H_{f_i}$$

S. No.	Substance	Formula	State	ΔH_{f}
1.	Oxygen	0	gas	249143
		LO ₂	gas	zero
2.	Water	H_2O	liquid	-285765
			vapour	-241783
3.	Carbon	С	gas	714852
			diamond	1900
			graphite	zero
4.	Carbon monoxide	CO	gas	-111508
5.	Carbon dioxide	CO2	gas	-393443
6.	Methane	CH_4	gas	-74855
7.	Methyl alcohol	CHOH	vapour	-240532
8.	Ethyl alcohol	C ₂ H ₅ OH	vapour	-281102
9.	Ethane	C_2H_6	gas	-83870
10.	Ethene	C_2H_4	gas	51780
11.	Propane	C ₃ H ₈	gas	- 102900
12.	Butane	C_4H_{10}	gas	- 125000
13.	Octane	C_8H_{18}	liquid	-247600
1	1	1		1

Typical values of ΔH_f for different substances at 25°C (298 K) in kJ/mole are given below :

ADIABATIC FLAME TEMPERATURE

In a given combustion process, that takes place *adiabatically* and with no work or changes in kinetic or potential energy involved, *the temperature of the products is referred to as the 'adiabatic flame temperature'*. With the assumptions of no work and no changes in kinetic or potential energy, this is the *maximum temperature* that can be achieved for the given reactants because any heat transfer from the reacting substances and any incomplete combustion would tend to lower the temperature of the products.

The following points are worthnoting :

(i) The maximum temperature achieved through adiabatic complete combustion varies with the type of reaction and per cent of theoretical air supplied.

An increase in the air-fuel ratio will effect a decrease in the maximum temperature.

(ii) For a given fuel and given pressure and temperature of the reactants, the *maximum adiabatic flame temperature that can be achieved is with a 'stoichiometric' mixture.*

(*iii*) The adiabatic flame temperature can be *controlled* by the *amount of excess air* that is used. This is important, for example, in *gas turbines*, where the maximum

permissible temperature is determined by *metallurgical considerations* in the turbine, and *close control of the temperature of the products is essential*.

Example 11.3. A coal sample gave the following analysis by weight, Carbon 85 per cent, Hydrogen 6 per cent, Oxygen 6 per cent, the remainder being incombustible. Determine minimum weight of air required per kg of coal for chemically correct composition. Solution.

Element, wt. (kg)	O ₂ required (kg)
C = 0.85	$0.85 \times \frac{8}{3} = 2.27$
$H_2 = 0.06$	$0.06 \times 8 = 0.48$
$O_2 = 0.06$	
	Total $O_2 = 2.75$
Weight of O_2 to be supplied	= Wt. of O_2 needed – Wt. of O_2 already present in fuel
	= 2.75 - 0.06 = 2.69 kg
Weight of air needed	$= 2.69 \times \frac{100}{23} = 11.70$ kg. (Ans.)

Example 11.4. The percentage composition of sample of liquid fuel by weight is, C = 84.8 per cent, and $H_2 = 15.2$ per cent. Calculate (i) the weight of air needed for the combustion of 1 kg of fuel; (ii) the volumetric composition of the products of combustion if 15 per cent excess air is supplied.

Solution.

Element, wt. (kg)	O ₂ used (kg)	Dry products (kg)
<i>C</i> = 0.848	$0.848 \times \frac{8}{3} = 2.261$	$\frac{0.848 \times 11}{3} = 3.109 \ (\text{CO}_2)$
$H_2 = 0.152$	$0.152 \times 8 = 1.216$	
	Total $O_2 = 3.477$	

(i) Minimum weight of air needed for combustion

	$= \frac{3.477 \times 100}{23} = 15.11 \text{ kg.} \text{ (Ans.)}$
Excess air supplied	$=\frac{1511\times15}{100}=2.266 \text{ kg}$
Wt. of oxygen in excess air	$=\frac{2.266\times23}{100}=0.521 \text{ kg}$
Total air supplied for combustion	= Minimum air + Excess air
	= 15.11 + 2.266 = 17.376 kg
$\therefore~$ Wt. of nitrogen $(N^{}_2)$ in flue gases	$=\frac{17.376 \times 77}{100}$ = 13.38 kg.

(ii) To get volumetric composition	of the product of con	nbustion let us use tabular	method.
---	-----------------------	-----------------------------	---------

Name of gas	Weight (x)	Molecular weight (y)	Proportional volume (z) = $\frac{(x)}{(y)}$	$\begin{array}{l} Percentage \ volume \\ = \frac{(z)}{\Sigma(z)} \times 100 \end{array}$
CO ₂ O ₂ N ₂	3.109 0.521 13.38	44 32 28	0.0707 0.0163 0.4780	12.51 per cent. (Ans.) 2.89 per cent. (Ans.) 84.60 per cent. (Ans.)
			$\Sigma z = 0.5650$	

Example 11.5. Percentage volumetric analysis of a sample of flue gases of a coal fired boiler gave $CO_2 = 10.4$; CO = 0.2; $O_2 = 7.8$ and $N_2 = 81.6$ (by difference). Gravemetric percentage analysis of coal was C = 78, $H_2 = 6$, $O_2 = 3$ and incombustible = 13. Estimate :

(i) Weight of dry flue gases per kg of fuel.

(ii) Weight of excess air per kg of fuel.

Solution.

Element, wt. (kg)	O2 reqd. (kg)	Dry products (kg)
C = 0.78 $H_2 = 0.06$	$0.78 \times \frac{8}{3} = 2.08$ $0.06 \times 8 = 0.48$	$0.78 \times \frac{11}{3} = 2.86 (CO_2)$
02=0.03	Total O ₂ = 2.56	

Minimum wt. of air needed for combustion = $(2.56 - .03) \times \frac{100}{23} = 11$ kg.

(i) Weight of dry flue gases per kg of fuel :

To determine the wt. of flue gases per kg of fuel let us use tabular method to convert volumetric analysis to analysis by weight.

Name of gas	Volume per m ³ of flue gas	Molecular weight	Relative volume $z = x \times y$	Weight per kg of flue gas
	<i>(x)</i>	(y)		$=\frac{Z}{\Sigma z}$
CO ₂	0.104	44	4.576	0.1525
co	0.002	28	0.056	0.0019
N ₂	0.816	28	22.848	0.7616
02	0.078	32	2.496	0.0832
			$\Sigma z = 29.976 \text{ (say 30)}$	

Amount of carbon present per kg of gases

= Amount of carbon in 0.1525 kg of CO_2 + Amount of carbon present in 0.0019 kg of CO

 $= \frac{3}{11} \times 0.1525 + \frac{3}{7} \times 0.0019 = 0.0416 + 0.0008 = 0.0424 \text{ kg}.$

Also carbon in the fuel = 0.78 kg.

... Weight of dry flue gas per kg of fuel

$$= \frac{\text{Weight of carbon in 1 kg of fuel}}{\text{Weight of carbon in 1 kg of flue gas}} = \frac{0.78}{0.0424} = 18.4 \text{ kg.} \text{ (Ans.)}$$

(ii) Weight of excess air per kg of fuel :

Weight of excess oxygen per kg of flue gas = $0.0832 - \frac{4}{7} \times .0019$

 $\begin{bmatrix} 2\text{CO} + \text{O}_2 = 2\text{CO}_2 \\ 56 & 32 & 88 \\ 1 \text{ kg} + \frac{4}{7} \text{ kg} = \frac{11}{7} \text{ kg} \end{bmatrix}$

= 0.0832 - 0.0011 (allowing for unburnt carbon monoxide) = 0.0821 kg.

Weight of excess O_2 per kg of fuel = $18.4 \times 0.0821 = 1.51$ kg

:. Weight of excess air per kg of fuel = $\frac{151 \times 100}{23}$ = 6.56 kg. (Ans.)

Example 11.6. A single cylinder was supplied with a gas having the following percentage volumetric analysis; CO = 5, $CO_2 = 10$, $H_2 = 50$, $CH_4 = 25$, $N_2 = 10$. The percentage volumetric analysis of dry gases was $CO_2 = 8$, $O_2 = 6$ and $N_2 = 86$. Determine the air-fuel ratio by volume.

Solution. Combustion equations are :

 $\begin{array}{l} 2\mathrm{H}_{2} + \mathrm{O}_{2} = 2\mathrm{H}_{2}\mathrm{O}\\ 1 \ \mathrm{vol} + 1/2 \ \mathrm{vol} = 1 \ \mathrm{vol}\\ 2\mathrm{CO} + \mathrm{O}_{2} = 2\mathrm{CO}_{2}\\ 1 \ \mathrm{vol} + 1/2 \ \mathrm{vol} = 1 \ \mathrm{vol}\\ \mathrm{CH}_{4} + 2\mathrm{O}_{2} = \mathrm{CO}_{2} + 2\mathrm{H}_{2}\mathrm{O}\\ 1 \ \mathrm{vol} + 2 \ \mathrm{vol} = 1 \ \mathrm{vol} + 2 \ \mathrm{vol} \end{array}$

Gas	Vol (m ³)	O_2 needed (m^3)	Products (m^3)	
			CO_2	N_2
CO	0.05	0.025	0.05	_
CO_2	0.10	—	0.10	_
H_2	0.50	0.25	_	_
CH_4	0.25	0.50	0.25	_
N_2	0.10	—	—	0.10
Total	1.0	0.775	0.4	0.10

Volume of air required

Volume of nitrogen in the air

 $= 0.775 \times 100/21 = 3.69 \text{ m}^3$ $= 3.69 \times 79/100 = 2.92 \text{ m}^3$

Dry combustion products of 1 m³ of gases (V) contain 0.4 m³ of CO₂ + 0.1 m³ of N₂ (as given in the table) + 2.92 m³ of N₂ (from air supplied for complete combustion) = 3.42 m³.

Excess air supplied	$= \frac{\mathbf{O}_2 \times \mathbf{V}}{(21 - \mathbf{O}_2)} = \frac{6.0 \times 3.42}{(21 - 6)} = \frac{20.52}{15} = 1.37 \text{ m}^3$
Total quantity of air supplied	$= 3.69 + 1.37 = 5.06 \text{ m}^3.$
Air fuel ratio	$= \frac{\text{Volume of air}}{\text{Volume of fuel}} = \frac{5.06}{1} = 5 \text{ (say).} \text{ (Ans.)}$

Example 11.7. The following is the ultimate analysis of a sample of petrol by weight : Carbon = 85 per cent; Hydrogen = 15 per cent.

Calculate the ratio of air to petrol consumption by weight if the volumetric analysis of the dry exhaust gas is :

 $CO_{\rm g}$ = 11.5 per cent ; CO = 1.2 per cent ; $O_{\rm g}$ = 0.9 per cent ; $N_{\rm g}$ = 86 per cent. Also find percentage excess air.

Solution.

Name of gas	Volume per m ³ of flue gas	Molecular weight	Relative weight	Weight per kg of flue gas
	(x)	(y)	$z = x \times y$	$\frac{z}{\Sigma z}$
CO ₂	0.115	44	5.06	0.1700
co	0.012	28	0.336	0.0113
O ₂	0.009	32	0.288	0.0096
\mathbf{N}_2	0.86	28	24.08	0.8091
			$\Sigma z = 29.76$	

... Weight of carbon per kg of flue gas

= Weight of carbon in 0.17 kg of $\rm CO_2$ + Weight of carbon in 0.0113 kg of CO

		$= \frac{3}{11} \times 0.17 + \frac{3}{7} \times 0.0113 = 0.0512 \text{ kg}$
	Weight of dry flue gas per ka	g of fuel = $\frac{0.85}{0.0512}$ = 16.6 kg
Vaj	pour of combustion	$= 9 \times 0.15 = 1.35 \text{ kg}$
Tot	al weight of gas	= 16.6 + 1.35 = 17.95 kg per kg of fuel
λ.	Air supplied	= (17.95 - 1) = 16.95 kg/kg of fuel
λ.	Ratio of air to petrol	= 16.95 : 1. (Ans.)
Sto	ichiometric air	$= \left[\left(0.85 \times \frac{8}{3} \right) + (0.15 \times 8) \right] \times \frac{100}{23}$
		= 15.07 kg per kg of fuel
Δ.	Excess air	= 16.95 - 15.07 = 1.88 kg
	Percentage excess air	$=\frac{1.88}{15.07}$ × 100 = 12.47%. (Ans.)

Example 11.8. A sample of fuel has the following percentage composition : Carbon = 86 per cent ; Hydrogen = 8 per cent ; Sulphur = 3 per cent ; Oxygen = 2 per cent ; Ash = 1 per cent.

For an air-fuel ratio of 12 : 1, calculate :

(i) Mixture strength as a percentage rich or weak.

(ii) Volumetric analysis of the dry products of combustion.

Solution.

Element, wt. (kg)	O ₂ reqd. (kg)
C = 0.86	$0.86 \times \frac{8}{3} = 2.29$
$H_2 = 0.08$	$0.08 \times 8 = 0.64$
S = 0.03	$0.03 \times \frac{1}{1} = 0.03$
O ₂ = 0.02	
	Total O ₂ = 2.96

Weight of oxygen to be supplied per kg of fuel = 2.96 - 0.02 = 294 kg.

Weight of minimum air required for complete combustion = $\frac{2.94 \times 100}{23}$ = 12.78 kg

Hence "correct" fuel-air ratio $= \frac{1}{12.78}$: 1

But actual ratio is $\frac{1}{12}$: 1.

...

(*i*) Mixture strength $= \frac{12.78}{12} \times 100 = 106.5\%$

This show that mixture is 6.5% rich. (Ans.)

Deficient amount of air = 12.78 - 12 = 0.78 kg

Amount of air saved by burning 1 kg of C to CO instead of CO₂

= Oxygen saved ×
$$\frac{100}{23}$$

= $\left[\frac{8}{3}(\text{CO}_2) - \frac{4}{3}(\text{CO})\right] \times \frac{100}{23}$ = 5.8 kg

Hence $\frac{0.78}{5.8} = 0.134$ kg of carbon burns to CO and as such 0.86 - 0.134 = 0.726 kg of carbon burns to CO₂.

CO formed	$= 0.134 \times \frac{7}{3} = 0.313 \text{ kg}$
CO_2 formed	$= 0.726 \times \frac{11}{3} = 2.662 \text{ kg}$
N_2 supplied	$= 12 \times 0.77 = 9.24$ kg
SO ₂ formed	$= 0.03 \times 2 = 0.06$ kg.

(ii) The percentage composition of dry flue gases is given as below :

Dry products	Weight (kg)	Molecular weight	Proportional volume	Percentage volume
	x	у	$z = \frac{x}{y}$	$\frac{z}{\Sigma z} \times 100$
CO	0.313	28	0.0112	2.78 per cent (Ans.)
CO2	2.662	44	0.0605	15.03 per cent (Ans.)
N ₂	9.24	28	0.3300	81.97 per cent (Ans.)
SO ₂	0.06	64	0.0009	0.22 per cent (Ans.)
			$\Sigma z = 0.4026$	

Example 11.9. A fuel $(C_{10}H_{22})$ is burnt using an air-fuel ratio of 13 : 1 by weight. Determine the complete volumetric analysis of the products of combustion, assuming that the whole amount of hydrogen burns to form water vapour and there is neither any free oxygen nor any free carbon. The carbon burns to CO_2 and CO.

Air contains 77% of nitrogen and 23% of oxygen by weight.

Solution. Combustion equation is :

$$\begin{aligned} & 2\mathbf{C}_{10}\mathbf{H}_{22} + 31\mathbf{O}_2 = 20\mathbf{CO}_2 + 22\mathbf{H}_2\mathbf{O} \\ & 2\times142 + 31\times32 = 20\times44 + 22\times18 \\ & 284 + 992 = 880 + 396 \end{aligned}$$

or

 \therefore Air required for complete combustion

		$= \frac{992 \times 100}{284 \times 23} = 15.2 \text{ kg/kg of fuel}$
	Air actually supplied	= 13 kg/kg of fuel
	∴ Deficiency of air	= 15.2 - 13 = 2.2 kg/kg of fuel
	Also 1 kg of C requires $\frac{4}{3} \times \frac{10}{23}$	$\frac{10}{3}$ = 5.8 kg of less air to burn to CO instead of CO ₂ .
	Hence $\frac{2.2}{5.8} = 0.379 \text{ kg C}$	is burnt to CO ;
and	$\left(\frac{12 \times 10}{142}\right) - 0.379 = 0.466 \text{ kg c}$	of C is burnt to CO ₂ .
	Weight of CO_2 formed	$= 0.466 \times \frac{11}{3} = 1.708 \text{ kg}$
	Weight of CO formed	$= 0.379 \times \frac{7}{3} = 0.884 \text{ kg}$
	Weight of $\rm H_{2}O$ formed	$=\left(\frac{22}{142}\right)\times9=1.394~\mathrm{kg}$
	Weight of N_2 from air	$= 13 \times 0.77 = 10.01$ kg.

The	percentage	composition	of	products	of	combustion
		.				

Products of combustion	Weight x	Molecular weight	Proportional volume	Percentage
		у	$z = \frac{x}{y}$	$=\frac{z}{\Sigma z} \times 100$
CO2	1.708	44	0.0388	7.678 per cent (Ans.)
CO	0.884	28	0.0316	6.254 per cent (Ans.)
H ₂ O	1.394	18	0.0774	15.317 per cent (Ans.)
N ₂	10.01	28	0.3575	70.750 per cent (Ans.)
			$\Sigma z = 0.5053$	

0.407

Example 11.20. Calculate the stoichiometric air-fuel ratio for the combustion of a sample of dry anthracite of the following composition by mass :

(i) Air-fuel ratio ;

(ii) Wet dry analysis of the products of combustion by volume.

Solution. Stoichiometric air-fuel (A/F) ratio :

In case of a fuel with several constituents a *tubular method* is advisable, as shown below. Each constituent is taken separately and the amount of oxygen required for complete combustion is found from the chemical equation. The oxygen in the fuel is included in the column headed 'oxygen required' as a negative quantity.

	Mass per hg coal	Combustion equation	Oxygen required per hg of coal	Products per hg of coal
С	0.88	$C + O_{g} \longrightarrow CO_{g}$	$0.88 \times \frac{32}{12} = 2.346 \text{ kg}$	$0.88 \times \frac{44}{12} = 3.23 \text{ kg CO}_{9}$
H,	0.04	$12 \text{ kg} + 32 \text{ kg} \longrightarrow 44 \text{ kg}$ $2H_g + O_g \longrightarrow 2H_gO$ $1 \text{ kg} + 8 \text{ kg} \longrightarrow 9 \text{ kg}$	$0.04 \times 8 = 0.32 \text{ kg}$	$0.04 \times 9 = 0.36 \text{ kg H}_{g}\text{O}$
0,	0.035		0.035 kg	_
N ₂	0.01	_	_	0.01 kg N ₂
s	0.005	$\mathbb{S} + \mathbb{O}_{g} {\longrightarrow} \mathbb{S} \mathbb{O}_{g}$	$0.005 \times \frac{32}{32} = 0.005 \text{kg}$	$0.005 \times \frac{64}{32} = 0.01 \text{ kg SO}_{9}$
		$32 \text{ kg} + 32 \text{ kg} \longrightarrow 64 \text{ kg}$		
Ash	0.03	_	-	-
			Total $O_{g} = 2.636$ kg	

From table :

O₂ required per kg of coal = 2.636 kg

$$\therefore$$
 Air required per kg of coal = $\frac{2.636}{0.233}$ = 11.31 kg

(where air is assumed to contain 23.3% O_2 by mass)

N₂ associated with this air = $0.767 \times 11.31 = 8.67$ kg

∴ Total N₂ in products = 8.67 + 0.01 = 8.68 kg

0.000

The stoichiometric A/F ratio = 11.81/1. (Ans.) When 30 per cent excess air is used : (i) Actual A/F ratio :

Actual A/F ratio = $11.31 + 11.31 \times \frac{30}{100} = 14.7/1$. Ans.

(ii) Wet and dry analyses of products of combustion by volume : As per actual A/F ratio, N₂ supplied = $0.767 \times 14.7 = 11.27$ kg Also O₂ supplied = $0.233 \times 14.7 = 3.42$ kg

(where air is assumed to contain $N_2 = 76.7\%$ and $O_2 = 23.3$)

In the products then, we have

 $N_2 = 11.27 + 0.01 = 11.28 \text{ kg}$

and

excess $O_2 = 3.42 - 2.636 = 0.78$ kg

- The products are entered in the following table and the analysis by volume is obtained : — In column 3 the percentage by mass is given by the mass of each product divided by the
 - total mass of 15.66 kg.
- In column 5 the moles per kg of coal are given by equation $n = \frac{m}{M}$. The total of column 5 gives the total moles of wet products per kg of coal, and by subtracting the moles of H₂O from this total, the total moles of dry products is obtained as 0.5008.
- Column 6 gives the proportion of each constituent of column 5 expressed as a percentage of the total moles of the wet products.

Product	Mass/kg coal	% by mass	М	Moles / kg coal	% by vol. wet	% by vol. dry
1	2	3	4	5	6	7
CO, H,O SO, O, N,	3.23 0.36 0.01 0.78 11.28	20.62 2.29 0.06 4.98 72.03	44 18 64 32 28	0.0734 0.0200 0.0002 (say) 0.0244 0.4028	14.10 3.84 0.04 4.68 77.34	14.66 0.04 4.87 80.43
	15.66 kg		-	Total wet = 0.5208 $-H_{9}O = 0.0200$ Total dry = 0.5008	3 100.00 3	100.00 (Ans.)

— Similarly column 7 gives the percentage by volume of the dry products.

INTERNAL ENERGY AND ENTHALPY OF COMBUSTION

Example 11.31. ΔH_0 (enthalpy of combustion at reference temperature T_0) for benzene vapour (C_6H_6) at 25°C is – 3301000 kJ/mole with the H_2O in the liquid phase. Calculate ΔH_0 for the H_2O in the vapour phase.

Solution. If H₂O remains as a vapour the heat transferred to the surroundings will be less than when the vapour condenses by the amount due to the change in enthalpy of the vapour during condensation at the reference temperature.

$$\Delta H_0 \text{ (vapour)} = \Delta H_0 \text{ (liquid)} + m_s h_{fee}$$

where, $m_s = Mass \text{ of } H_2O$ formed, and

 $h_{f\theta^0}$ = Change in enthalpy of steam between saturated liquid and saturated vapour at the reference temperature T_0

= 2441.8 kJ at 25°C

For the reaction :

$$C_6H_6 + 7.5O_2 \longrightarrow 6CO_2 + 3H_2O_2$$

3 moles of H₂O are formed on combustion of 1 mole of C₆H₆; 3 moles of H₂O

= 3 × 18 = 54 kg H₂O

 $\therefore \Delta H_0 (vapour) = -3301000 + 54 \times 2441.8 = -3169143 \text{ kJ/mole.} (Ans.)$

Example 11.32. Calculate ΔU_0 in kJlkg for the combustion of benzene (C_6H_6) vapour at 25°C given that $\Delta H_0 = -3169100$ kJ/mole and the H_2O is in the vapour phase.

Solution. Given : $\Delta H_0 = -3169100 \text{ kJ}$

The combustion equation is written as

$$C_6H_6 + 7.5O_2 \longrightarrow 6CO_2 + 3H_2O$$
 (vapour)
 $n_R = 1 + 7.5 = 8.5, n_P = 6 + 3 = 9$

Using the relation,

$$\Delta U_0 = \Delta H_0 - (n_P - n_R)R_0T_0$$

= - 3169100 - (9 - 8.5) × 8.314 × (25 + 273)

= - 3169100 - 1239 = - 3170339 kJ/mole

(It may be noted that ΔU_0 is negligibly different from $\Delta H_0)$

1 mole of $C_6H_6 = 6 \times 12 + 1 \times 6 = 78 \text{ kg}$

$$\Delta U_0 = \frac{-3170339}{78} = -40645 \text{ kJ/kg.} \text{ (Ans.)}$$

Example 11.33. ΔH_0 for CO at 60°C is given as – 285200 kJ/mole. Calculate ΔH_0 at 2500°C given that the enthalpies of gases concerned in kJ/mole are as follows :

Gas		60°C	2500°C
co		9705	94080
0		9696	99790
CO_2		10760	149100
	10 A. 10 A. 10		

Solution. The reaction equation is given by

$$CO + \frac{1}{2}O_2 \longrightarrow CO_2$$

Refer Fig. 11.7.

It can be seen from the property diagram of Fig. 11.7 that the enthalpy of combustion at temperature T, ΔH_T can be obtained from ΔH_0 and T_0 by the relationship

 $\begin{array}{ll} &-\Delta H_T=-\Delta H_0+(H_{R_T}-H_{R_0})-(H_{P_T}-H_{P_0}) & \ldots(i) \\ \text{where} & H_{R_T}-H_{R_0} = \text{ increase in enthalpy of the } reactants \\ \text{from } T_0 \text{ to } T \end{array}$

and $H_{p_T} - H_{p_0}$ = increase in enthalpy of the *products* from T_0 to T.

Now, from the given data, we have

$$H_{R_0} = 1 \times 9705 + \frac{1}{2} \times 9696 = 14553 \text{ kJ}$$

 $H_{R_0} = 1 \times 94080 + \frac{1}{2} \times 99790 = 143975 \text{ kJ}$

$$\pi_{R_T} = 1 \times 94080 + \frac{1}{2} \times 99790 = 143975, k_{e}$$

 $H_{P_0} = 1 \times 10760 = 10760 \text{ kJ}$

 $H_{P_{\pi}} = 1 \times 149100 \text{ kJ} = 149100 \text{ kJ}$

Using equation (i), we get

...

$$-\Delta H_T = + 285200 + (143975 - 14553) - (149100 - 10760)$$

= 285200 + 129422 - 138340 = 276282
$$\Delta H_T = - 276282 \text{ kJ/mole.} \quad (Ans.)$$





MODULE II

Vapor Power Cycles

We know that the Carnot cycle is most efficient cycle operating between two specified temperature limits. However; the Carnot cycle is not a suitable model for steam power cycle since:

- The turbine has to handle steam with low quality which will cause erosion and wear in turbine blades.
- It is impractical to design a compressor that handles two phase.
- It is difficult to control the condensation process that precisely as to end up with the desired at point 4.



Fig. 1: T-s diagram for two Carnot vapor cycle.

Ideal Rankine Cycle

The Rankine cycle is the ideal cycle for vapor power plants; it includes the following four reversible processes:

1-2:	Isentropic compression	Water enters the pump as state 1 as saturated liquid and is compressed isentropically to the operating pressure of the boiler.
2-3:	Const P heat addition	Saturated water enters the boiler and leaves it as superheated vapor at state 3
3-4:	Isentropic expansion	Superheated vapor expands isentropically in turbine and produces work.
4-1:	Const P heat rejection	High quality steam is condensed in the condenser



Fig. 2: The ideal Rankine cycle.

Energy Analysis for the Cycle

All four components of the Rankine cycle are steady-state steady-flow devices. The potential and kinetic energy effects can be neglected. The first law per unit mass of steam can be written as:

Pump	q = 0	$w_{pump,in} = h_2 - h_1$
Boiler	w = 0	$q_{in} = h_3 - h_2$
Turbine	q = 0	$w_{turbine,out} = h_3 - h_4$
Condenser	w = 0	$q_{out} = h_4 - h_1$

The thermal efficiency of the cycle is determined from:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

where

 $w_{\mathit{net}} = q_{\mathit{in}} - q_{\mathit{out}} = w_{\mathit{turbine,out}} - w_{\mathit{pump,in}}$

If we consider the fluid to be incompressible, the work input to the pump will be:

$$(h_2 - h_1) = v(P_2 - P_1)$$

where $h_1 = h_{f@Pl} & v = v_1 = v_{f@Pl}$

$$\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \qquad \eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

Deviation of Actual Vapor Power Cycle from Ideal Cycle

As a result of irreversibilities in various components such as fluid friction and heat loss to the surroundings, the actual cycle deviates from the ideal Rankine cycle. The deviations of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies defined as:



Fig. 3: Deviation from ideal Rankine cycle.

Increasing the Efficiency of Rankine Cycle

We know that the efficiency is proportional to:

$$\eta_{th} \propto 1 - \frac{T_L}{T_H}$$

That is, to increase the efficiency one should increase the average temperature at which heat is transferred to the working fluid in the boiler, and/or decrease the average temperature at which heat is rejected from the working fluid in the condenser.

Decreasing the of Condenser Pressure (Lower TL)

Lowering the condenser pressure will increase the area enclosed by the cycle on a *T*-*s* diagram which indicates that the net work will increase. Thus, the thermal efficiency of the cycle will be increased.



Fig. 4: Effect of lowering the condenser pressure on ideal Rankine cycle.

The condenser pressure cannot be lowered than the saturated pressure corresponding to the temperature of the cooling medium. We are generally limited by the thermal reservoir temperature such as lake, river, etc. Allow a temperature difference of 10°C for effective heat transfer in the condenser.

For instance take (a) $15^{\circ}C + \Delta T (10^{\circ}C) = 25^{\circ}C$.

The steam saturation pressure (or the condenser pressure) then will be \Rightarrow P_{sat} = 3.2 *kPa*.

Superheating the Steam to High Temperatures (Increase TH)

Superheating the steam will increase the net work output and the efficiency of the cycle. It also decreases the moisture contents of the steam at the turbine exit. The temperature to which steam can be superheated is limited by metallurgical considerations (~ 620° C).



Fig. 5: The effect of increasing the boiler pressure on the ideal Rankine cycle.

Increasing the Boiler Pressure (Increase T_H)

Increasing the operating pressure of the boiler leads to an increase in the temperature at which heat is transferred to the steam and thus raises the efficiency of the cycle.



Fig.6: The effect of increasing the boiler pressure on the ideal cycle.

Note that for a fixed turbine inlet temperature, the cycle shifts to the left and the moisture content of the steam at the turbine exit increases. This undesirable side effect can be corrected by *reheating* the steam.

The Ideal Reheat Rankine Cycle

To take advantage of the increased efficiencies at higher boiler pressure without facing the excessive moisture at the final stages of the turbine, reheating is used. In the ideal reheating cycle, the expansion process takes place in two stages, i.e., the high-pressure and low-pressure turbines.



Fig. 7: The ideal reheat Rankine cycle.

The total heat input and total turbine work output for a reheat cycle become:

 $\begin{aligned} q_{in} &= q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4) \\ w_{nurbine,out} &= w_{H-P \ nurbine} + w_{L-P \ nurbine} = (h_3 - h_4) + (h_5 - h_6) \end{aligned}$

efficiency by 4 to 5 percent by increasing the average temperature at which heat is transferred to the steam.
The Ideal Regenerative Rankine Cycle

The regeneration process in steam power plants is accomplished by extracting (or bleeding) steam from turbine at various stages and feed that steam in heat exchanger where the feed water is heated. These heat exchangers are called regenerator or feedwater heater (FWH). FWH also help removing the air that leaks in at the condenser (deaerating the feedwater). There are two types of FWH's, open and closed.

Open (Direct-Contact) Feedwater Heaters

An open FWH is basically a mixing chamber where the steam extracted from the turbine mixes with the feed water exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.



Fig. 8: The ideal regenerative Rankine cycle with an open FWH.

Using Fig. 8, the heat and work interactions of a regenerative Rankine cycle with one FWH can be expressed per unit mass of steam flowing through the boiler as:

 $\begin{aligned} q_{in} &= h_5 - h_4 \\ q_{out} &= (1 - y)(h_7 - h_1) \\ w_{turbine,out} &= (h_5 - h_6) + (1 - y)(h_6 - h_7) \\ w_{pump,in} &= (1 - y)w_{PumpI} + w_{PumpII} \\ \text{where} \\ y &= \dot{m}_6 / \dot{m}_5 \\ w_{PumpI} &= v_1(P_2 - P_1) \\ w_{PumpII} &= v_3(P_4 - P_3) \end{aligned}$

Thermal efficiency of the Rankine cycle increases as a result of regeneration since FWH raises the average temperature of the water before it enters the boiler. Many large power plants have as many as 8 FWH's.

Closed Feedwater Heaters

In closed FWH, heat is transferred from the extracted steam to the feedwater without any mixing taking place. Thus; two streams can be at different pressures, since they don't mix. In an ideal closed FWH, the feedwater is heated to the exit temperature of the extracted steam, which ideally leaves the heater as a saturated liquid at the extraction pressure.





Fig.9: Ideal regenerative Rankine cycle with a closed FWH.

Open FWH	Closed FWH

- more complex (internal tubing) simple inexpensive less effective (no mixing) good heat transfer characteristics bring feedwater to the saturation FWH state
- a pump is required for each FWH ٠
- do not require a pump for each

Cogeneration

Many system and industries require energy input in the form of heat, called *process heat*. Some industries such as chemical, pulp and paper rely heavily on process heat. The process heat is typically supplied by steam at 5 to 7 atm and 150 to 200 C. These plants also require large amount of electric power. Therefore, it makes economical and engineering sense to use the already-existing work potential (in the steam entering the condenser) to use as process heat. This is called cogeneration.



Fig. 10: A cogeneration plant with adjustable loads.

In the cogeneration cycle shown in the above figure, at times of high demands for process

heat, all the steam is routed to the process heating unit and none to the condenser.

Combined Gas-Vapor Power Cycle

Gas-turbine cycles typically operate at considerably higher temperatures than steam cycles. The maximum fluid temperature at the turbine inlet is about 620C for modern steam power plants, but over 1425C for gas-turbine power plants. It is over 1500C at the burner exit of turbojet engines.

It makes engineering sense to take advantage of the very desirable characteristics of the gas-turbine cycle at high-temperature and to use the high temperature exhaust gases as the energy source for the bottoming cycle as a steam power cycle. This is called *combined cycle*. Combined cycles can achieve high thermal efficiencies, some of recent ones have η about 60%.

Gas Power Cycles:

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. *The efficiency of engine using air as the working medium is known as an "Air standard efficiency"*. This efficiency is oftenly called *ideal efficiency*. The actual efficiency of a cycle is always *less* than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "*Relative efficiency*"

which is defined as :

 $\eta_{\text{relative}} = \frac{Actual \ thermal \ efficiency}{Air \ standard \ efficiency}$

The analysis of all air standard cycles is based upon the following *assumptions* : Assumptions :

1. The gas in the engine cylinder is a *perfect gas i.e.*, it obeys the gas laws and has constant specific heats.

2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures *i.e.*, the molecular weight of cylinder gas is 29. Cp = 1.005 kJ/kg-K, Cv = 0.718 kJ/kg-K.

3. The compression and expansion processes are adiabatic and they take place without internal friction, *i.e.*, these processes are *isentropic*.

4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.

5. The cycle is considered closed with the same 'air' always remaining in the cylinder to repeat the cycle.

THE CARNOT CYCLE

This cycle has the *highest possible efficiency* and consists of four simple operations namely,

- (a) Isothermal expansion
- (b) Adiabatic expansion
- (c) Isothermal compression
- (d) Adiabatic compression.



Fig. 13.1. Carnot cycle.

Stage (1). Line 1-2 [Fig. 13.1 (*a*)] represents the isothermal expansion which takes place at temperature T1 when source of heat H is applied to the end of cylinder. Heat supplied in this case is given by $RT1 \log e r$ and where r is the ratio of expansion.

Stage (2). Line 2-3 represents the application of non-conducting cover to the end of the cylinder. This is followed by the adiabatic expansion and the temperature falls from T1 to T2.

Stage(3). Line 3-4 represents the isothermal compression which takes place when sump 'S' is applied to the end of cylinder. Heat is rejected during this operation whose value is given by $RT2 \log e r$ where r is the ratio of compression.

Stage (4). Line 4-1 represents repeated application of non-conducting cover and adiabatic compression due to which temperature increases from T2 to T1.

It may be noted that ratio of expansion during isotherm 1-2 and ratio of compression during isotherm 3-4 must be equal to get a closed cycle.

Fig. 13.1 (*b*) represents the Carnot cycle on *T*-*s* coordinates. Now according to law of conservation of energy, Heat supplied = Work done + Heat rejected

Work done = Heat supplied – Heat rejected

 $= RT1 \cdot \log e r - RT2 \log e r$

Efficiency of cycle $= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{R \log_e r (T_1 - T_2)}{RT_1 \cdot \log_e r}$ $= \frac{T_1 - T_2}{T_1}$

From this equation, it is quite obvious that if temperature T2 decreases efficiency increases and it becomes 100% if T2 becomes absolute zero which, of course is impossible to attain. Further more it is not possible to produce an engine that should work on Carnot's cycle as it would necessitate the piston to travel very slowly during first portion of the forward stroke (isothermal expansion) and to travel more quickly during the remainder of the stroke (adiabatic expansion) which however is not practicable.

CONSTANT VOLUME OR OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines. Figs. 13.5 (*a*) and (*b*) shows the theoretical p-V diagram and T-s diagrams of this cycle respectively.

- The point 1 represents that cylinder is full of air with volume V1, pressure p1 and absolute temperature T1.
- Line 1-2 represents the *adiabatic compression* of air due to which p1, V1 and T1 change to p2, V2 and T2, respectively.
- Line 2-3 shows the supply of heat to the air at constant volume so that p2 and T2 change to p3 and T3 (V3 being the same as V2).
- Line 3-4 represents the *adiabatic expansion* of the air. During expansion p3, V3 and T3change to a final value of p4, V4 or V1 and T4, respectively.
- Line 4-1 shows the *rejection of heat* by air at *constant volume* till original state (point 1) reaches. Consider 1 kg of air (working substance) :
 Heat supplied at constant volume = cv(T3 T2).
 Heat rejected at constant volume = cv (T4 T1).

But, work done = Heat supplied – Heat rejected

$$= Cv (T3 - T2) - Cv (T4 - T1)$$

Efficiency = $\frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v (T_3 - T_2) - c_v (T_4 - T_1)}{c_v (T_3 - T_2)}$
= $1 - \frac{T_4 - T_1}{T_3 - T_2}$



Let compression ratio, $r_{\rm c} (= r) = \frac{v_1}{v_2}$

 $r_e (= r) = \frac{v_4}{v_3}$ expansion ratio, and

(These two ratios are same in this cycle)

As
$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

Then,

Similarly,

 $T_{3}=T_{4}\ .\ (r)^{\gamma -1}$ Inserting the values of T_{2} and T_{3} in equation (i), we get

 $T_2 = T_1 \cdot (r)^{\gamma - 1}$

 $\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1}$

$$\begin{split} \eta_{otto} &= 1 - \frac{T_4 - T_1}{T_4 \cdot (r)^{\gamma - 1} - T_1 \cdot (r)^{\gamma - 1}} = 1 - \frac{T_4 - T_1}{r^{\gamma - 1}(T_4 - T_1)} \\ &= 1 - \frac{1}{(r)^{\gamma - 1}} \end{split}$$

This expression is known as the air standard efficiency of the Otto cycle.

It is clear from the above expression that efficiency increases with the increase in the value of r, which means we can have maximum efficiency by increasing r to a considerable extent, but due to practical difficulties its value is limited to about 8. The net work done per kg in the Otto cycle can also be expressed in terms of p, v. If p is expressed in bar i.e., 105 N/m2, then work done

or

$$W = \left(\frac{p_3v_3 - p_4v_4}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}\right) \times 10^2 \text{ kJ}$$
Also
$$\frac{p_3}{p_4} = r^{\gamma} = \frac{p_2}{p_1}$$

$$\therefore \qquad \frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$$
re r_p stands for pressure ratio.

when nds for p р

and

$$v_{1} = rv_{2} = v_{4} = rv_{3}$$

$$W = \frac{1}{\gamma - 1} \left[p_{4}v_{4} \left(\frac{p_{3}v_{3}}{p_{4}v_{4}} - 1 \right) - p_{1}v_{1} \left(\frac{p_{2}v_{2}}{p_{1}v_{1}} - 1 \right) \right]$$

$$= \frac{1}{\gamma - 1} \left[p_{4}v_{4} \left(\frac{p_{3}}{p_{4}r} - 1 \right) - p_{1}v_{1} \left(\frac{p_{2}}{p_{1}r} - 1 \right) \right]$$

$$= \frac{v_{1}}{\gamma - 1} \left[p_{4} \left(r^{\gamma - 1} - 1 \right) - p_{1} \left(r^{\gamma - 1} - 1 \right) \right]$$

$$\begin{split} &= \frac{v_1}{\gamma - 1} \bigg[(r^{\gamma - 1} - 1)(p_4 - p_1) \bigg] \\ &= \frac{p_1 v_1}{\gamma - 1} \bigg[(r^{\gamma - 1} - 1)(r_p - 1) \bigg] \end{split}$$

Mean effective pressure (p_m) is given by :

$$\begin{split} p_m &= \left[\left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \right) \div (v_1 - v_2) \right] \text{ bar} \\ p_m &= \frac{\left[\frac{p_1 v_1}{\gamma - 1} (r^{\gamma - 1} - 1) (r_p - 1) \right]}{(v_1 - v_2)} \\ &= \frac{\frac{p_1 v_1}{\gamma - 1} \left[(r^{\gamma - 1} - 1) (r_p - 1) \right]}{v_1 - \frac{v_1}{r}} \\ &= \frac{\frac{p_1 v_1}{\gamma - 1} \left[(r^{\gamma - 1} - 1) (r_p - 1) \right]}{v_1 \left(\frac{r - 1}{r} \right)} \\ p_m &= \frac{p_1 r \left[(r^{\gamma - 1} - 1) (r_p - 1) \right]}{(\gamma - 1)(r - 1)} \end{split}$$

Also

i.e.,

Example 13.9. The minimum pressure and temperature in an Otto cycle are 100 kPa and 27°C. The amount of heat added to the air per cycle is 1500 kJ/kg.

 (i) Determine the pressures and temperatures at all points of the air standard Otto cycle.
 (ii) Also calculate the specific work and thermal efficiency of the cycle for a compression ratio of 8 : 1.

Take for air : $c_v = 0.72 \text{ kJ/kg K}$, and $\gamma = 1.4$. (GATE, 1998)

Solution. Refer Fig. 13.7. Given : $p_1 = 100 \text{ kPa} = 10^5 \text{ N/m}^2 \text{ or } 1 \text{ bar}$;

 $T_1 = 27 + 273 = 300 \text{ K}$; Heat added = 1500 kJ/kg;

r = 8 : 1; $c_v = 0.72$ kJ/kg; $\gamma = 1.4$.

Consider 1 kg of air.





(i) Pressures and temperatures at all points :

Adiabatic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (8)^{14-1} = 2.297$$
$$T_2 = 300 \times 2.297 = 689.1 \text{ K.} \quad (\text{Ans.})$$

Also

Δ.

λ.

$$p_1 v_1^{T} = p_2 v_2^{T}$$

or

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^2 = (8)^{14} = 18.379$$

 $\mathbf{p}_2 = 1 \times 18.379 = 18.379$ bar. (Ans.)

Constant volume process 2-3 :

Heat added during the process,

$$c_v (T_3 - T_2) = 1500$$

$$0.72 (T_2 - 689.1) = 1500$$

or

or

$$T_3 = \frac{1500}{0.72} + 689.1 = 2772.4 \text{ K.}$$
 (Ans.)

Also

Δ.

b,
$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow \mathbf{p}_3 = \frac{p_2 T_3}{T_2} = \frac{18.379 \times 2772.4}{6891} = 73.94 \text{ bar.} \text{ (Ans.)}$$

Adiabatic Expansion process 3-4 :

$$\begin{aligned} \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = (r)^{\gamma-1} = (8)^{14-1} = 2.297\\ \mathbf{T_4} = \frac{T_3}{2.297} = \frac{2772.4}{2.297} = \mathbf{1206.9} \ \mathbf{K}. \end{aligned} (Ans.)$$

Also,
$$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \implies \mathbf{p}_4 = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma} = 73.94 \times \left(\frac{1}{8}\right)^{14} = 4.023$$
 bar. (Ans.)

(ii) Specific work and thermal efficiency :

$$= c_v (T_3 - T_2) - c_v (T_4 - T_1) = c_v [(T_3 - T_2) - (T_4 - T_1)]$$

= 0.72 [(2772.4 - 689.1) - (1206.9 - 300)] = 847 kJ/kg. (Ans.)

Thermal efficiency, $\eta_{th} = 1 - \frac{1}{(r)^{\gamma-1}}$

$$= 1 - \frac{1}{(8)^{14-1}} = 0.5647$$
 or 56.47%. (Ans.)

Example 13.10. An air standard Otto cycle has a volumetric compression ratio of 6, the lowest cycle pressure of 0.1 MPa and operates between temperature limits of 27°C and 1569°C.

 (i) Calculate the temperature and pressure after the isentropic expansion (ratio of specific heats = 1.4).

(ii) Since it is observed that values in (i) are well above the lowest cycle operating conditions, the expansion process was allowed to continue down to a pressure of 0.1 MPa. Which process is required to complete the cycle ? Name the cycle so obtained.

(iii) Determine by what percentage the cycle efficiency has been improved. (GATE, 1994)

Solution. Refer Fig. 13.8. Given : $\frac{v_1}{v_2} = \frac{v_4}{v_3} = r = 6$; $p_1 = 0.1$ MPa = 1 bar; $T_1 = 27 + 273$

= 300 K ; $T_3 = 1569 + 273 = 1842$ K ; $\gamma = 1.4$.

(i) Temperature and pressure after the isentropic expansion, T_4 , p_4 : Consider 1 kg of air :

For the compression process 1-2 :

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \implies p_2 = p_1 \times \left(\frac{v_1}{v_2}\right)^{\gamma} = 1 \times (6)^{14} = 12.8 \text{ bar}$$

Also

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (6)^{14-1} = 2.048$$

 $T_2 = 300 \times 2.048 = 614.4 \text{ K}$

2

For the constant volume process 2-3 :



For the expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1} = (6)^{14 - 1} = 2.048$$
$$T_4 = \frac{T_3}{2.048} = \frac{1842}{2.048} = 900 \text{ K.} \quad (\text{Ans.})$$

Also

...

$$\mathbf{p}_4 = 36.9 \times \left(\frac{1}{6}\right)^{1.4} = 3$$
 bar. (Ans.)

or

(ii) Process required to complete the cycle :

Process required to complete the cycle is the constant pressure scavenging.

 $p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \implies p_4 = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma}$

The cycle is called Atkinson cycle (Refer Fig. 13.9).

(iii) Percentage improvement/increase in efficiency :

$$\begin{split} \eta_{\text{Otto}} &= 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(6)^{14 - 1}} = 0.5116 \quad \text{or} \quad 51.16\%. \quad (\text{Ans.}) \\ \eta_{\text{Atkinson}} &= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} \\ &= \frac{c_v(T_3 - T_2) - c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{\gamma(T_5 - T_1)}{(T_3 - T_2)} \end{split}$$

Example 13.15. (a) An engine working on Otto cycle, in which the salient points are 1, 2, 3 and 4, has upper and lower temperature limits T_3 and T_1 . If the maximum work per kg of air is to be done, show that the intermediate temperature is given by

$$T_2 = T_4 = \sqrt{T_1 T_3}$$

(b) If an engine works on Otto cycle between temperature limits 1450 K and 310 K, find the maximum power developed by the engine assuming the circulation of air per minute as 0.38 kg.

Solution. (a) Refer Fig. 13.13 (Example 13.14).

Using the equation (iii) of example 13.14.

$$W = c_v \left[T_3 - T_1 \cdot (r)^{\gamma - 1} - \frac{T_3}{(r)^{\gamma - 1}} + T_1 \right]$$

and differentiating W w.r.t. r and equating to zero

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

$$T_2 = T_1(r)^{\gamma-1}$$
 and $T_4 = T_3/(r)^{\gamma-1}$

Substituting the value of r in the above equation, we have

$$\begin{split} T_2 &= T_1 \left[\left(\frac{T_3}{T_1} \right)^{\nu_2(\gamma-1)} \right]^{\gamma-1} = T_1 \left(\frac{T_3}{T_1} \right)^{\nu_2} = \sqrt{T_1 T_3} \\ T_4 &= \frac{T_3}{\left[\left(\frac{T_3}{T_1} \right)^{\nu_2(\gamma-1)} \right]^{\gamma-1}} = \frac{T_3}{\left(\frac{T_3}{T_1} \right)^{\nu_2}} = \sqrt{T_3 T_1} \end{split}$$

Similarly,

.

 $T_2=T_4=\sqrt{T_1T_3}$. Proved.

(b) Power developed, P :

$$\begin{array}{ll} T_1 = 310 \text{ K} \\ T_3 = 1450 \text{ K} \\ m = 0.38 \text{ kg} \end{array} \qquad \dots (\text{given}) \\ m = 0.38 \text{ kg} \end{array} \\ \end{array} \\ \text{Work done} \qquad \qquad W = c_v \left[(T_3 - T_2) - (T_4 - T_1) \right] \\ T_2 = T_4 = \sqrt{T_1 T_3} = \sqrt{310 \times 1450} = 670.4 \text{ K} \\ \therefore \qquad \qquad W = 0.71 \left[(1450 - 670.4) - (670.4 - 310) \right] \\ = 0.71 \left[(779.6 - 360.4) = 297.6 \text{ kJ/kg} \\ \end{array} \\ \text{Work done per second} \qquad = 297.6 \times (0.38/60) = 1.88 \text{ kJ/s} \\ \text{Hence power developed, } \mathbf{P} = 1.88 \text{ kW}, \quad (\text{Ans.}) \end{array}$$

CONSTANT PRESSURE OR DIESEL CYCLE:

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle in that *heat is supplied at constant pressure instead of at constant volume*. Fig. 13.15 (*a* and *b*) shows the *p*-*v* and *T*-*s* diagrams of this cycle respectively.

This cycle comprises of the following *operations*:

- (i) 1-2.....Adiabatic compression.
- (ii) 2-3.....Addition of heat at constant pressure.
- (iii) 3-4.....Adiabatic expansion.

(iv) 4-1.....Rejection of heat at constant volume.

Point 1 represents that the cylinder is full of air. Let p1, V1 and T1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (*i.e.* $pV\gamma$ = constant) till the values become p2, V2 and T2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let



is called the *point of cut-off*. The air then expands adiabatically to the conditions p4, V4 and T4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

Consider 1 kg of air.

Heat supplied at constant pressure = Cp(T3 - T2)

Heat rejected at constant volume = Cv(T4 - T1)

Work done = Heat supplied – heat rejected

$$\begin{split} &= c_p(T_3 - T_2) - c_v(T_4 - T_1) \\ &\eta_{\text{diesel}} = \frac{\text{Work done}}{\text{Heat supplied}} \\ &= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} \\ &= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \qquad \qquad \dots (i) \left[\because \frac{c_p}{c_v} = \gamma \right] \end{split}$$

Let compression ratio, $r = \frac{v_1}{v_2}$, and cut-off ratio, $\rho = \frac{v_3}{v_2}$ *i.e.*, $\frac{\text{Volume at cut-off}}{\text{Clearance volume}}$ Now, during *adiabatic compression 1-2*,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} \text{ or } T_2 = T_1 \cdot (r)^{\gamma-1}$$

During constant pressure process 2-3,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho \quad \text{or} \quad T_3 = \rho \ . \ T_2 = \rho \ . \ T_1 \ . \ (r)^{\gamma - 1}$$

During adiabatic expansion 3-4

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma-1} \qquad \qquad \left(\because \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho}\right)$$
$$T_4 = \frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = \frac{\rho \cdot T_1(r)^{\gamma-1}}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = T_1 \cdot \rho^{\gamma}$$

By inserting values of $T_{\rm 2},\,T_{\rm 3}$ and $T_{\rm 4}$ in eqn. $(i),\,{\rm we}$ get

$$\begin{split} \eta_{\text{diesel}} &= 1 - \frac{(T_1 \cdot \rho^{\gamma} - T_1)}{\gamma \left(\rho \cdot T_1 \cdot (r)^{\gamma - 1} - T_1 \cdot (r)^{\gamma - 1}\right)} = 1 - \frac{(\rho^{\gamma} - 1)}{\gamma (r)^{\gamma - 1} (\rho - 1)} \\ \eta_{\text{diesel}} &= 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] \qquad \dots (13.7) \end{split}$$

It may be observed that eqn. (13.7) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $\rho > 1$. Hence *for a given compression ratio, the Otto cycle is more efficient*.

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The *net work* for diesel cycle can be expressed in terms of pv as follows:

$$\begin{split} W &= p_{2}(v_{3} - v_{2}) + \frac{p_{3}v_{3} - p_{4}v_{4}}{\gamma - 1} - \frac{p_{2}v_{2} - p_{1}v_{1}}{\gamma - 1} \\ &= p_{2} \left(\rho v_{2} - v_{2}\right) + \frac{p_{3}\rho v_{2} - p_{4}rv_{2}}{\gamma - 1} - \frac{p_{2}v_{2} - p_{1}rv_{2}}{\gamma - 1} \\ &\left[\because \frac{v_{3}}{v_{2}} = \rho \quad \therefore \quad v_{3} = \rho v_{2} \text{ and } \frac{v_{1}}{v_{2}} = r \quad \therefore \quad v_{1} = rv_{2} \\ &\text{But } v_{4} = v_{1} \quad \therefore \quad v_{4} = rv_{2} \\ \end{bmatrix} \\ &= p_{2}v_{2} \left(\rho - 1\right) + \frac{p_{3}\rho v_{2} - p_{4}rv_{2}}{\gamma - 1} - \frac{p_{2}v_{2} - p_{1}rv_{2}}{\gamma - 1} \\ &= \frac{v_{2}\left[p_{2}\left(\rho - 1\right)(\gamma - 1) + p_{3}\rho - p_{4}r - \left(p_{2} - p_{1}r\right)\right]}{\gamma - 1} \\ &= \frac{v_{2}\left[p_{2}\left(\rho - 1\right)(\gamma - 1\right) + p_{3}\left(\rho - \frac{p_{4}r}{p_{3}}\right) - p_{2}\left(1 - \frac{p_{1}r}{p_{2}}\right)\right]}{\gamma - 1} \\ &= \frac{p_{2}v_{2}\left[\left(\rho - 1\right)(\gamma - 1\right) + \rho - \rho^{\gamma} \cdot r^{1 - \gamma} - \left(1 - r^{1 - \gamma}\right)}{\gamma - 1} \\ &\left[\because \frac{p_{4}}{p_{3}} = \left(\frac{v_{3}}{v_{4}}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} = \rho^{\gamma} r^{-\gamma} \right] \\ &= \frac{p_{1}v_{1}r^{\gamma - 1}\left[\left(\rho - 1\right)(\gamma - 1\right) + \rho - \rho^{\gamma}r^{1 - \gamma} - \left(1 - r^{1 - \gamma}\right)}{\gamma - 1} \\ &\left[\because \frac{p_{2}}{p_{1}} = \left(\frac{v_{1}}{v_{2}}\right)^{\gamma} \quad \text{or} \quad p_{2} = p_{1} \cdot r^{\gamma} \text{ and } \frac{v_{1}}{v_{2}} = r \quad \text{or} \quad v_{2} = v_{1}r^{-1} \right] \\ &= \frac{p_{1}v_{2}r^{\gamma - 1}\left[\gamma(\rho - 1) - r^{1 - \gamma}\left(\rho^{\gamma} - 1\right)\right]}{(\gamma - 1)} & \dots (13.8) \end{split}$$

Mean effective pressure \boldsymbol{p}_m is given by :

$$p_m = \frac{p_1 v_1 r^{\gamma - 1} \left[\gamma(\rho - 1) - r^{1 - \gamma} \left(\rho^{\gamma} - 1\right)\right]}{(\gamma - 1) v_1 \left(\frac{r - 1}{r}\right)}$$
$$p_m = \frac{p_1 r^{\gamma} \left[\gamma(\rho - 1) - r^{1 - \gamma} \left(\rho^{\gamma} - 1\right)\right]}{(\gamma - 1)(r - 1)} .$$

Example 13.21. An engine with 200 mm cylinder diameter and 300 mm stroke works on theoretical Diesel cycle. The initial pressure and temperature of air used are 1 bar and 27°C. The cut-off is 8% of the stroke. Determine :

(i) Pressures and temperatures at all salient points.

- (ii) Theoretical air standard efficiency.
- (iii) Mean effective pressure.
- (iv) Power of the engine if the working cycles per minute are 380.
- Assume that compression ratio is 15 and working fluid is air.

Consider all conditions to be ideal.

Solution. Refer Fig. 13.17.



Fig. 13.17

Cylinder diameter, D = 200 mm or 0.2 mStroke length, L = 300 mm or 0.3 m $p_1 = 1.0$ bar Initial pressure, $T_1 = 27 + 273 = 300 \text{ K}$ Initial temperature, $=\frac{8}{100}V_{s}=0.08V_{s}$

Cut-off

Now, stroke volume,

(i) Pressures and temperatures at salient points :

 $V_s = \pi/4 \ D^2 L = \pi/4 \times 0.2^2 \times 0.3 = 0.00942 \ m^3$

$$\begin{split} V_1 &= V_s + V_c = V_s + \frac{V_s}{r-1} & \left[\because V_c = \frac{V_s}{r-1} \right] \\ &= V_s \left(1 + \frac{1}{r-1} \right) = \frac{r}{r-1} \times V_s \\ V_1 &= \frac{15}{15-1} \times V_s = \frac{15}{14} \times 0.00942 = 0.0101 \text{ m}^8. \end{split}$$
 (Ans.)

Mass of the air in the cylinder can be calculated by using the gas equation,

$$\begin{split} p_1 V_1 &= \ mRT_1 \\ m &= \ \frac{p_1 V_1}{RT_1} = \frac{1 \times 10^5 \times 0.0101}{287 \times 300} \ = \ 0.0117 \ \text{kg/cycle} \end{split}$$

For the adiabatic (or isentropic) process 1-2

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \text{ or } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma}$$

$$p_2 = p_1 \cdot (r)^{\gamma} = 1 \times (15)^{1.4} = 44.31 \text{ bar. (Ans.)}$$
Also,
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.4-1} = 2.954$$

$$\therefore \qquad T_2 = T_1 \times 2.954 = 300 \times 2.954 = 886.2 \text{ K. (Ans.)}$$

$$V_2 = V_c = \frac{V_s}{r-1} = \frac{0.00942}{15-1} = 0.0006728 \text{ m}^3. \text{ (Ans.)}$$

$$p_2 = p_3 = 44.31 \text{ bar. (Ans.)}$$

$$(4ns.) = \frac{\rho - 1}{r-1}$$

$$\frac{8}{100} = \frac{\rho - 1}{15-1}$$

$$\rho = 0.08 \times 14 + 1 = 2.12$$

$$V_3 = \rho V_2 = 2.12 \times 0.0006728 = 0.001426 \text{ m}^3. \text{ (Ans.)}$$

$$\left[\begin{array}{c} V_3 \text{ can also be calculated as follows:}\\ V_3 = 0.08V_s + V_c = 0.08 \times 0.00942 + 0.0006728 = 0.001426 \text{ m}^3 \end{array}\right]$$

For the constant pressure process 2-3,

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$
$$T_8 = T_2 \times \frac{V_3}{V_2} = 886.2 \times \frac{0.001426}{0.0006728} = 1878.3 \text{ K.} \quad (\text{Ans.})$$

For the isentropic process 3-4,

$$p_{3}V_{3}^{\gamma} = p_{4}V_{4}^{\gamma}$$

$$p_{4} = p_{3} \times \left(\frac{V_{3}}{V_{4}}\right)^{\gamma} = p_{3} \times \frac{1}{(7.07)^{14}}$$

$$= \frac{44.31}{(7.07)^{14}} = 2.866 \text{ bar. (Ans.)}$$

$$\begin{bmatrix} \because \frac{V_{4}}{V_{3}} = \frac{V_{4}}{V_{2}} \times \frac{V_{2}}{V_{3}} = \frac{V_{1}}{V_{2}} \times \frac{V_{2}}{V_{3}} \\ = \frac{r}{\rho}, \quad \because \quad V_{4} = V_{1} = \frac{15}{2.12} = 7.07 \end{bmatrix}$$

(Ans.)

Also,

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$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{7.07}\right)^{14-1} = 0.457$$
$$T_4 = T_3 \times 0.457 = 1878.3 \times 0.457 = 858.38 \text{ K.}$$

 $V_4 = V_1 = 0.0101$ m⁸. (Ans.) (ii) Theoretical air standard efficiency :

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] = 1 - \frac{1}{14 (15)^{14-1}} \left[\frac{(2.12)^{14} - 1}{2.12 - 1} \right]$$
$$= 1 - 0.2418 \times 1.663 = 0.598 \text{ or } 59.8\%. \quad \text{(Ans.)}$$

(iii) Mean effective pressure, p_m :

Mean effective pressure of Diesel cycle is given by

$$p_m = \frac{p_1(r)^{\gamma}[\gamma(\rho-1) - r^{1-\gamma}(\rho^{\gamma} - 1)}{(\gamma - 1)(r - 1)}$$
$$= \frac{1 \times (15)^{1.4}[1.4(2.12 - 1) - (15)^{1-1.4}(2.12^{1.4} - 1)]}{(1.4 - 1)(15 - 1)}$$
$$= \frac{44.31[1.568 - 0.338 \times 1.863]}{0.4 \times 14} = 7.424 \text{ bar.} \text{ (Ans.)}$$

(iv) Power of the engine, P :

Work done per cycle	$= p_m V_s = \frac{7.424 \times 10^5 \times 0.00942}{10^3} = 6.99 \text{ kJ/cycle}$
Work done per second	= Work done per cycle \times no. of cycles per second
	$= 6.99 \times 380/60 = 44.27 \text{ kJ/s} = 44.27 \text{ kW}$
Hence power of the engine	= 44.27 kW. (Ans.)

Example 13.18. The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m³ and fuel injection takes place at constant pressure for 5 per cent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.

Solution. Refer Fig. 13.3	16.
Length of stroke,	L = 250 mm = 0.25 m
Diameter of cylinder,	D = 150 mm = 0.15 m
Clearance volume,	$V_2 = 0.0004 \text{ m}^3$
Swept volume,	$V_s = \pi/4 \ D^2 L = \pi/4 \times 0.15^2 \times 0.25 = 0.004418 \ \text{m}^3$
Total cylinder volume	= Swept volume + clearance volume
	$= 0.004418 + 0.0004 = 0.004818 \text{ m}^3$
Volume at point of cut-off	$V_3 = V_2 + \frac{5}{100} V_s$
	$= 0.0004 + \frac{5}{100} \times 0.004418 = 0.000621 \text{ m}^3$
∴ Cut-off ratio,	$\rho = \frac{V_3}{V_2} = \frac{0.000621}{0.0004} = 1.55$
Compression ratio,	$r = \frac{V_1}{V_2} = \frac{V_s + V_2}{V_2} = \frac{0.004418 + 0.0004}{0.0004} = 12.04$
Hence,	$\eta_{diesel} = 1 - \frac{1}{\gamma(r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4 \times (12.04)^{1.4 - 1}} \left[\frac{(1.55)^{1.4} - 1}{1.55 - 1} \right]$
	$= 1 - 0.264 \times 1.54 = 0.593$ or 59.3%. (Ans.)

Example 13.19. Calculate the percentage loss in the ideal efficiency of a diesel engine with compression ratio 14 if the fuel cut-off is delayed from 5% to 8%.

Solution. Let the clearance volume (V_2) be unity.

Then, compression ratio, r = 14

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Now, when the fuel is cut off at 5%, we have

 $\frac{\rho - 1}{r - 1} = \frac{5}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = 0.05 \quad \text{or} \quad \rho - 1 = 13 \times 0.05 = 0.65$ $\rho = 1.65$ $\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4 \times (14)^{14 - 1}} \left[\frac{(1.65)^{1.4} - 1}{1.65 - 1} \right]$

 $= 1 - 0.248 \times 1.563 = 0.612$ or 61.2%

When the fuel is cut-off at 8%, we have

$$\frac{\rho - 1}{r - 1} = \frac{8}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = \frac{8}{100} = 0.08$$

$$\rho = 1 + 1.04 = 2.04$$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4 \times (14)^{1.4 - 1}} \left[\frac{(2.04)^{1.4} - 1}{2.04 - 1} \right]$$

$$= 1 - 0.248 \times 1.647 = 0.591$$
 or 59.1%.

Hence percentage loss in efficiency due to delay in fuel cut off = 61.2 - 59.1 = 2.1%. (Ans.)

DUAL COMBUSTION CYCLE

This cycle (also called the limited pressure cycle or mixed cycle) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.

The dual combustion cycle (Fig. 13.19) consists of the following operations :

- (i) 1-2—Adiabatic compression
- (ii) 2-3—Addition of heat at constant volume
- (iii) 3-4—Addition of heat at constant pressure
- (iv) 4-5—Adiabatic expansion
- (v) 5-1—*Rejection of heat at constant volume.*



Consider 1 kg of air. Total heat supplied

= Heat supplied during the operation 2-3

+ heat supplied during the operation 3-4

$$= c_{p}(T_{2} - T_{2}) + c_{p}(T_{4} - T_{2})$$

Heat rejected during operation $5-1 = c_v(T_5 - T_1)$ Work done = Heat supplied – heat rejected

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$$

$$\begin{split} \eta_{\text{dual}} &= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v \left(T_3 - T_2\right) + c_p \left(T_4 - T_3\right) - c_v \left(T_5 - T_1\right)}{c_v \left(T_3 - T_2\right) + c_p \left(T_4 - T_3\right)} \\ &= 1 - \frac{c_v \left(T_5 - T_1\right)}{c_v \left(T_3 - T_2\right) + c_p \left(T_4 - T_3\right)} \\ &= 1 - \frac{c_v \left(T_5 - T_1\right)}{\left(T_3 - T_2\right) + \gamma \left(T_4 - T_3\right)} \\ & \dots(i) \quad \left(\because \quad \gamma = \frac{c_p}{c_v}\right) \\ r &= \frac{v_1}{v_2} \end{split}$$

Compression ratio, r

During adiabatic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \qquad \dots (ii)$$

During constant volume heating process,

$$\begin{array}{l} \frac{p_3}{T_3} = \frac{p_2}{T_2} \\ \\ \frac{T_3}{T_2} = \frac{p_3}{p_2} = \beta, \mbox{ where } \beta \mbox{ is known as pressure or explosion ratio.} \\ \\ T_2 = \frac{T_3}{\beta} \mbox{ ...(iii)} \end{array}$$

During adiabatic expansion process,

$$\left(\because \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_2} \times \frac{v_2}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{r}{\rho}, \rho \text{ being the cut-off ratio} \right)$$

During constant pressure heating process,

$$\begin{array}{l} \frac{v_3}{T_3} = \frac{v_4}{T_4} \\ T_4 = T_3 \ \frac{v_4}{v_3} = \rho \ T_3 \end{array} \qquad \dots (v) \\ \end{array}$$

Putting the value of $T_{\rm 4}$ in the eqn. (iv), we get

$$\frac{\rho T_3}{T_5} = \left(\frac{r}{\rho}\right)^{\gamma - 1} \quad \text{or} \quad T_5 = \rho \cdot T_3 \cdot \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

Putting the value of $T_{\rm 2}$ in eqn. (ii), we get

$$\frac{\frac{T_3}{\beta}}{T_1} = (r)^{\gamma - 1}$$
$$T_1 = \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma - 1}}$$

Now inserting the values of T_1 , T_2 , T_4 and T_5 in eqn. (i), we get

Work done is given by,

$$\begin{split} W &= p_3(v_4 - v_3) + \frac{p_4v_4 - p_5v_5}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1} \\ &= p_3v_3(\rho - 1) + \frac{(p_4\rho v_3 - p_5rv_3) - (p_2v_3 - p_1rv_3)}{\gamma - 1} \\ &= \frac{p_3v_3(\rho - 1)(\gamma - 1) + p_4v_3\left(\rho - \frac{p_5}{p_4}r\right) - p_2v_3\left(1 - \frac{p_1}{p_2}r\right)}{\gamma - 1} \\ \frac{p_5}{p_4} &= \left(\frac{v_4}{v_5}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} \quad \text{and} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma} \\ p_2 &= p_{41}v_2 = v_{21}v_5 = v_1 \end{split}$$

Also

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also,

i.e.,

$$\begin{split} p_{3} &= p_{4}, v_{2} = v_{3}, v_{5} = v_{1} \\ W &= \frac{v_{3}[p_{3}(\rho - 1)(\gamma - 1) + p_{3}(\rho - \rho^{\gamma}r^{1 - \gamma}) - p_{2}(1 - r^{1 - \gamma})]}{(\gamma - 1)} \\ &= \frac{p_{2}v_{2}[\beta(\rho - 1)(\gamma - 1) + \beta(\rho - \rho^{\gamma}r^{1 - \gamma}) - (1 - r^{1 - \gamma})]}{(\gamma - 1)} \\ &= \frac{p_{1}(r)^{\gamma}v_{1}h[\beta\gamma(\rho - 1) + (\beta - 1) - r^{1 - \gamma}(\beta\rho^{\gamma} - 1)]}{\gamma - 1} \\ &= \frac{p_{1}v_{1}r^{\gamma - 1}[\beta\gamma(\rho - 1) + (\beta - 1) - r^{\gamma - 1}(\beta\rho^{\gamma} - 1)]}{\gamma - 1} \qquad ...(13.11) \end{split}$$

Mean effective pressure (\boldsymbol{p}_{m}) is given by,

$$p_{m} = \frac{W}{v_{1} - v_{2}} = \frac{W}{v_{1} \left(\frac{r-1}{r}\right)} = \frac{p_{1}v_{1}[r^{1-\gamma}\beta\gamma(\rho-1) + (\beta-1) - r^{1-\gamma}(\beta\rho^{\gamma}-1)]}{(\gamma-1)v_{1}\left(\frac{r-1}{r}\right)}$$
$$p_{m} = \frac{p_{1}(r)^{\gamma}[\beta(\rho-1) + (\beta-1) - r^{1-\gamma}(\beta\rho^{\gamma}-1)]}{(\gamma-1)(r-1)} \qquad \dots(13.12)$$

COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the *important variable factors which are used as a basis for comparison of the cycles:*

- Compression ratio.
- Maximum pressure
- Heat supplied
- ➢ Heat rejected
- > Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

13.7.1. Efficiency Versus Compression Ratio

Fig. 13.26 shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut-off ratio for the Diesel and Dual combustion cycles. It is evident from the Fig. 13.26 that the air standard efficiencies *increase with the increase in the compression ratio*. For a given compression ratio Otto cycle is the most efficient while the Diesel cycle is the least efficient. (notto > ndual > ndiesel).

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

13.7.2. For the Same Compression Ratio and the Same Heat Input

A comparison of the cycles (Otto, Diesel and Dual) on the *p*-*v* and *T*-*s* diagrams for the *same compression ratio and heat supplied* is shown in the Fig. 13.27.



Fig. 13.27. (a) p-v diagram, (b) T-s diagram.



Fig. 13.26. Comparison of efficiency at various compression ratios.

Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of *heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram.* As is evident from the eqn. (13.13) the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

For Constant Maximum Pressure and Heat Supplied

Fig. 13.28 shows the Otto and Diesel cycles on p-v and T-s diagrams for constant maximum pressure and heat input respectively.



For the maximum pressure the points 3 and 3' must lie on a constant pressure line. On T-s diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4' to 1; hence the Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied.

BRAYTON CYCLE

Brayton cycle is a constant pressure cycle for a perfect gas. It is also called **Joule** cycle. The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas turbine plant would perform the processes that make up a Brayton cycle. The cycle is shown in the Fig. 13.33 (*a*) and it is represented on p-v and T-s diagrams as shown in Figs. 13.33 (*b*) and (*c*).

Operation 1-2. The air is compressed isentropically from the lower pressure p1 to the upper pressure P_2 , the temperature rising from T1 to T2.

Operation 2-3. Heat flows into the system increasing the volume from V2 to V3 and temperature from T2 to T3 whilst the pressure remains constant at p2.

Heat received = mcp (T3 - T2).

Operation 3-4. The air is expanded isentropically from p2 to p1, the temperature falling from T3 to T4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V4 to V1 and the temperature from T4 to T1 whilst the pressure remains constant at p1.



Heat rejected = $mC_p (T4 - T1)$.

Fig. 13.33. Brayton cycle : (a) Basic components of a gas turbine power plant (b) p-V diagram (c) T-s diagram.



$$\begin{split} \eta_{\text{air-standard}} &= \frac{\text{Work done}}{\text{Heat received}} \\ &= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}} \\ &= \frac{mc_p \left(T_3 - T_2\right) - mc_p \left(T_4 - T_1\right)}{mc_p \left(T_3 - T_2\right)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \end{split}$$

Now, from isentropic expansion,



Fig. 13.34. Effect of pressure ratio on the efficiency of Brayton cycle.

The eqn. (13.16) shows that the efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.

Pressure Ratio for Maximum Work

Now we shall prove that the pressure ratio for maximum work is a function of the limiting temperature ratio.

Work output during the cycle = Heat received/cycle – heat rejected/cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle} \\ = mc_p (T_3 - T_2) - mc_p (T_4 - T_1) \\ = mc_p (T_3 - T_4) - mc_p (T_2 - T_1) \\ = mc_p T_3 \left(1 - \frac{T_4}{T_3} \right) - T_1 \left(\frac{T_2}{T_1} - 1 \right) \\ \end{cases}$$

In case of a given turbine the minimum temperature T1 and the maximum temperature T3 are prescribed, T1 being the temperature of the atmosphere and T3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure cp to be constant. Then

 $\frac{T_3}{T_4} = (r_p)^{\frac{\gamma - 1}{\gamma}} = \frac{T_2}{T_1}$

Using the constant $z' = \frac{\gamma - 1}{\gamma}$,

we have, work output/cycle

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$$W = K \left[T_{3} \left(1 - \frac{1}{r_{p}^{z}} \right) - T_{1} \left(r_{p}^{z} - 1 \right) \right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K \left[T_3 \times \frac{z}{r_p(z+1)} - T_1 z r_p^{(z-1)} \right] = 0 \text{ for a maximum}$$
$$\frac{zT_3}{r_p^{(z+1)}} = T_1 z (r_p)^{(z-1)}$$
$$r_p^{-2z} = \frac{T_3}{T_1}$$

$$r_p = (T_3/T_1)^{D/2} \quad i.e., \quad r_p = (T_3/T_1)^{2(\gamma-1)} \qquad \dots (13.17)$$

Thus, the pressure ratio for maximum work is a function of the limiting temperature ratio.

Work Ratio

Work ratio is defined as the ratio of net work output to the work done by the turbine.

Work ratio =
$$\frac{W_T - W_C}{W_T}$$

[where, W_T = Work obtained from this turbine,
and W_C = Work supplied to the compressor.]
= $\frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$
= $1 - \frac{T_1}{T_3} \left[\frac{\frac{\gamma - 1}{\gamma}}{1 - \frac{1}{(r_p)^{\gamma}} - 1}}{1 - \frac{1}{(r_p)^{\gamma}}} \right] = 1 - \frac{T_1}{T_3} (r_p)^{\frac{\gamma - 1}{\gamma}}$

Methods for Improvement of Thermal Efficiency of Open Cycle Gas

Turbine Plant

The following methods are employed to increase the specific output and thermal efficiency of the plant :

1. Intercooling 2. Reheating 3. Regeneration.

1. **Intercooling.** A compressor in a gas turbine cycle utilises the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and incorporating an intercooler between the two as shown in Fig. 13.37. The corresponding T-s diagram for the unit is shown in Fig. 13.38. The actual processes take place as follows :

- ▶ 1-2′ ... L.P. (Low pressure) compression
- ➤ 2'-3 … Intercooling
- ➢ 3-4′ ... H.P. (High pressure) compression
- ▶ 4′-5 ... C.C. (Combustion chamber)-heating
- ► 5-6′ ... T (Turbine)-expansion



Fig. 13.37. Turbine plant with intercooler.

The ideal cycle for this arrangement is 1-2-3-4-5-6; the compression process without intercooling is shown as 1-L' in the actual case, and 1-L in the ideal isentropic case.

Now,

Work input (with intercooling)

$$= c_p (T_2' - T_1) + c_p (T_4' - T_3) \qquad \dots (13.21)$$
 Work input (without intercooling)

$$= c_n (T_L' - T_1) = c_n (T_2' - T_1) + c_n (T_L' - T_2') \qquad \dots (13.22)$$

By comparing equation (13.22) with equation (13.21) it can be observed that the work input with intercooling is less than the work input with no intercooling, when $c_p (T_4' - T_3)$ is less than $c_p (T_L' - T_2')$. This is so if it is assumed that isentropic efficiencies of the two compressors,

operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then $(T_4' - T_3) < (T_L' - T_2')$ since the pressure lines diverge on the *T*-s diagram from left to the right.



From this we may conclude that when the compressor work input is reduced then the work ratio is increased. However the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by,

Heat supplied with intercooling = cp(T5 - T4')

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature T5, is given by

Heat supplied without intercooling = cp (T5 - TL')

Thus, the heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease. When intercooling is used a supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

2. Reheating

The output of a gas turbine can be amply improved by expanding the gases in two stages with a *reheater* between the two as shown in Fig. 13.39. The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding T-s diagram is shown in Fig. 13.40. The line 4'-L' represents the expansion in the L.P. turbine if reheating is *not* employed.



Fig. 13.39. Gas turbine with reheater.



Fig. 13.40. T-s diagram for the unit.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor

i.e., cpa (T2' - T1) = cpg (T3 - T4')

The work output (net output) of L.P. turbine is given by,

Net work output (with reheating) = cpg (T5 - T6')

and Net work output (without reheating) = cpg (T4' - TL')

Since the pressure lines diverge to the right on T-s diagram it can be seen that the temperature difference (T5 - T6') is always greater than (T4' - TL'), so that reheating increases the net work output.

Although net work is increased by reheating the heat to be supplied is also increased, and the net effect can be to reduce the thermal efficiency

Heat supplied = cpg (T3 - T2') + cpg (T5 - T4').

Note. cpa and cpg stand for specific heats of air and gas respectively at constant pressure.

3. Regeneration

The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber. Fig. 13.41 shows a gas turbine plant with a regenerator. The corresponding T-s diagram is shown in Fig. 13.42. 2'-3 represents the heat flow into the compressed air during its passage through the heat exchanger and 3-4 represents the heat taken in from the combustion of fuel. Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy. The effectiveness of the heat exchanger is given by:



(assuming $c_{pa} \mbox{ and } c_{pg} \mbox{ to be equal})$

A heat exchanger is usually used in large gas turbine units for marine propulsion or industrial power.



Fig. 13.42. T-s diagram for the unit.

SIMPLE VAPOUR COMPRESSION SYSTEM

Out of all refrigeration systems, the vapour compression system is the most important system from the view point of commercial and domestic utility. It is the most practical form of refrigeration. In this system the working fluid is a vapour. It readily evaporates and condenses or changes alternately between the vapour and liquid phases without leaving the refrigerating plant. During evaporation, it absorbs heat from the cold body. This heat is used as its latent heat for converting it from the liquid to vapour. In condensing or cooling or liquifying, it rejects heat to external body, thus creating a cooling effect in the working fluid. This refrigeration system thus acts as a latent heat pump since it pumps its latent heat from the cold body or brine and rejects it or delivers it to the external hot body or cooling medium. The principle upon which the vapour compression system works apply to all the vapours for which tables of Thermodynamic properties are available.

Vapour Compression Cycle:

In a simple vapour compression system fundamental processes are completed in one cycle. These are : 1. Compression 2. Condensation 3. Expansion 4. Vapourisation. The flow diagram of such a cycle is shown in Fig. 14.9.



Fig. 14.9. Vapour compression system.

The vapour at low temperature and pressure (state '2') enters the "compressor" where it is compressed isentropically and subsequently its temperature and pressure increase considerably (state '3'). This vapour after leaving the compressor enters the "condenser" where it is condensed into high pressure liquid (state '4') and is collected in a "receiver tank". From receiver tank it passes through the "expansion valve", here it is throttled down to a lower pressure and has a low temperature (state '1'). After finding its way through expansion "valve" it finally passes on to "evaporator" where it extracts heat from the surroundings or circulating fluid being refrigerated and vapourises to low pressure vapour (state '2').

Merits and demerits of vapor compression system over Air refrigeration system: Merits:

1. C.O.P. is quite high as the working of the cycle is very near to that of reversed Carnot cycle.

2. When used on ground level the running cost of vapour-compression refrigeration system is only 1/5th of air refrigeration system.

3. For the same refrigerating effect the size of the evaporator is smaller.

4. The required temperature of the evaporator can be achieved simply by adjusting the Throttle valve of the same unit.

Demerits:

1. Initial cost is high.

2. The major disadvantages are inflammability, leakage of vapours and toxity. These have been overcome to a great extent by improvement in design.

14.3.3. Functions of Parts of a Simple Vapour Compression System

Here follows the brief description of various parts of a simple vapour compression system shown in Fig. 14.9.

1. Compressor. The function of a compressor is to remove the vapour from the evaporator, and to raise its temperature and pressure to a point such that it (vapour) can be condensed with available condensing media.

2. Discharge line (or hot gas line). A hot gas or discharge line delivers the highpressure, high-temperature vapour from the discharge of the compressor to the condenser. **3.** Condenser. The function of a condenser is to provide a heat transfer surface through which heat passes from the hot refrigerant vapour to the condensing medium.

4. Receiver tank. A receiver tank is used to provide storage for a condensed liquid so that a constant supply of liquid is available to the evaporator as required.

5. Liquid line. A liquid line carries the liquid refrigerant from the receiver tank to the refrigerant flow control.

6. Expansion valve (refrigerant flow control). Its function is to meter the proper amount
of refrigerant to the evaporator and to reduce the pressure of liquid entering the evaporator so that liquid will vapourize in the evaporator at the desired low temperature and take out sufficient amount of heat.

7. Evaporator. An evaporator provides a heat transfer surface through which heat can pass from the refrigerated space into the vapourizing refrigerant.

8. Suction line. The suction line conveys the low pressure vapour from the evaporator to the suction inlet of the compressor.

Vapour Compression Cycle on Temperature-Entropy (T-s) Diagram

We shall consider the following three cases :

1. When the vapor is dry and saturated at the end of compression. Fig. 14.10 represents the vapour compression cycle, on *T*-*s* diagram the points 1, 2, 3 and 4 correspond to the state points 1, 2, 3 and 4 in Fig. 14.9.



At point '2' the vapour which is at low temperature (T2) and low pressure enters the compressor's cylinder and is compressed adiabatically to '3' when its temperature increases to the temperature T1. It is then condensed in the condenser (line 3-4) where it gives up its latent heat to the condensing medium. It then undergoes throttling expansion while passing through the expansion valve and its again reduces to T2, it is represented by the line 4-1. From the T-s diagram it may be noted that due to this expansion the liquid partially evaporates, as its dryness fraction is represented by the

ratio

At '1' it enters the evaporator where it is further evaporated at constant pressure and constant temperature to the point '2' and the cycle is completed.

Work done by the compressor = W = Area '2-3-4-b-2'

$$\therefore \qquad C.O.P. = \frac{\text{Heat extracted or refrigerating effect}}{\text{Work done}} = \frac{\text{Area '2-1-g-f-2'}}{\text{Area '2-3-4-b-2'}}$$
or
$$C.O.P = \frac{h_2 - h_1}{h_3 - h_2} \qquad \dots [14.10 \ (a)]$$

$$h_2 - h_4$$

 $=\frac{h_2 - h_4}{h_3 - h_2} \qquad \dots [14.10 \ (b)]$

 $(\cdot, h_1 = h_4, \text{ since during the throttling expansion 4-1 the total heat content remains unchanged})$

2. When the vapour is superheated after compression. If the compression of the vapour is continued after it has become dry, the vapour will be superheated, its effect on *T*-*s* diagram is shown in Fig. 14.11. The vapour enters the compressor at condition '2' and is compressed to '3' where it is superheated to temperature *Tsup*. Then it enters the condenser. Here firstly superheated vapour cools to temperature *T*1 (represented by line 3-3') and then it condenses at constant temperature along the line 3'-4; the remaining of the cycle ; however is the same as before.



Now, Work done = Area '2-3-3'-4-*b*-2' and Heat extracted/absorbed = Area '2-1-*g*-*f*-2'

C.O.P. =
$$\frac{\text{Heat extracted}}{\text{Work done}} = \frac{\text{Area} \cdot 2 \cdot 1 \cdot g \cdot f \cdot 2}{\text{Area} \cdot 2 \cdot 3 \cdot 3 \cdot 4 \cdot 5 \cdot 2} = \frac{h_2 - h_1}{h_3 - h_2} \dots [14.10 \ (c)]$$

In this case $h_3 = h_3' + c_p (T_{sup.} - T_{sat.})$ and $h_3' = \text{total heat of dry and saturated vapour at the point '3''.$

When the vapour is wet after compression. Refer Fig. 14.12.
 Work done by the compressor = Area "2-3-4-b-2"

Heat extracted = Area "2-1-g-f-2"

C.O.P. =
$$\frac{\text{Heat extracted}}{\text{Work done}} = \frac{\text{Area} \cdot 2 \cdot 1 \cdot g \cdot f \cdot 2}{\text{Area} \cdot 2 \cdot 3 \cdot 4 \cdot b \cdot 2} = \frac{h_2 - h_1}{h_3 - h_2}$$
 ...[14.10(d)]

Note. If the vapour is not superheated after compression, the operation is called 'WET COMPRESSION' and if the vapour is superheated at the end of compression, it is known as 'DRY COMPRESSION'. Dry compression, in actual practice is always preferred as it gives higher volumetric efficiency and mechanical efficiency and there are less chances of compressor damage.



Pressure-Enthalpy (p-h) Chart

4

The diagram commonly used in the analysis of the refrigeration cycle are:

(*i*) Pressure-enthalpy (*p*-*h*) chart (*ii*) Temperature-entropy (*T*-*s*) chart.

Of the two, the pressure-enthalpy diagram seems to be the more useful. The condition of the refrigerant in any thermodynamic state can be represented as a point on the p-h chart. The point on the p-h chart that represents the condition of the refrigerant in any one particular thermodynamic state may be located if any two properties of the

refrigerant for that state are known, the other properties of the refrigerant for that state can be determined directly from the chart for studying the performance of the machines.



Fig. 14.13. Pressure enthalpy (p-h) chart.

Refer Fig. 14.13. The chart is dividing into three areas that are separated from each other by the saturated liquid and saturated vapour lines. The region on the chart to the *left* of the saturated liquid line is called the *sub-cooled region*. At any point in the subcooled region the refrigerant is in the liquid phase and its temperature is below the saturation temperature corresponding to its pressure. The area to the *right* of the saturated vapour line is superheated region and the refrigerant is in the form of a superheated vapour. The section of the chart between the saturated liquid and saturated vapour lines is the two phase region and represents the change in phase of the refrigerant between liquid and vapour phases. At any point between two saturation lines the refrigerant is in the form of a liquid vapour mixture. The distance between the two lines along any constant pressure line, as read on the enthalpy scale at the bottom of the chart, is the latent heat of vapourisation of the refrigerant at that pressure. The horizontal lines extending across the chart are lines of 'constant pressure' and the vertical lines are lines of constant enthalpy. The lines of 'constant temperature' in the sub-cooled region are almost vertical on the chart and parallel to the lines of constant enthalpy. In the centre section, since the refrigerant changes state at a constant temperature and pressure, the lines of constant temperature are parallel to

and coincide with the lines of constant pressure. At the saturated vapour line the lines of constant temperature change direction again and, in the superheated vapour region, fall of sharply toward the bottom of the chart. The straight lines which extend diagonally and almost vertically across the superheated vapour region are lines of constant entropy. The curved, nearly horizontal lines crossing the superheated vapour region are lines of constant volume. p-h chart gives directly the changes in enthalpy and pressure during a process for thermodynamic analysis.

14.3.6. Simple Vapour Compression Cycle on p-h Chart

Fig. 14.14 shows a simple vapour compression cycle on a p-h chart. The points 1, 2, 3 and 4 correspond to the points marked in Fig. 14.9.



Fig. 14.14. Simple vapour compression cycle on p-h chart.

The dry saturated vapour (at state 2) is drawn by the compressor from evaporator at lower pressure p1 and then it (vapour) is compressed isentropically to the upper pressure p2. The isentropic compression is shown by the line 2-3. Since the vapour is dry and saturated at the start of compression it becomes superheated at the end of compression as given by point 3. The process of *condensation which takes place at constant pressure* is given by the line 3-4. The vapour now reduced to saturated liquid is throttled through the expansion valve and the process is shown by the line 4-1. At the point 1 a mixture of vapour and liquid enters the evaporator where it gets dry saturated as shown by the point 2. Heat extracted (or refrigerating effect produced),

$$\begin{array}{ll} R_n = h_2 - h_1 \\ Work \mbox{ done,} & W = h_3 - h_2 \\ \vdots & C.O.P. = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} \end{array}$$

The values of h_1 , h_2 and h_3 can be directly read from p-h chart.

REFRIGERANTS

A **'refrigerant'** is defined as any substance that absorbs heat through expansion or vaporisation and loses it through condensation in a refrigeration system. The term 'refrigerant' in the broadest sense is also applied to such secondary cooling mediums as cold water or brine, solutions. Usually refrigerants include only those working mediums which pass through the cycle of evaporation, recovery, compression, condensation and liquification. These substances absorb heat at one place at low temperature level and reject the same at some other place having higher temperature and pressure. The rejection of heat takes place at the cost of some mechanical work. Thus circulating cold mediums and cooling mediums (such as ice and solid carbondioxide) are not primary refrigerants. In the early days only four refrigerants, Air, ammonia (NH3), Carbon dioxide (CO2), Sulphur dioxide (SO2), possessing chemical, physical and thermodynamic properties permitting their efficient application and service in the practical design of refrigeration equipment were used. All the refrigerants change from liquid state to vapour state during the process.

14.5.1. Classification of Refrigerants

The refrigerants are classified as follows :

1. Primary refrigerants.

2. Secondary refrigerants.

1. **Primary refrigerants** are those working mediums or heat carriers which directly take part in the refrigeration system and cool the substance by the absorption of latent heat e.g. Ammonia, Carbon dioxide, Sulphur dioxide, Methyl chloride, Methylene chloride, Ethyl chloride and Freon group etc.

2. Secondary refrigerants are those circulating substances which are first cooled with the help of the primary refrigerants and are then employed for cooling purposes, e.g. ice, solid carbon dioxide etc. These refrigerants cool substances by absorption of their sensible heat.

The primary refrigerants are grouped as follows:

(i) **Halocarbon compounds.** In 1928, Charles Kettening and Dr. Thomas Mighey invented and developed this group of refrigerant. In this group are included refrigerants which contain one or more of three halogens, chlorine and bromine and they are sold in the market under the names as Freon, Genetron, Isotron, and Areton. Since the refrigerants belonging to this application in domestic, and industrial purposes. The list of the halocarbon-refrigerants commonly used is given below :

R-10 — Carbon tetrachloride (CCl4)

- R-11 Trichloro-monofluoro methane (CCl3F)
- R-12 Dichloro-difluoro methane (CCl2F2)
- R-13 Mono-bromotrifluoro methane (CBrF3)
- R-21 Dichloro monofluoro methane (CHCl2F)
- R-22 Mono chloro difluoro methane (CHClF2)
- R-30 Methylene-chloride (CH2Cl2)
- R-40 Methyle chloride (CH3Cl)
- R-41 Methyle fluoride (CH3F)
- R-100—Ethyl chloride (C2H5Cl)
- R-113— Trichloro trifluoroethane (C2F3Cl3)
- R-114— Tetra-fluoro dichloroethane (Cl2F4Cl2)
- R-152— Difluoro-ethane (C2H6F2)

(ii) **Azeotropes.** The refrigerants belonging to this group consists of mixtures of different substances. These substances cannot be separated into components by distillations. They possess fixed thermodynamic properties and do not undergo any separation with changes in temperature and pressure. An azeotrope behaves like a simple substance. **Example** R-500 It contains 73.8% of (R-12) and 26.2% of (R-152).

(iii) **Hydrocarbons.** Most of the refrigerants of this group are organic compounds. Several hydrocarbons are used successfully in commercial and industrial installations. Most of them possess satisfactory thermodynamic properties but are highly inflammable. Some of the important refrigerants of this group are :

- R-50 Methane (CH4)
- R-170—Ethane (C2H6)
- R-290—Propane (C2H8)
- R-600—Butane (C4H10)
- R-601— Isobentane [CH(CH3)3]

(iv) **Inorganic compounds.** Before the introduction of hydrocarbon group these refrigerants were most commonly used for all purposes. The important refrigerants of this group are :

R-717— Ammonia (NH3)

R-718—Water (H2O)

R-729— Air (mixture of O2, N2, CO2 etc.)

R-744—Carbon dioxide (CO2)

R-764—Sulphur dioxide (SO2)

(v) **Unsaturated organic compound.** The refrigerants belonging to this group possess ethylene or propylene as their constituents. They are

R-1120 — Trichloroethylene (C3H4Cl3)

R-1130 — Dichloroethylene (C2H4Cl2)

R-1150 — Ethylene (C3H6)

R-1270 — Propylene.

Desirable properties of an ideal refrigerant

An ideal refrigerant should possess the following properties :

1. Thermodynamic properties :

(i) Low boiling point

(ii) Low freezing point

(iii) Positive pressures (but not very high) in condenser and evaporator.

(iv) High saturation temperature

(v) High latent heat of vapourisation.

2. Chemical Properties :

(i) Non-toxicity

(ii) Non-flammable and non-explosive

(iii) Non-corrosiveness

(iv) Chemical stability in reacting

(v) No effect on the quality of stored (food and other) products like flowers, with other materials i.e., furs and fabrics.

(vi) Non-irritating and odourless.

3. Physical Properties :

- (i) Low specific volume of vapour
- (ii) Low specific heat

- (iii) High thermal conductivity
- (iv) Low viscosity
- (v) High electrical insulation.

4. Other Properties :

- (i) Ease of leakage location
- (ii) Availability and low cost
- (iii) Ease of handling
- (iv) High C.O.P.
- (v) Low power consumption per tonne of refrigeration.
- (vi) Low pressure ratio and pressure difference.

Some important properties (mentioned above) are discussed below :

Freezing point. As the refrigerant must operate in the cycle above its freezing point, it is evident that the same for the refrigerant must be lower than system temperatures. It is found that except in the case of water for which the freezing point is 0° C, other refrigerants have reasonably low values. Water, therefore, can be used only in airconditioning applications which are above 0° C.

Condenser and evaporator pressures: The evaporating pressure should be as near atmospheric as possible. If it is too low, it would result in a large volume of the suction vapour. If it is too high, overall high pressures including condenser pressure would result necessitating stronger equipment and consequently higher cost. A positive pressure is required in order to eliminate the possibility of the entry of air and moisture into the system. The normal boiling point of the refrigerant should, therefore, be lower than the refrigerant temperature.

Critical temperature and pressure. Generally, for high C.O.P. the critical temperature should be very high so that the condenser temperature line on p-h diagram is far removed from the critical point. This ensures reasonable refrigerating effect as it is very small with the state of liquid before expansion near the critical point. The critical pressure should be low so as to give low condensing pressure.

Latent heat of vaporizations: It should be as large as possible to reduce the weight of the refrigerant to be circulated in the system. This reduces initial cost of the refrigerant. The size of the system will also be small and hence low initial cost.

Toxicity. Taking into consideration comparative hazard to life due to gases and vapours underwriters Laboratories have divided the compounds into six groups. Group six contains compounds

with a very low degree of toxicity. It includes R12, R114, R13, etc. Group one, at the other end of the scale, includes the most toxic substances such as SO2. Ammonia is not used in comfort air-conditioning and in domestic refrigeration because of inflammability and toxicity.

Inflammability. Hydrocarbons (e.g. methane, ethane etc.) are highly explosive and inflammable. Fluorocarbons are neither explosive nor inflammable. Ammonia is explosive in a mixture with air in concentration of 16 to 25% by volume of ammonia.

Volume of suction vapour. The size of the compressor depends on the volume of suction vapour per unit (say per tonne) of refrigeration. Reciprocating compressors are used with refrigerants with high pressures and small volumes of the suction vapour. Centrifugal or turbocompressors are used with refrigerants with low pressures and large volumes of the suction vapour. A high volume flow rate for a given capacity is required for centrifugal compressors to permit flow passages of sufficient width to minimize drag and obtain high efficiency.

Thermal conductivity. For a high heat transfer co-efficient a high thermal conductivity is desirable. R22 has better heat transfer characteristics than R12 ; R21 is still better, R13 has poor heat transfer characteristics.

Viscosity. For a high heat transfer co-efficient a low viscosity is desirable.

Leak tendency. The refrigerants should have low leak tendency. The greatest drawback of fluorocarbons is the fact that they are odourless. This, at times, results in a complete loss of costly gas from leaks without being detected. An ammonia leak can be very easily detected by pungent odour.

Refrigerant cost. The cost factor is only relevant to the extent of the price of the initial charge of the refrigerant which is very small compared to the total cost of the plant and its installation. The cost of losses due to leakage is also important. In small-capacity units requiring only a small charge of the refrigerant, the cost of refrigerant is immaterial. The cheapest refrigerant is Ammonia. R12 is slightly cheaper than R22. R12 and R22 have replaced ammonia in the dairy and frozen food industry (and even in cold storages) because of the tendency of ammonia to attack some food products.

Co-efficient of performance and power per tonne. Practically all common refrigerants have approximately same C.O.P. and power requirement.

Properties and Uses of Commonly Used Refrigerants

1. **Air**

Properties :

(i) No cost involved ; easily available.

(ii) Completely non-toxic.

(iii) Completely safe.

(iv) The C.O.P. of air cycle operating between temperatures of 80° C and -15° C is 1.67.

Uses :

(i) Air is one of the earliest refrigerants and was widely used even as late as World War I wherever a completely non-toxic medium was needed.

(ii) Because of low C.O.P., it is used only where operating efficiency is secondary as in aircraft refrigeration.

2. Ammonia (NH3)

Properties :

(i) It is highly toxic and flammable.

(ii) It has the excellent thermal properties.

(iii) It has the highest refrigerating effect per kg of refrigerant.

(iv) Low volumetric displacement.

(v) Low cost.

(vi) Low weight of liquid circulated per tonne of refrigeration.

(vii) High efficiency.

(viii) The evaporator and condenser pressures are 3.5 bar abs. and 13 bar abs. (app.) respectively at standard conditions of -15° C and 30°C.

Uses :

(i) It is widely used in large industrial and commercial reciprocating compression systems where high toxicity is secondary.

It is extensively used in ice plants, packing plants, large cold storages and skating rinks etc.

(ii) It is widely used as the refrigerant in absorption systems.

The following points are worth noting :

• Ammonia should never be used with copper, brass and other copper alloys ; iron and steel should be used in ammonia systems instead.

• In ammonia systems, to detect the leakage a sulphur candle is used which gives off a

dense white smoke when ammonia vapour is present.

3. Sulphur dioxide (SO2)

Properties :

(i) It is a colourless gas or liquid.

(ii) It is extremely toxic and has a pungent irritating odour.

(iii) It is non-explosive and non-flammable.

(iv) It has a liquid specific gravity of 1.36.

(v) Works at low pressures.

(vi) Possesses small latent heat of vapourisation.

Uses :

It finds little use these days. However its use was made in small machines in early days.

• The leakage of sulphur dioxide may be detected by bringing aqueous ammonia near the leak, this gives off a white smoke.

4. Carbon dioxide (CO2)

Properties :

(i) It is a colourless and odourless gas, and is heavier than air.

(ii) It has liquid specific gravity of 1.56.

- (iii) It is non-toxic and non-flammable.
- (iv) It is non-explosive and non-corrosive.
- (v) It has extremely high operating pressures.
- (vi) It gives very low refrigerating effect.

Uses :

This refrigerant has received only limited use because of the high power requirements per tonne of refrigeration and the high operating pressures. In former years it was selected for marine refrigeration, for theater air-conditioning systems, and for hotel and institutional refrigeration instead of ammonia because it is non-toxic.

At the present-time its use is limited primarily to the manufacture of dry ice (solid carbon dioxide).

• The leak detection of CO2 is done by soap solution.

5. Methyl Chloride (CH3Cl)

Properties :

- (i) It is a colourless liquid with a faint, sweet, non-irritating odour.
- (ii) It has liquid specific gravity of 1.002 at atmospheric pressure.

(iii) It is neither flammable nor toxic.

Uses :

It has been used in the past in both domestic and commercial applications. It should never be used with aluminium.

6. R-11 (Trichloro monofluoro methane)

Properties : (i) It is composed of one carbon, three chlorine and one fluorine atoms (or parts by weight) and is non-corrosive, non-toxic and non-flammable.

(ii) It dissolves natural rubber.

(iii) It has a boiling point of -24° C.

(iv) It mixes completely with mineral lubricating oil under all conditions.

Uses : It is employed for 50 tonnes capacity and over in small office buildings and factories. A

centrifugal compressor is used in the plants employing this refrigerant.

• Its leakage is detected by a halide torch.

7. R-12 (Dichloro-difluoro methane) or Freon-12

Properties :

(i) It is non-toxic, non-flammable, and non-explosive, therefore it is most suitable refrigerant.

(ii) It is fully oil miscible therefore it simplifies the problem of oil return.

(iii) The operating pressures of R-12 in evaporator and condenser under standard tonne of refrigeration are 1.9 bar abs. and 7.6 bar abs. (app.).

(iv) Its latent heat at -15° C is 161.6 kJ/kg.

(v) C.O.P. = 4.61.

(vi) It does not break even under the extreme operating conditions.

(vii) It condenses at moderate pressure and under atmospheric conditions.

Uses :

1. It is suitable for high, medium and low temperature applications.

2. It is used for domestic applications.

3. It is excellent electric insulator therefore it is universally used in sealed type compressors.

8. R-22 (Monochloro-difluoro methane) or Freon-22

R-22 refrigerant is superior to R-12 in many respects. It has the following properties and uses :

Properties :

(i) The compressor displacement per tonne of refrigeration with R-22 is 60% less than the compressor displacement with R-12 as refrigerant.

(ii) R-22 is miscible with oil at condenser temperature but tries to separate at evaporator temperature when the system is used for very low temperature applications (– 90°C). Oil separators must be incorporated to return the oil from the evaporator when the system is used for such low temperature applications.

(iii) The pressures in the evaporator and condenser at standard tonne of refrigeration are 2.9 bar abs. and 11.9 bar abs. (app.).

(iv) The latent heat at -15° C is low and is 89 kJ/kg. The major disadvantage of R-22 compared with R-12 is the high discharge temperature which requires water cooling of the compressor head and cylinder.

Uses :

R-22 is universally used in commercial and industrial low temperature systems.

MODULE 3 PSYCHROMETRY

UNIT-III

PSYCHROMETRY

3.1 INTRODUCTION

The psychrometric is that branch of engineering science which deals with the study of moist air i.e., dry air mixed with water vapour or humidity. It also includes the study of behavior of dry air and water vapour mixture under various sets of conditions. Though the earth's atmosphere is a mixture of gases including nitrogen (N_2), oxygen (O_2), argon (Ar) and carbon dioxide (CO₂), yet for the purpose of psychrometric, it is considered to be a mixture of dry air and water vapour only.

3.2 PSYCHOMETRIC TERMS

Though there are many psychometric terms, yet the following are important from the subject point of view :

1. *Dry air*. The pure dry air is a mixture of a number of gases such as nitrogen, oxygen, carbon dioxide, hydrogen, argon, neon, helium etc. But the nitrogen and oxygen have the major portion of the combination. The dry air is considered to have the composition as given in the following table:

S.No.	Constituent	By volume	By mass	Molecular Mass
1	Nitrogen (N ₂)	78.03%	75.47%	28
2	Oxygen (O ₂)	20.99%	23.19%	32
3	Argon (Ar)	0.94%	1.29%	40
4	Carbon dioxide (CO ₂)	0.03%	0.05%	44
5	Hydrogen (H ₂)	0.01%	-	2

Table .1 Composition of dry air

The molecular mass of dry air is taken as 28.966 and the gas constant of air (R_a) is equal 0.287 kJ / kg K or 287 J/kg K.

The molecular mass of water vapour is taken as 18.016 and the gas constant for water vapour (k) is equal to 0.461-kJ/kg K or 461 J/kg K.

Notes: (a) The pure dry air does not ordinarily exist in nature because it always contains some water vapout

(b) The term air, wherever used in this text, means dry air containing moisture in the vapour form.

(c) Both dry air and water vapour can be considered as perfect gases because both exist in the atmosphere at low pressure. Thus all the perfect gas terms can be applied to them individually.

(d) The density of dry air is taken as 1.293 kg/m3 at pressure 1.0135 bar or 101.35 $1(11/m2 \text{ and at temperature 0}^{\circ}C (273 \text{ K}).$

2. *Moist air*. It is a mixture of dry air and water vapour. The amount of water vapour present. in the air depends upon the absolute pressure and temperature of the mixture.

3. *Saturated air*. It is mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it. The water vapours, usually, occur in the form of superheated steam as an invisible gas. However, when the saturated air is cooled, the water vapour in the air starts condensing, and the same may be visible in the form of moist, fog or condensation on cold surfaces.

4. *Degree of saturation*. It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.

5. *Humidity*. It is the mass of water vapour present in 1 kg of dry air, and is generally expressed in terms of gram per kg of dry air (g / kg of dry air). It is also called specific humidity or humidity ratio.

6. *Absolute humidity*. It is the mass of water vapour present in 1 m3 of dry air, and is generally expressed in terms of gram per cubic metre of dry air (g /m3 of dry air). It is also expressed in terms of grains per cubic metre of dry air. Mathematically, one kg of water vapour is equal to 15 430 grains.

7. *Relative humidity*. It is the ratio of actual mass of water vapour in a given_volume of moist air to the mass of water vapour in the same volume of saturated air at the same temperature and pressure. It is briefly written as RH.

8. *Dry bulb temperature*. It is the temperature of air recorded by a thermometer, when it is not affected by the moisture present in the air. The dry bulb temperature (briefly written as DBT) is generally denoted by t_d or t_{db} .

9. Wet bulb temperature. It is the temperature of air recorded by a thermometer, when its bulb is surrounded by a wet cloth exposed to the air. Such a thermometer is called *wet bulb thermometer. The wet bulb temperature (briefly written as WBT) is generally denoted by t_w or t_{wb} .

10. *Wet bulb depression*. It is the difference between dry bulb temperature and wet bulb temperature at any point. The wet bulb depression indicates relative humidity of the air.

11. *Dew point temperature*. It is the temperature of air recorded by a thermometer, when the moisture (water vapour) present in it begins to condense. In other words, the dew point temperature is the saturation temperature (t_{sat}). corresponding to the partial pressure of water vapour (P_v) It is, usually, denoted by t_{dp} . Since p_v is very small, therefore the saturation temperature by water vapour at pv is also low (less than the atmospheric or dry bulb temperature). Thus the water vapour in air exists in the superheated state and the moist air containing moisture in such a form (i.e., superheated state) is said to be unsaturated air. This condition is shown by point A on temperature-entropy (T-s) diagram as shown in Fig.1. When the partial pressure of water vapour (Pv) is equal to the saturation pressure (Ps) the water vapour is in dry condition and the air will be saturated air



Fig.1. T-s diagram

If a sample of unsaturated air, containing superheated water vapour, is cooled at constant pressure, the partial pressure (pr) of each constituent remains constant until the water vapour reaches the saturated state as shown by point B in Fig.1. At this point 8, the first drop of dew will' be formed and hence the temperature at point B is called dew paint temperature. Further cooling will cause condensation of water vapour.

From the above we see that the dew point temperature is the temperature at which the water vapour begins to condense.

Note: For saturated air, the dry bulb temperature, wet bulb temperature and dew point temperature is same.

12. *Dew point depression*. It is the difference between the dry bulb temperature and dew point temperature of air.

13. **Psychrometer**. There are many types of psychrometers, but the sling psychrometer, as shown in Fig..2, is widely used. It consists of a dry bulb thermometer and a wet bulb thermometer mounted side by side in a protective case that is attached to a handle by a swivel connection so that the case can be easily rotated. The dry bulb thermometer is directly exposed to air and measures the actual temperature of the air. The bulb of the wet bulb thermometer is covered; by a wick thoroughly wetted by distilled water. The temperature measured by this wick covered bulb of a thermometer is the temperature of liquid water in the wick and is called wet Nib j temperature.

The sling psychrometer is rotated in the air for approximately one minute after which HO readings from both the thermometers are taken. This process is repeated several times to assure': that the lowest possible wet bulb temperature is recorded.



Fig.2, Sling psychrometer

3.3 DALTON'S LAW OF PARTIAL PRESSURES

It states, The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent Fould exert, if it occupied the same space by itself. In other words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Mathematically, barometric pressure of the mixture,

 P_a = Partial pressure of dry air, and

where

 $P_b = P_a + P_{\nu},$

 P_{v} = Partial pressure of water vapour.

3.4 PSYCHROMETRIC RELATIONS

We have already discussed some psychrometric terms in Art. These terms have some relations between one another. The following psychrometric relations are important from the subject point of view:

1. *Specific humidity*, humidity ratio or moisture content. It is the mass of water vapour present in 1 kg of dry air (in the air-vapour mixture) and is generally expressed in g /kg of dry air. It may also be defined as the ratio of mass of water vapour to the mass of dry air in a given volume of the air-vapour mixture.

Let P_a , V_a , Ta, m_a and R_a = Pressure, volume, absolute temperature, mass and gas constant

respectively for dry air, and

 P_{v} , V_{v} , m_{v} and R_{v} = Corresponding values for the water vapour.

Assuming that the dry air and water vapour behave as perfect gases, we have for dry air,

$$P_a v_a = m_a R_a T_a$$

and for water vapour, $P_v v_v = m_v R_v T_v$,

Also

 $v_a = v_v$

and

 $T_a = T_{v} = T_d \dots$ (where Td is dry bulb temperature)

From equations (i) and (ii), we have

$$\frac{p_{\nu}}{p_a} = \frac{m_{\nu} R_{\nu}}{m_a R_a}$$

. Humidity ratio,

$$W = \frac{m_v}{m_a} = \frac{R_a p_v}{R_v p_a}$$

Substituting $R_a = 0.287$ kJ/kg K for dry air and $R_v = 0.461$ kJ/kg K for water vapour in the above equation, we have



 $W = \frac{0.287 \times p_{\nu}}{0.461 \times p_{a}} = 0.622 \times \frac{p_{\nu}}{p_{a}} = 0.622 \times \frac{p_{\nu}}{p_{b} - p_{\nu}}$

 $\dots (:: p_b = p_a + p_v)$

Fig.3 T-s diagram

Consider unsaturated air containing superheated vapour at dry bulb temperature t_d and partial pressure p_v as shown by point A on the *T*-*s* diagram in Fig. 3. If water is added into this unsaturated air, the water will evaporate which will increase the moisture content (specific humidity) of the air and the partial pressure p_v increases. This will continue until the water vapour becomes saturated at that temperature, as shown by point C in Fig.3, and there will be more evaporation of water. The partial pressure p_v , increases to the saturation pressure p_s and it is maximum partial pressure of water vapour at temperature t_d . The air containing moisture in such a state (point *C*) is called saturated air.

For saturated air (i.e. when the air is holding maximum amount of water vapour), the humidity ratio or maximum specific humidity,

$$W_s = W_{max} = 0.622 \times \frac{p_s}{p_b - p_s}$$

where P_s = Partial pressure of air corresponding to saturation temperature (i.e. dry bulb temperature td).

2. Degree of saturation or percentage humidity. We have already discussed that the degree of saturation is the ratio of vapour in a unit mass of water air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature (dry bulb temperature), it may be defined as the ratio of actual specific humidity to the specific humidity of saturated air at the same dry bulb temperature. It is, usually, denoted by μ . Mathematically, degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{\frac{0.622 p_v}{p_b - p_v}}{\frac{0.622 p_s}{p_b - p_s}} = \frac{p_v}{p_s} \left(\frac{p_b - p_s}{p_b - p_v}\right) = \frac{p_v}{p_s} \left[\frac{1 - \frac{p_s}{p_b}}{1 - \frac{p_v}{p_b}}\right]$$

Notes: (a) The partial pressure of saturated air (P_s) is obtained from the steam tables corresponding to dry bulb temperature t_d .

(b) If the relative humidity, \emptyset) = Pv / Ps is equal to zero, then the humidity ratio, W = 0, i.e. for dry air, $\mu = 0$.

(c) If the relative humidity, \emptyset) Pv / Ps is equal to 1, then W = Ws and $\mu = 1$. Thus p. varies between 0 and 1.

3. *Relative humidity*. We have already discussed that the relative humidity is the ratio of actual mass of water vapour (m_v) in a given volume of moist air to the mass of water vapour (m_s) in the same volume of saturated air at the same temperature and pressure. It is usually denoted by \emptyset . Mathematically, relative humidity,

$$\phi = \frac{m_v}{m_s}$$

Let p_v , v_v , T_v , m_v and R_v = Pressure, volume, temperature, mass and gas constant respectively for

water vapour in actual conditions, and

 p_s , v_s , T_s , m_s and Rs = Corresponding values for water vapour in saturated air.

We know that for water vapour in actual conditions,

$$P_v v_v = m_v R_v T_v \qquad \dots (i)$$

Similarly, for water vapour in saturated air,

$$Ps vs = ms Rs Ts$$
 ...(ii)

According to the definitions,

$$v_v = v_s$$

 $Tv = Ts$
 $Rv = Rs = 0.461 \text{ kJ/kg K}$

Also

: From equations (i) and (ii), relative humidity,

$$\phi = \frac{m_v}{m_s} = \frac{p_v}{p_s}$$

Thus, the relative humidity may also be defined as the ratio of actual partial pressure of water vapour in moist air at a given temperature (dry bulb temperature) to the saturation pressure of water vapour (or partial pressure of water vapour in saturated air) at the same temperature.

The relative humidity may also be obtained as discussed below:

We know that degree of saturation,

$$\mu = \frac{p_{\nu}}{p_s} \left[\frac{1 - \frac{p_s}{p_b}}{1 - \frac{p_{\nu}}{p_b}} \right] = \phi \left[\frac{1 - \frac{p_s}{p_b}}{1 - \phi \times \frac{p_s}{p_b}} \right] \qquad \cdots \left(\because \phi = \frac{p_{\nu}}{p_s} \right)$$
$$\phi = \frac{\mu}{1 - (1 - \mu) \frac{p_s}{p_b}}$$

4. *Pressure of water vapour*. According to Carrier's equation, the partial pressure of water vapours,

$$p_{v} = p_{w} - \frac{(p_{b} - p_{w})(t_{d} - t_{w})}{1544 - 1.44 t_{w}}$$

Where

 p_w , = Saturation pressure corresponding to wet bulb temperature (from steam tables),

 P_b = Barometric pressure,

 t_d = Dry bulb temperature, and

 t_w = Wet bulb temperature.

5. *Vapour density or absolute humidity*. We have already discussed that the vapour density or absolute humidity is the mass of water vapour present in 1 m^3 of dry air.

Let $v_v =$ Volume of water vapour in m³/kg of dry air at its partial pressure,

 v_a = Volume of dry air in m³/kg of dry air at its partial pressure,

 $\rho_{v,}$ = Density of water vapour in kg/m³ corresponding to its partial pressure and dry bulb

temperature t_d , and

 ρ_a = Density of dry air in kg/m³ of dry air.

We know that mass of water vapour,

	$m_v = v_v \rho_v$
and mass of dry air,	$m_a = v_a \rho_a$
Dividing equation (i)	by equation (ii),

$$\frac{m_v}{m_a} = \frac{v_v \rho_v}{v_a \rho_a}$$

Since $v_a = v_v$, therefore humidity ratio,

$$W = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a} \quad \text{or} \quad \rho_v = W \rho_a$$
$$p_a v_a = m_a R_a T_d$$

We know that

Since $v_a = \frac{1}{\rho_a}$ and $m_a = 1$ kg, therefore substituting these values we get

$$p_a \times \frac{1}{\rho_a} = R_a T_d$$
 or $\rho_a = \frac{p_a}{R_a T}$

Substituting the value of ρ_a in equation (iii), we have

$$\rho_{v} = \frac{W p_{a}}{R_{a} T_{d}} = \frac{W (p_{b} - p_{v})}{R_{a} T_{d}} \qquad \dots (\because p_{b} = p_{a} + p_{v})$$

where

 p_a = Pressure of air in kN/m²,

 R_a = Gas constant for air = 0.287 kJ/ kg K, and

 T_d = Dry bulb temperature in K.

Example.1. The readings from a sling psychrometer are as follows ry bulb temperature $= 30^{\circ}$ C; Barometer reading 740mm of Hg Using steam tables, determine : I. Dew point temperature ; 2. Relative humidity ; 3. Specific humidity ; 4. Degree of-saturation ; 5. Vapour density ; and 6. Enthalpy of mixture per kg of dry air.

Solution given: $t_d = 30^{\circ}C$; $t_w = 20^{\circ}C$; $P_4 = 740$ mm of Hg **1.Dew point temperature**

First of all, let us find the partial pressure of water vapour (P_v) .

From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 20° C is

$$P_w = 0.023 \ 37 \ bar$$

We know that barometric pressure,

ph = 740 mm of Hg ... (Given)
= 740 x 133.3 = 98 642 N/m² ... (
$$\because$$
 mm of Hg =
133.3 N/m²)
= 0.986 42 bar \because 1 bar = 10⁵

 N/m^2)

∴ Partial pressure of water vapour,

$$p_{v} = p_{w} - \frac{(p_{b} - p_{w})(t_{d} - t_{w})}{1544 - 1.44 t_{w}}$$

= 0.023 37 - $\frac{(0.986 \ 42 - 0.02337)(30 - 20)}{1544 - 1.44 \times 20}$
= 0.023 37 - 0.006 36 = 0.017 01 bar

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (P_v), therefore from steam tables, we find that corresponding to pressure 0.017 01 bar, the dew point temperature is

$$t_{dp} = 15^{\circ}C$$
 Ans

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

$$Ps = 0.042 \ 42 \ bar$$

We know the relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{0.01701}{0.04242} = 0.40$$
 or 40% Ans.

3. Specific humidity

We know that specific humidity,

$$W = \frac{0.622 \ p_{\nu}}{p_b - p_{\nu}} = \frac{0.622 \times 0.01701}{0.986 \ 42 - 0.01701}$$
$$= \frac{0.01058}{0.96941} = 0.010 \ 914 \ \text{kg/kg of dry air}$$
$$= 10.914 \ \text{g/kg of dry air Ans.}$$

4. Degree of saturation

We know that specific humidity of saturated air,

$$W_s = \frac{0.622 \, p_s}{p_b - p_s} = \frac{0.622 \times 0.04242}{0.98642 - 0.04242}$$
$$= \frac{0.02638}{0.944} = 0.027 \, 945 \, \text{kg/kg of dry air}$$

We know that degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{0.010\ 914}{0.027\ 945} = 0.391$$
 or 39.1% Ans.

Note : The degree of saturation (μ) may also be calculated from the following relation :

$$\mu = \frac{p_v}{p_s} \left(\frac{p_b - p_s}{p_b - p_v} \right)$$
$$= \frac{0.01701}{0.04242} \left[\frac{0.98642 - 0.04242}{0.98642 - 0.01701} \right]$$
$$= 0.391 \text{ or } 39.1\% \text{ Ans.}$$

5. Vapour density

We know that vapour density,

$$\rho_{\nu} = \frac{W(p_b - p_{\nu})}{R_a T_d} = \frac{0.010\,914\,(0.986\,42 - 0.017\,01)\,10^5}{287\,(273 + 30)}$$

= 0.012 16 kg/m³ of dry air Ans.

6. Enthalpy of mixture per kg of dry air

From steam tables, we find that the latent heat of vaporisation of water at dew point temperature of 15°C is

$$h_{fedn} = 2466.1 \text{ kJ/kg}$$

: Enthalpy of mixture per kg of dry air,

$$h = 1.022 t_d + W [h_{f_g dp} + 2.3 t_{dp}]$$

= 1.022 × 30 + 0.010 914 [2466.1 + 2.3 × 15]
= 30.66 + 27.29 = 57.95 kJ/kg of dry air Ans.

'n.,

Example.2: On a particular day, the atmospheric air was found to have a dry bulb temperature of 30°C and a wet bulb temperature of 18°C. The barometric pressure was observed to b 756mm of Hg. Using the tables of psychrometric properties of air, determine the relative humidity, the specific humidity, the dew point temperature, the enthalpy of air per kg of dry air and the volume of mixture per kg of dry air.

Solution: Given:
$$t_d = 30^{\circ}$$
C; $t_w - 18^{\circ}$ C; $P_b = 756 mm$ of Hg

Relative humidity

First of all, let us find the partial pressure of water vapour (p_v) . From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 18°C is.

 $p_w = 0.020 \ 62 \ bar = 0.020 \ 62 \times 10^5 = 2062 \ \text{N/m}^2$

 $= \frac{2062}{133.3} = 15.47 \text{ mm of Hg} \qquad \dots (\because 1 \text{ mm of Hg} = 133.3 \text{ N/m}^2)$

We know that
$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44 t}$$

$$= 15.47 - \frac{(756 - 15.47)(30 - 18)}{1544 - 144 \times 18} \text{ mm of Hg}$$

$$= 15.47 - 5.85 = 9.62 \text{ mm of Hg}$$

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

 $\dot{p}_s = 0.042 \ 42 \ \text{bar} = 0.042 \ 42 \times 10^5 = 4242 \ \text{N/m}^2$ = $\frac{4242}{133.3} = 31.8 \ \text{mm of Hg}$

We know that the relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{9.62}{31.8} = 0.3022 \text{ or } 30.22\%$$

Specific humidity

We know that specific humidity,

$$W = \frac{0.622 \ p_v}{p_b - p_v} = \frac{0.622 \times 9.62}{756 - 9.62} = 0.008 \ \text{kg/kg of dry air Ans.}$$

Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (P_v) , therefore from steam tables, we find that corresponding

to 9.62 mm of Hg or 9.62 x $133.3 = 1282.3 \text{ N/m}^2 = 0.012 823$ bar, the dew point temperature is,

$$t_{dp} = 10.6^{\circ} \text{ C Ans.}$$

Enthalpy of air per kg of dry air

From steam tables, we also find that latent heat of vaporization of water at dew point temperature of 10.6°C,

$$h_{\rm fgdp} = 2476.5 \text{ kJ/kg}$$

We know that enthalpy of air per kg of dry air,

$$h = 1.022 t_d + W (h_{fgdp} + 2.3 t_{dp})$$

= 1.022 x 30 + 0.008 (2476.5 + 2.3 x 10.6)
= 30.66 + 20 = 50.66 kJ/kg of dry air Ans.

Volume of the mixture per kg of dry air

From psychrometric tables, we find that specific volume of the dry air at 760 mm of Hg and 30°C dry bulb temperature is 0.8585 m³/kg of dry air. We know that one kg of dry air at a partial pressure of (756 — 9.62) mm of Hg occupies the same volume as W = 0.008 kg of vapour at its partial pressure of 9.62 mm of Hg. Moreover, the mixture occupies the same volume but at a total pressure of 756 mm of Hg.

 \therefore Volume of the mixture (v) at a dry bulb temperature of 30°C and a pressure of 9.62 mm of Hg

= Volume of 1 kg of dry air (v_a) at a pressure of (756 — 9.62) or

746.38 mm of Hg

$$= 0.8585 \times \frac{760}{746.38} = 0.8741$$
 kg/kg of dry air Ans.

Note : The volume of mixture per kg of dry air may be calculated as discussed below :

We know that
$$v = v_a = \frac{R_a T_d}{p_a}$$

where

 $R_a = \text{Gas constant for air} = 287 \text{ J/kg K}$

$$T_d$$
 = Dry bulb temperature in K

= 30 + 273 = 303 K, and

 p_a = Pressure of air in N/m²

$$= P_b - P_v = 756 - 9.62 = 746.38 \text{ mm of Hg}$$
$$= 746.38 \text{ x } 133.3 = 994 \text{ 92 N/m}^2$$

Substituting the values in the above equation,

$$v = \frac{287 \times 303}{99492} = 0.8741 \text{ m}^3/\text{kg of dry air Ans.}$$

Example.3. The humidity ratio of atmospheric air at 28°C dry bulb temperature and 760 mm of mercury is 0.016 kg / kg of dry air. Determine: 1. partial pressure of Water vapour; 2.relative humidity; 3. dew point temperature; 4. specific enthalpy; and 5. vapour density.

Solution: Given: $t_d = 28^{\circ}$ C; $P_b = 760$ mm of Hg; W = 0.016 kg/kg of dry air

1. Partial pressure of water vapour

Let P_v = Partial pressure of water vapour. We know that humidity ratio (W),

$$0.016 = \frac{0.622 \, p_v}{p_b - p_v} = \frac{0.622 \, p_v}{760 - p_v}$$

12.16 - 0.016 $P_v = 0.622 P_v$ or 0.638 $P_v = 12.16$ $P_v = 12.16/0.638 = 19.06 \text{ mm of Hg}$ $= 19.06 \text{ x } 133.3 = 2540.6 \text{ N/m}^2 \text{ Ans.}$

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 28°C is

$$P_s = 0.03778 \text{ bar} = 3778 \text{ N/m}^2$$

∴ Relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{2540.6}{3778} = 0.672$$
 or 67.2% Ans.

3. Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (P_v), therefore from steam tables, we find that corresponding to a pressure of 2540.6 N/m² (0.025406 bar), the dew point temperature is,

$$t_{dp} = 21.1^{\circ} \text{ C Ans.}$$

4. Specific enthalpy

From steam tables, latent heat of vaporization of water corresponding to a dew point temperature of 21.1° C,

$$h_{fgdp} = 2451.76 \text{ kJ/kg}$$

We know that specific enthalpy.

$$h = 1.022 t_d + W (h_{fgdp} + 2.3 t_{dp})$$

= 1.022 x 28 + 0.016 (2451.76 + 2.3 x 21_1)
= 28.62 + 40 - 68.62 kJ/kg of dry air Ans.

5. Vapour density

We know that vapour density,

$$\rho_{v} = \frac{W(p_{b} - p_{v})'}{R_{a} T_{d}} = \frac{0.016(760 - 19.06)133.3}{287(273 + 28)}$$

 $= 0.0183 \text{ kg/m}^3 \text{ of dry air.}$

3.5 THERMODYNAMIC WET BULB TEMPERATURE OR ADIABATIC SATURATION TEMPERATURE

The thermodynamic wet bulb temperature or adiabatic saturation temperature is the temperature at which the air can be brought to saturation state, adiabatically, by the evaporation of water into the flowing air.



Fig.4 Adiabatic saturation of air.

The equipment used for the adiabatic saturation of air, in its simplest form, consists of an insulated chamber containing adequate quantity of water. There is also an arrangement for extra water (known as make-up water) to flow into the chamber from its top, as shown in Fig.4. Let the unsaturated air enters the chamber at section 1. As the air passes through the chamber over a long sheet of water, the water evaporates which is carried with the flowing stream of air, and the specific humidity of the air increases. The make-up water is added to the chamber at this temperature to make the water level constant. Both the air and water are cooled as the evaporation takes place. This process continues until the energy transferred from the air to the water is equal to the energy required to vaporize the water. When steady conditions are reached, the air flowing at section 2 is saturated with water vapour. The temperature of the saturated air at section 2 is known as *thermodynamic wet bulb temperature or adiabatic saturation temperature*.

The adiabatic saturation process can be represented on T-s diagram as shown by the curve 1-2 in Fig.5.



Fig.5. T-s diagram for adiabatic saturation process

During the adiabatic saturation process, the partial pressure of vapour increases, although the total ressure of the air-vapour mixture. The unsaturated air initially at dry bulb temperature t_{d2} , is coo e adiabatically to dry bulb temperature td, which is equal to the adiabatic saturation temperature t_w . It may be noted that the adiabatic saturation temperature is taken equal to the wet bulb temperature for all practical purposes.

Let h_1 = Enthalpy of unsaturated air at section 1,

 W_1 = Specific humidity of air at section 1,

 h_2, W_2 = Corresponding values of saturated air at section 2, and

 h_{fw} = Sensible heat of water at adiabatic saturation temperature.

Balancing the enthalpies of air at inlet and outlet (i.e. at sections 1 and 2),

$h_1 + ($	$(W_2 - W_1) h_{fw} = h_2$	(i)
or	$h_1 - W_1 h_{fw} = h_2 - W_2 h_{fw}$	(ii)
The term	$(h_2 - W_2 h_{fw})$ is known as sigma heat and remains constant	during the adiabatic
process.		
We know	that $h_1 = h_{a1} + W_1 h_{s1}$	
and	$h_2 = h_{a2} + W_2 h_{s2}$	
where	$h_{a1} =$ Enthalpy of 1 kg of dry air at dry bulb temp	erature t _{d1} ,
	$*h_{s1}$ = Enthalpy of superheated vapour at t_{d1} per kg	of vapour,
	h_{a2} = Enthalpy of 1 kg of air at wet bulb temperate	ure t_w , and
	h_{s2} = Enthalpy of saturated vapour at wet bulb tem vapour.	perature t_w per kg of
Now the e	equation (ii) may be written as :	
(h _{a1}	$+ W_1 h_{s1} - W_1 h_{fw} = (h_{a2} + W_2 h_{s2}) - W_2 h_{fw}$	
	$W_1 (h_{s1} - h_{fw}) = W_2 (h_{s2} - h_{fw}) + h_{a2} - h_{a1}$	
:	$W_{1} = \frac{W_{2}(h_{s2} - h_{fw}) + h_{a2} - h_{a1}}{h_{s1} - h_{fw}}$	

3.6 PSYCHROMETRIC CHART

It is a graphical representation of the various thermodynamic properties of moist air. The psychrometric chart is very useful for finding out the properties of air (which are required in the field of air conditioning) and eliminate lot of calculations. There is a slight variation in the charts prepared by different air-conditioning manufactures but basically they are all alike. The psychrometric chart is normally drawn for standard atmospheric pressure of 760 mm of Hg (or 1.01325 bar).



Fig. 6 Psychrometric chart.

In a psychrometric chart, dry bulb temperature is taken as abscissa and specific humidity i.e. moisture contents as ordinate, as shown in Fig. 6. Now the saturation curve is

drawn by plotting the various saturation points at corresponding dry bulb temperatures. The saturation curve represents 100% relative humidity at various dry bulb temperatures. It also represents the wet bulb and dew point temperatures.

Though the psychrometric chart has a number of details, yet the following lines are important frpm the subject point of view :

1. Dry bulb temperature lines. The dry bulb temperature lines are vertical i.e. parallel to the ordinate and uniformly spaced as shown in Fig. 7. Generally the temperature range of these lines on psychrometric chart is from - 6° C to 45° C. The dry bulb temperature lines are drawn with difference of every 5°C and up to the saturation curve as shown in the figure. The values of dry bulb temperatures are also shown on the saturation curve.

2. Specific humidity or moisture content lines. The specific humidity (moisture content) lines are horizontal i.e. parallel to the abscissa and are also uniformly spaced as shown in Fig. 16.8. Generally, moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry air (or from 0 to 0.030 kg / kg of dry air). The moisture content lines are drawn with a difference of every 1 g (or 0.001 kg) and up to the saturation curve as shown in the figure.



Fig.7. Dry bulb temperature lines.

Fig. 8. Specific humidity lines.

3. *Dew point temperature lines*. The dew point temperature lines are horizontal i.e. parallel to the abscissa and non-uniformly spaced as shown in Fig. 16.9. At any point on the saturation curve, the dry bulb and dew point temperatures are equal.

The values of dew point temperatures are generally given along the saturation curve of the chart as shown in the figure.



Fig. 9 Dew point temperature lines. Fig. 10 Wet bulb temperature lines.

4. *Wet bulb temperature lines*. The wet bulb temperature lines are inclined straight lines and non-uniformly spaced as shown in Fig.10. At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.

The values of wet bulb temperatures are generally given along the saturation curve of the chart as shown in the figure.

5. *Enthalpy (total heat) lines*. The enthalpy (or total heat) lines are inclined straight lines and uniformly spaced as shown in Fig.11. These lines are parallel to the wet bulb temperature lines, and are drawn up to the saturation curve. Some of these lines coincide with the wet bulb temperature lines also.

The values of total enthalpy are given on a scale above the saturation curve as shown in the figure.

6. *Specific volume lines.* The specific volume lines are obliquely inclined straight lines and uniformly spaced as shown in Fig.12. These lines are drawn up to the saturation curve. The values of volume lines are generally given at the base of the chart.



Fig. 11. Enthalpy lines.

Fig. 12. Specific volume lines.

7. *Vapour pressure lines*. The vapour pressure lines are horizontal and uniformly spaced. Generally, the vapour pressure lines are not drawn in the main chart. But a scale showing vapour pressure in mm of Hg is given on the extreme left side of the chart as shown in Fig.13.



Fig. 13. Vapour pressure lines.

Fig. 14. Relative humidity

lines.

8. *Relative humidity lines*. The relative humidity lines are curved lines and follow the saturation curve. Generally, these lines are drawn with values 10%, 20%, 30% etc. and up to 100%. The saturation curve represents 100% relative humidity. The values of relative humidity lines are generally given along the lines themselves as shown in Fig. 14.

3.7 PSYCHROMETRIC PROCESSES

The various psychrometric processes involved in air conditioning to vary the psychrometric properties of air according to the requirement are as follows:

1. Sensible heating, 2. Sensible cooling, 3. Humidification and dehumidification, 4. Cooling and adiabatic humidification, 5. Cooling and humidification by water injection, 6. Heating and humidification, 7. Humidification by steam injection, 8. Adiabatic chemical dehumidification, 9. Adiabatic mixing of air streams.

We shall now discuss these psychrometric processes, in detail, in the following pages.

3.71 Sensible Heating

The heating of air, without any-change in its specific humidity, is known as sensible heating. Let air at temperature td, passes over a heating coil of temperature t_{d3} , as shown in Fig. 15 (a). It may be noted that the temperature of air leaving the heating coil (t_{d2}) will be less than t_{d3} . The process of sensible heating, on the psychrometric chart, is shown by a horizontal line 1-2 extending from left to right as shown in Fig.15 (b). The point 3 represents the surface temperature of the heating coil.

The heat absorbed by the air during sensible heating may be obtained from the psychrometric chart by the enthalpy difference $(h_2 - h_1)$ as shown in Fig. 15 (b). It may be noted that the specific humidity during the sensible heating remains constant (i.e. $W_1 = W_2$).

The dry bulb temperature increases from t_{d1} , to t_{d2} and relative humidity reduces from \emptyset_1 , to \emptyset_2 as shown in Fig. 15 (b). The amount of heat added during sensible heating may also be obtained from the relation:

Heat added,

$$q = h_2 - h_1$$

$$= c_{pa} (t_{d2} - t_{d1}) + W c_{ps} (t_{d2} - t_{d1})$$

$$= (c_{pa} + W c_{ps}) (t_{d2} - t_{d1}) = c_{pm} (t_{d2} - \tilde{t}_{d1})$$

The term $(c_{pa} + W c_{ps})$ is called *humid specific heat* (c_{pm}) and its value is taken as 1.022 kJ /kg K.



Fig.15 Sensible heating

Notes: 1. For sensible heating, steam or hot water is passed through the heating coil. The heating coil may be electric resistance coil.

2. The sensible heating of moist air can be done to any desired temperature.

3.72 Sensible Cooling

The cooling of air without any change in its specific humidity, is known as sensible cooling. Let air at temperature rd, passes over a cooling coil of temperature t_{d3} as shown in Fig. 16 (a). It may be noted that the temperature of air leaving the cooling coil (t_{d2}) will be more than t_{d3} . The process of sensible cooling, on the psychrometric chart, is shown by a horizontal line 1-2 extending from right to left as shown in Fig. 16



Fig. 16 Sensible cooling.

The heat rejected by air during sensible cooling may be obtained from the psychrometric chart by the enthalpy difference (h1 - h) as shown in Fig. 16(b).

It may be noted that the specific humidity during the sensible cooling remains constant (i.e. $W_1 = W_2$). The dry bulb temperature reduces from t_{d1} to t_{d2} and relative humidity increases from ϕ_1 to ϕ_2 as shown in Fig. 16(b). The amount of heat rejected during sensible cooling may also be obtained from the relation:

Heat rejected, $q = h_1 - h_2$ $= C_{pa} (t_{d1} - t_{d2}) + W C_{ps}(t_{d1} - t_{d2})$ $= (C_{pa} + W C_{ps}) (t_{d1} - t_{d2}) = C_{pm} (t_{d1} - t_{d2})$

The term $(C_{pa} + W C_{ps})$ is called humid specific heat (C_{pm}) and its value is taken as 1.022 kJ /kg K.

: Heat rejected, $q = 1.022 (t_{d1} - t_{d2}) \text{ kJ/kg}$

For air conditioning purposes, the sensible heat per minute is given as

$$SH = m_a C_{pm} \Delta t = v\rho C_{pm} \Delta t \text{ kJ/min} \qquad \dots (:: m = v\rho)$$

where

v = Rate of dry air flowing in m³/min,

 ρ = Density of moist air at 20° C and 50% relative humidity

= 1.2 kg / m3 of dry air,

 $C_{\rm pm}$ = Humid specific heat = 1.022 kJ /kg K, and

 $\Delta t = t_{d1} - t_{d2}$ = Difference of dry bulb temperatures between

the entering and leaving conditions of air in ° C.

Substituting the values of p and cp,,, in the above expression, we get

SH = v x 1.2 x 1.022 x
$$\Delta t$$
 = 1.2264 v x Δt kJ/min
= $\frac{1.2264 \text{ v} \times \Delta t}{60}$ = 0.020 44 v × Δt kJ/s or kW
... (\because 1 kJ/s = 1 kW)

3.73 By-pass Factor of Heating and Cooling Coil

The temperature of the air corning out of the apparatus (t_{d2}) will be less than $*t_{d3}$ in case the coil is a heating coil and more than t_{d3} in case the coil is a cooling coil.

Let 1 kg of air at temperature t_{d1} is passed over the coil having its temperature (i.e. coil surface temperature) t_{d3} as shown in Fig. 17.
A little consideration will show that when air passes over a coil, some of it (say x kg) just by-passes unaffected while the remaining (1 - x) kg comes in direct contact with the coil. This by-pass process of air is measured in terms of a by-pass factor. The amount of air that by-passes or the by-pass factor depends upon the following factors :

1. The number of fins provided in a unit length i.e. the pitch of the cooling coil fins ;

- 2. The number of rows in a coil in the direction of flow; and
- 3. The velocity of flow of air.

It may be noted that the by-pass factor of a cooling coil decreases with decrease in fin spacing and increase in number of rows.



Fig.17. By-pass factor

Balancing the enthalpies, we get

or

...

$$x c_{pm} t_{d1} + (1 - x) c_{pm} t_{d3}$$

= 1 × c_{pm} t_{d2} ... (where c_{pm}
x (t_{d3} - t_{d1}) = t_{d3} - t_{d2}

$$x = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

where x is called the *by-pass factor* of the coil and is generally written as *BPF*. Therefore, by-pass factor for heating coil,

$$BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

Similarly, *by-pass factor for cooling coil,

I

x

$$3PF = \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}}$$



= Specific humid heat)

The by-pass factor for heating or cooling coil may also be obtained as discussed below :

Let the air passes over a heating coil. Since the temperature distribution of air passing through the heating coil is as shown in Fig.18. therefore sensible heat given out by the coil.



where

U = Overall heat transfer coefficient,

 $A_{\rm c}$ = Surface area of the coil, and

 $t_{\rm m}$ = Logarithmic mean temperature difference.

We know that logarithmic mean temperature difference,

$$t_m = \frac{t_{d2} - t_{d1}}{\log_e \left[\frac{t_{d3} - t_{d1}}{t_{d3} - t_{d2}}\right]}$$
, and $BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$

...

$$t_m = \frac{t_{d2} - t_{d1}}{\log_e (1/BPF)}$$

Now the equation (i) may be written as

$$Q_s = U \times A_c \times \frac{t_{d2} - t_{d1}}{\log_e (1/BPF)} \qquad \dots (ii)$$

	$Q_s = m_a c_{pm} \left(t_{d2} - t_{d1} \right)$	(iii)
vhere	c_{pm} = Humid specific heat = 1.022 kJ/kg K, and	
	m_a = Mass of air passing over the coil.	
Equatin	ng equations (ii) and (iii), we have	
	$UA_c = m_a c_{pm} \log_e (1/BPF)$	
	$\log_e \left(\frac{1}{BPF}\right) = \frac{UA_c}{m_a c_{pm}}$	
or	$\log_e (BPF) = -\frac{UA_c}{m_a c_{pm}}$	
	$-\left(\frac{UA_c}{UA_c}\right) - \left(\frac{UA_c}{UA_c}\right)$	
	$BPF = e^{\binom{m_a c_{pm}}{2}} = e^{\binom{1.022 m_a}{2}}$	(iv)

Proceeding in the same• way as discussed above, we can derive the equation (iv) for a cooling coil.

Note: The performance of a heating or cooling coil is measured in terms of a by-pass factor. A coil with low by-pass factor has better performance.

3.74 Efficiency of Heating and Cooling Coils

The term (1 - BPF) is known as efficiency of coil or contact factor.

 \therefore Efficiency of the heating coil,

$$\eta_{\rm H} = 1 - BPF = 1 - \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}} = \frac{t_{d2} - t_{d1}}{t_{d3} - t_{d1}}$$

Similarly, efficiency of the cooling coil,

$$\eta_{\rm C} = 1 - \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}} = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}}$$

3.75 Humidification and Dehumidification

The addition of moisture to the air, without change in its dry bulb temperature, is known as *humidification*. Similarly, removal of moisture from the air, without change in its dry bulb temperature, is known as *dehumidification*. The heat added during humidification process and heat removed during dehumidification process is shown on the psychrometric chart in Fig. 19 (a) and (b) respectively.



Ultrasonic humidification system

It may be noted that in humidification, the relative humidity increases from \emptyset_1 to \emptyset_2 and specific humidity also increases from W_1 to W_2 as shown in Fig. 19 (a). Similarly, in dehumidification, the relative humidity decreases from \emptyset_1 to \emptyset_2 and specific humidity also decreases from W_1 to W_2 as shown in Fig. 19 (b).



Fig. 19 Humidification and dehumidification

It may be noted that in humidification, change in enthalpy is shown by the intercept $(h_2 - h_1)$ on the psychrometric chart. Since the dry bulb temperature of air during the humidification remains constant, therefore its sensible heat also remains constant. It is thus obvious that the change in enthalpy per kg of dry air due to the increased moisture content equal to $(W_2 - W_1)$ kg per kg of dry air is considered to cause a latent heat transfer (LH). Mathematically,



Multiple small plate dehumidification system

LH = $(h_2 - h_1) = h_{fg} (W_2 - W_1)$ where h_{fg} is the latent heat of vaporization at dry bulb temperature (t_{dt}) .

Notes: 1. For dehumidification, the above equation may be written as:

$$LH = (h_1 - h_2) = h_{fg} (W_1 - W_2)$$

2. Absolute humidification and dehumidification processes are rarely found in practice. These are always accompanied by heating or cooling processes.

3. In air conditioning, the latent heat load per minute is given as

LH =
$$m_a \Delta h = m_a h_{fg} \Delta W = v \rho h_{fg} \Delta W$$
 ... (:: $m_a = v \rho$)

 $v = Rate of dry air flowing in m^3/min,$

 ρ = Density of moist air = 1.2 kg/m³ of dry air,

 h_{fg} = Latent heat of vaporization = 2500 kJ /kg, and

AW = Difference of specific humidity between the entering and leaving conditions of

air = $(W_2 - W_1)$ for humidification and $(W_1 - W_2)$ for dehumidification.

Substituting these values in the above expression, we get

LH = v x 1.2 x 2500 x ΔW = 3000 v x ΔW kJ/min = $\frac{3000 v \times \Delta W}{60}$ = 50 v × ΔW kJ/s or kW

3.8 Methods of Obtaining Humidification and Dehumidification

The humidification is achieved either by supplying or spraying steam or hot water or cold water into the air. The humidification may be obtained by the following two methods:

1. **Direct method**. In this method, the water is sprayed in a highly atomized state into the room to be air-conditioned. This method of obtaining humidification is not very effective.

2. **Indirect method**. In this method, the water is introduced into the air in the airconditioning plant, with the help of an air-washer, as shown in Fig. 20. This -conditioned air is then supplied to the room to be air-conditioned. The air-washer humidification may be accomplished in the following three ways:



Fig. 20. Air-washer.

(a) by using re-circulated spray water without prior heating of air,

(b) by pre-heating the air and then washing if with re-circulated water, and

where

(c) by using heated spray water.

The dehumidification may be accomplished with the help of an air-washer or by using chemicals. In the air-washer system the outside or entering air is cooled below its dew point temperature so that it loses moisture by condensation. The moisture removal is also accomplished when the spray water is chilled water and its temperature is lower than the dew point temperature of the entering air. Since the air leaving the air-washer has its dry bulb temperature much below the desired temperature in the room, therefore a heating coil is placed after the air-washer. The dehumidification may also be achieved by using chemicals which have the capacity to absorb moisture in them. Two types of chemicals known as absorbents (such as calcium chloride) and adsorbents (such as silica gel and activated alumina) are commonly used for this purpose.

Sensible Heat Factor

As a matter of fact, the heat added during a psychrometric process may be split up into sensible heat and latent heat. The ratio of the *sensible heat to the total heat is known us *sensible heat factor* (briefly written as SHF) or *sensible heat ratio* (briefly written as SHR). Mathematically,

	QUE	Sensible heat	SH
	SHF =	Total heat	$=$ $\frac{1}{SH + LH}$
where		SH = Sensi	ble heat, and
		LH = Later	nt heat.

The sensible heat factor scale is shown on the right hand side of the psychrometric chart.

3.9 Cooling and Dehumidification

This process is generally used in summer air conditioning to cool and dehumidify the air. The air is passed over a cooling coil or through a cold water spray. In this process, the dry bulb temperature as well as the specific humidity of air decreases. The final relative humidity of the air is generally higher than that of the entering air. The dehumidification of air is only possible wh6n the effective surface temperature of the cooling coil (i.e. t_{d4}) is less than the dew point temperature of the air entering the coil (i.e., t_{dpt} .). The effective surface temperature of the cooling and dehumidification process is shown in Fig. 21.



 t_{d1} = Dry bulb temperature of air entering the coil,

 t_{dpl} = Dew point temperature of the entering air = t_{d3} and

 t_{d4} = Effective surface temperature or ADP of the coil.

Under ideal conditions, the dry bulb temperature of the air leaving the cooling coil (i.e. t_{d4}) should be equal to the surface temperature of the cooling coil (i.e. ADP), but it is never possible due to inefficiency of the cooling coil. Therefore, the resulting condition of air coming out of the coil is shown by a point 2 on the straight line joining the points 1 and 4. The by-pass factor in this case is given by

Also

$$BPF = \frac{t_{d2} - t_{d4}}{t_{d1} - t_{d4}} = \frac{t_{d2} - ADP}{t_{d1} - ADP}$$
$$BPF = \frac{W_2 - W_4}{W_1 - W_4} = \frac{h_2 - h_4}{h_1 - h_4}$$

Actually, the cooling and dehumidification process follows the path as shown by a dotted curve in Fig. 21(a), but for the calculation of psychrometric properties, only end points are important. Thus the cooling and dehumidification process shown by a line 1-2 may be assumed to have followed a path 1-A (i.e. dehumidification) and A-2 (i.e. cooling) as shown in Fig. 21 (a). We see that the total heat removed from the air during the cooling and dehumidification process is

$$q = h_1 - h_2 = (h_1 - h_A) + (h_A - h_2) = LH + SH$$

where $LH = h_1 - h_A = Latent$ heat removed due to condensation of vapour of the reduced moisture content (W₁ – W₂), and

$$SH = h_A - h_2 = Sensible heat removed.$$

We know that sensible heat factor,

$$SHF = \frac{Sensible heat}{Total heat} = \frac{SH}{LH + SH} = \frac{h_A - h_2}{h_1 - h_2}$$

Note: The line 1-4 (i.e. the line joining the point of entering air and the apparatus dew point) in Fig. 21 (b) is known as sensible heat factor line.

Example 1: In a cooling application, moist air enters a refrigeration coil at the rate of 100 kg of dry air per minute at 35° C and 50% RH. The apparatus dew point of coil is 5° C and by-pass factor is 0.15. Determine the outlet state of moist air and cooling capacity of coil in TR.

Solution Given: $m_a = 100 \text{ kg/min}$; $t_{dt} = 35^{\circ}\text{C}$; $\emptyset = 50\%$; ADP = 5°C; BPF = 0.15

Outlet state of moist air

Let t_{d2} , and ϕ_2 = Temperature and relative humidity of air leaving the cooling coil.

First of all, mark the initial condition of air, i.e. 35° C dry bulb temperature and 50% relative humidity on the psychrometric chart at point 1, as shown in Fig. 22. From the psychrometric chart, we find that the dew point temperature of the entering air at point 1,

$$t_{\rm dpt} = 23^{\circ}{\rm C}$$

Since the coil or apparatus dew point (ADP) is less than the dew point temperature of entering air, therefore it is a process of cooling and dehumidification.

We know that by-pass factor,



Fig.22

From the psychrometric chart, we find that the relative humidity corresponding to a dry bulb temperature (t_{d2}) of 9.5°Con the line 1-4 is $\phi_2 = 99\%$. Ans.

Cooling capacity of the coil

The resulting condition of the air coming out of the coil is shown by point 2, on the line joining the points 1 and 4, as shown in Fig. 22. The line 1-2 represents the cooling and dehumidification process which may be assumed to have followed the path 1-A (i.e. dehumidification) and A-2 (i.e. cooling). Now from the psychrometric chart, we find that enthalpy of entering air at point 1,

and enthalpy of air at point 2,

 $h_2 = 28 \text{ kJ/kg of dry air}$

We know that cooling capacity of the coil

$$= m_{a}(h_{1}-h_{2}) = 100 (81 - 28) = 5300 \text{ kJ/min}$$
$$= 5300/210 = 25.24 \text{ TR Ans. } \dots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

Example 2. 39.6 m³/min of a mixture of re-circulated room air and outdoor air enters cooling coil at 31°C dry bulb temperature and 18.5°C wet bulb temperature. The effective surface temperature of the coil is 4.4°C. The surface area of the coil is such as would give 12.5 kW of refrigeration with the given entering air state. Determine the dry and wet bulb temperatures of the air leaving the coil and the by-pass factor.

Solution: Given: v_1 = 39.6 m³/min; t_{dt} = 31°C; t_{wt} = I8.5°C; ADP= t_{d4} = 4.4°C; Q= 12.5 kW = 12.5 kJ/s = 12.5 x 60 kJ/min

Dry and wet bulb temperature of the air leaving the coil

Let t_{d2} and t_{w2} = Dry and wet bulb temperature of the air leaving the coil.

First of all, mark the initial condition of air, i.e. 31°C dry bulb temperature and 18.5°C wet bulb temperature on the psychrometric chart at point 1, as shown in Fig. 23. Now mark the effective surface temperature (ADP) of the coil at 4.4°C at point 4.

From the psychrometric chart, we find that enthalpy at point 1

 $h_1 = 52.5 \text{ kJ} / \text{kg of dry air}$

Enthalpy at point 4,

 $h_4 = 17.7 \text{kJ/kg}$ of dry air

Specific humidity at point 1

 $W_1 = 0.0082 \text{ kg} / \text{kg of dry air}$

Specific humidity at point 4,

 $W_4 = 0.00525 \text{ kg} / \text{kg of dry air}$

Specific volume at point

$$v_{s1}$$
, = 0.872m³/ kg

We know that mass flow rate of dry air at point 1,



 $m_{a} = \frac{v_{1}}{v_{s1}} = \frac{39.6}{0.872} = 44.41 \text{ kg/min}$ and cooling capacity of the coil, $Q = m_{a}(h_{1} - h_{2})$ or $h_{1} - h_{2} = \frac{Q}{m_{a}} = \frac{12.5 \times 60}{44.41} = 16.89 \text{ kJ / kg of dry air}$ $h_{2} = h_{1} - 16.89 = 52.5 - 16.89 = 35.61 \text{ kJ / kg of dry air}$ The equation for the condition line 1-2-4 is given as $\frac{W_{2} - W_{4}}{W_{1} - W_{4}} = \frac{h_{2} - h_{4}}{h_{1} - h_{4}}$ $\frac{W_{2} - 0.005 25}{0.0082 - 0.005 25} = \frac{35.61 - 17.7}{52.5 - 17.7}$ $W_{2} = 0.006 77 \text{ kg / kg of dry air}$

Now plot point 2 on the psychrometric chart such as enthalpy, $h_2 = 35.61$ kJ/kg of dry air and specific humidity, $W_2 = 0.00677$ kg/kg of dry air. At point 2, we find that

$$t_{d2} = 18.5^{\circ}$$
C; and $t_{w2} = 12.5^{\circ}$ C Ans.

By-pass factor

We know that by-pass factor,

$$BPF = \frac{h_2 - h_4}{h_1 - h_4} = \frac{35.61 - 17.7}{52.5 - 17.7} = 0.5146 \text{ Ans}.$$

3.10 Heating and Humidification

This process is generally used in winter air conditioning to warm and humidify the air. It is the reverse process of cooling and -- dehumidification. When air is passed through a humidifier having spray water temperature higher than the dry bulb temperature of the entering air, the unsaturated air will reach the condition of saturation and thus the air becomes hot. The heat of vaporization of water is absorbed from the spray water itself and hence it gets cooled. In this way, the air becomes heated and humidified. The process of heating and humidification is shown by line 1-2 on the psychrometric chart as shown in Fig. 24.



The air enters at condition 1 and leaves at condition 2. In this process, the dry bulb temperature as well as specific humidity of air increases. The final relative humidity of the air can be lower or higher than that of the entering air.



Fig.24 heating and humidification

Let m_{w1} and m_{w2} = Mass of spray water entering and leaving the humidifier in kg, h_{fw1} and h_{fw2} = Enthalpy of spray water entering and leaving the humidifier in kJ/kg, W_1 and W_2 = Specific humidity of the entering and leaving air in kg/kg of dry h_1 and h_2 = Enthalpy of entering and leaving air in kJ/kg of dry air, and m_a = Mass of dry air entering in kg.

For mass balance of spray water,

$$(m_{w1} - m_{w2}) = m_a (W_2 - W_1)$$

 $m_{w2} = m_{w1} - m_a (W_2 - W_1)$ (i)

or and for enthalpy balance,

$$m_{w1} h f_{w1} = m_{w2} h_{fw2} = m_a (h_2 - h_1)$$
(ii)

Substituting the value of mw2 from equation (i), we have

$$m_{w1} h_{fw1} - [m_{w1} - m_a (W_2 - W_1)] h_{fw2}$$
$$= m_a (h_2 - h_1)$$
$$h_2 - h_1 = \frac{m_{w1}}{m_a} (h_{fw1} - h_{fw2}) + (W_2 - W_1) h_{fw2}$$

The temperatures t_{s1} and t_{s2} shown in Fig. 24 (a) denote the temperatures of entering and leaving spray water respectively. The temperature 13 is the mean temperature of the spray water which the entering air may be assumed to approach.

Actually, the heating and humidification process follows the path as shown by dotted curve in Fig. 24(b), but for the calculation of psychrometric properties, only the end points are important. Thus, the heating and humidification process shown by a line 1-2 on the psychrometric chart may be assumed to have followed the path 1-A (i.e. heating) and A-2

(i.e. humidification), as shown in Fig. 24(b). We see that the total heat added to the air during heating and humidification is

$$q = h_2 - h_1 = (h_2 - h_1) + (h_A - h_i) = q_t + q_s$$

where moisture

$$q_t = (h_2 - h_A) =$$
 Latent heat of vaporization of the increased

content
$$(W_2 - W_1)$$
, and

$$q_{\rm s} = (h_{\rm A} - h_{\rm i}) =$$
 Sensible heat added

We know that sensible heat factor,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{q_{\text{S}}}{q} = \frac{q_{\text{S}}}{q_{\text{S}} + q_{\text{L}}} = \frac{h_{\text{A}} - h_{\text{I}}}{h_{2} - h_{\text{I}}}$$

Note: The line 1-2 in Fig. 24 (b) is called sensible heat factor line.

3.11 Heating and Humidification by Steam Injection

The steam is normally injected into the air in order to increase its specific humidity as shown in Fig. 25 (a). This process is used for the air conditioning of textile mills where high humidity is to be maintained. The dry bulb temperature of air changes very little during this process, as shown on the psychrometric chart in Fig. 25 (b).

Let $m_{\rm s} = {\rm Mass}$ of steam supplied,

 $m_{\rm a} = {\rm Mass}$ of dry air entering,



Fig.25 heating and humidification by steam injection

W₁= Specific humidity of air entering,

 $W_2 =$ Specific humidity of air leaving,

 h_1 = Enthalpy of air entering,

 h_2 = Enthalpy of air leaving, and

 $h_{\rm s}$ = Enthalpy of steam injected into the air.

Now for the mass balance,

$$W_2 = W_1 + \frac{m_s}{m_a} \qquad \dots (i)$$

and for the heat balance,

$$h_2 = h_1 + \frac{m_s}{m_a} \times h_s = h_1 + (W_2 - W_1) h_s$$
 ... [From equation (i)]

Example 3: Atmospheric air at a dry bulb temperature of 16° C and 25% relative humidity passes through a furnace and then through a humidifier, in such a way that the final dry bulb temperature is 30° C and 50% relative humidity. Find the heat and moisture added to the air. Also determine the sensible heat factor of the-process.

Solution: Given:
$$t_{dt} = 16^{\circ}$$
 C; $\phi_1 = 25\%$; $t_{d2} = 30^{\circ}$ C; $42 = 50\%$

Heat added to the air

First of all, mark the initial condition of air i.e. at 16° C dry bulb temperature and 25% relative humidity on the psychrometric chart at point 1, as shown in Fig. 16.47. Then mark the final condition of air at 30° C dry bulb temperature and 50% relative humidity on the psychrometric chart at point 2. Now locate the point A by drawing horizontal line through point 1 and vertical line through point 2. From the psychrometric chart, we find that enthalpy of air at point 1,

$$h_1 = 23 \text{ k}1/\text{kg}$$
 of dry air



Fig.26

Enthalpy of air at point A,

$$h_{\rm A} = 38 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 2,

$$h_2 = 64 \text{ la/kg of dry air}$$

 \therefore Heat added to the air

 $= h_2 - h_1 = 64 - 23 = 41$ kJ/kg of dry air Ans.

Moisture added to the air

From the psychrometric chart, we find that the specific humidity in the air at point 1,

 $W_1 = 0.0026 \text{ kg/kg}$ of dry air

and specific humidity in the air at point 2,

 $W_2 = 0.0132 \text{ kg}/\text{kg}$ of dry air

 \therefore Moisture added to the air

 $= W_2 - W_1 = 0.0132 - 0.0026 = 0.0106 \text{ kg/kg of dry air Ans.}$

Sensible heat factor of the process

We know that sensible heat factor of the process,

$$SHF = \frac{h_{\rm A} - h_{\rm 1}}{h_{\rm 2} - h_{\rm 1}} = \frac{38 - 23}{64 - 23} = 0.366$$
 Ans.

Example 4: Air at 10°C dry bulb temperature and 90% relative humidity is to be ated and humidified to 35°C dry bath temperature and 22.5°C wet bulb temperature. The air is pre-heated sensibly before passing to the air washer in which water is recirculated. The relative humidity of the air coming out of the air washer is 90%. This air is again reheated sensibly to obtain the final desired condition. Find: 1. the temperature to which the air should be preheated. 2. the total heating required; 3. the makeup water required in the air washer ; and 4. the humidifying efficiency of the air washer.

Solution: Given :
$$t_{dt} = 10^{\circ}$$
C; $\phi_1 = 90\%$; t_{d2} ,=35°C; t_{w2} =22.5°C

First of all, mark the initial condition of air i.e. at 10°C dry bulb temperature and 90% relative humidity, on the psychrometric chart at point 1, as shown in Fig. 16.48. Now mark the final condition of air i.e. at 35° C dry bulb temperature and 22.5° C wet bulb temperature at point 2.

From point I, draw a horizontal line to represent sensible heating and from point 2 draw horizontal line to intersect 90% relative humidity curve at point B. Now from point B, draw a constant wet bulb temperature line which intersects the horizontal line drawn through point 1 at point A. The line 1-A represents preheating of air, line AB represents humidification and line 8-2 represents reheating to final condition.





1. Temperature to which the air should be preheated

From the psychrometric chart, the temperature to which the air should be preheated (corresponding to point A) is 4 t_{dA} = 32.6°C Ans.

2. Total heating required

From the psychrometric chart, we find that enthalpy of,air at point 1.

 $h_1 = 27.2 \text{ kJ} / \text{kg of dry air}$

Enthalpy of air at point A,

 $h_{\rm A} = 51 \text{ kJ/kg of dry air}$

and enthalpy of air at point 2,

 $h_2 = 68 \text{ kJ/kg of dry air}$

We know that heat required for preheating of air

 $= h_{\text{A}} - h_1 = 51 - 27.2 = 23.8 \text{ kJ/kg of dry air}$

and heat required for reheating of air

 $= h_2 - h_B = 68 - 51 = 17 \text{ kJ/kg of dry air}$

 \therefore Total heat required = 23.8+ 17 = 40.8 kJ/kg of dry air Ans.

3. Make up water required in the air washer

From the psychrometric chart, we find that specific humidity of entering air,

 $W_1 = 0.0068 \text{ kg}/\text{kg}$ of dry air

and specific humidity of leaving air,

$$W_2 = 0.0122 \text{ kg}/\text{kg}$$
 of dry air

: Make up water required in the air washer

$$= W_{B} - W_{A} = W_{2} - W_{1}$$

= 0.0122 - 0.0068 = 0.0054 kg/kg of dry air Ans.

4. Humidifying efficiency of the air washer

From the psychrometric chart, we find that

$$t_{\rm dB} = 19.1^{\circ}{\rm C}$$
 and $t_{\rm dB} = 18^{\circ}{\rm C}$

We know that humidifying efficiency of the air washer,

$$\eta_{\rm H} = \frac{\text{Actual drop in DBT}}{\text{Ideal drop in DBT}} = \frac{t_{dA} - t_{dB}}{t_{dA} - t_{dB'}}$$
$$= \frac{32.6 - 19.1}{32.6 - 18} = \frac{13.5}{14.6} = 0.924 \text{ or } 92.4\% \text{ Ans.}$$

3.12 Heating and Dehumidification -Adiabatic Chemical Dehumidification

This process is mainly used in industrial air conditioning and can also be used for some comfort air conditioning installations requiring either a low relative humidity or low dew point temperature in the room.

In this process, the air is passed over chemicals which have an affinity for moisture. As the air comes in contact with these chemicals, the moisture gets condensed out of the air and gives up its latent heat. Due to the condensation, the specific humidity decreases and the heat of condensation supplies sensible heat for heating the air and thus increasing its dry bulb temperature.



Fig.28

The process, which is the reverse of adiabatic saturation process, is shown by the line 1-2 on the psychrometric chart as shown in Fig. 28. The path followed during the process is along the constant wet bulb temperature line or-constant enthalpy line.

The effectiveness or efficiency of the dehumidifier is given as

$$\eta_{\rm H} = \frac{\text{Actual increase in dry bulb temperature}}{\text{Ideal increase in dry bulb temperature}} = \frac{t_{d3} - t_{d1}}{t_{d2} - t_{d1}}$$

Notes: 1. In actual practice, the process is accompanied with a release of heat called heat of adsorption, which is very large. Thus the sensible heat gain of air exceeds the loss of latent heat and the process is shown above the constant wet bulb temperature line in Fig. 28.

2. Two types of chemicals used for dehumidification are absorbents and adsorbents. The absorbents are substances which can take up moisture from air and during this process change it chemically, physically or in both respects. These include water solutions or brines of calcium chloride, lithium chloride, lithium bromide and ethylene glycol. These are used as air dehydrators by spraying or otherwise exposing a large surface of the solution in the air stream.

The adsorbents are substances in the solid state which can take up moisture from the air and during this process do not change it chemically or physically. These include silca gel (which is a form of silicon dioxide prepared by mixing fused sodium silicate and sulphuric acid) and activated alumina (which is a porous amorphous form of aluminum oxide).

Example 5: Saturated air at 21° C is passed through a drier so that its final relative humidity is 20%. The drier uses silica gel adsorbent. The air is then passed through a cooler until its final temperature is 21° C without a change in specific humidity. Determine : 1. the temperature of air at the end of the drying process; 2. the heat rejected during the cooling process ; 3. the relative humidity at the end of cooling process; 4. the dew point temperature at the end of the drying process ; and 5. the moisture removed during the drying process.

Solution: Given: t_{d1} , = t_{d3} = 21°C; ϕ_2 = 20%

1. Temperature of air at the end of drying process

First of all, mark the initial condition of air i.e. at 21°C dry bulb temperature upto the saturation curve (because the air is saturated) on the psychrometric chart at point 1, as shown in Fig. 29. Since the drying process is a chemical dehumidification process, therefore-. it follows a path along-the-constant wet bulb temperature or the constant enthalpy line as shown by the line 1- 2 in Fig. 29. Now mark the point 2 at relative humidity of 20%. From the psychrometric chart, the temperature at the end of drying process at point 2, $t_{d2} = 38.5$ °C Ans.



Fig.29

2. Heat rejected during the cooling process

The cooling process is shown by the line 2-3 on the psychrometric chart as shown in Fig.29. From the psychrometric chart. we find that enthalpy of air at point 2.

 $h_2 = 61 \text{ kJ} / \text{kg of dry air}$

and enthalpy of air at point 3,

 $h_3 = 43 \text{ kJ/kg of dry air}$

: Heat rejected during the cooling process

$$= h_2 - h_3 = 61 - 43 = 18 \text{ kJ/kg of dry air A ns.}$$

3. Relative humidity at the end of cooling process

From the psychrometric chart, we find that relative humidity at the end of cooling process (i.e. at point 3),

 $\phi_3 = 55\%$ Ans.

4. Dew point temperature at the end of drying process

From the psychrometric chart, we find that the dew point temperature at the end of the drying process,

$$t_{\rm dp2} = 11.6^{\circ} \text{ C Ans.}$$

5. Moisture removed during the drying process

From the psychrometric chart, we find that moisture in air before the drying process at point 1,

$$W_1 = 0.0157 \text{ kg/kg of dry air}$$

and moisture in air after the drying process at point 2,

$$W_2 = 0.0084 \text{ kg/kg of dry air}$$

MODULE 4 COMPRESSIBLE FLOW

INTRODUCTION

A compressible flow is that flow in which the density of the fluid changes during flow. All real fluids are compressible to some extent and therefore their density will change with change in pressure or temperature. If the relative change in density $\Delta \rho / \rho$ is small, the fluid can be treated as incompressible. A compressible fluid, such as air, can be considered as incompressible with constant ρ if changes in elevation are small, acceleration is small, and/or temperature changes are negligible. In other words, if Mach's number U/C, where C is the sonic velocity, is small, compressible fluid can be treated as incompressible.

• The gases are treated as compressible fluids and study of this type of flow is often referred to as 'Gas dynamics'. • Some important problems where compressibility effect has to be considered are : (i) Flow of gases through nozzles, orifices ; (ii) Compressors ; (iii) Flight of aeroplanes and projectiles moving at higher altitudes ; (iv) Water hammer and acoustics.

• 'Compressibility' affects the drag coefficients of bodies by formation of shock waves,

discharge coefficients of measuring devices such as orificemeters, venturimeters and pitot tubes, stagnation pressure and flows in converging-diverging sections.

BASIC EQUATIONS OF COMPRESSIBLE FLUID FLOW

The basic equations of compressible fluid flow are : (i) Continuity equation, (ii) Momentum equation, (iii) Energy equation, and (iv) Equation of state.

16.2.1. Continuity Equation

In case of one-dimensional flow, mass per second = Pav (where ρ = mass density, A = area of cross-section, V = velocity)

Since the mass or mass per second is constant according to law of conservation of mass, therefore, $\rho AV = constant$

Differentiating the above equation, we get

$$d(\rho AV) = 0 \text{ or } \rho d(AV) + AVd\rho = 0$$

 $\rho(AdV + VdA) + AVd\rho = 0 \text{ or } \rho AdV + \rho VdA + AVd\rho = 0$

Dividing both sides by ρAV and rearranging, we get

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad \dots (16.2)$$

Eqn. (16.2) is also known as equation of continuity in differential form.

16.2.2. Momentum Equation

The momentum equation for compressible fluids is similar to the one for incompressible fluids. This is because in momentum equation the change in momentum flux is equated to force required to cause this change.

Momentum flux = mass flux × velocity =
$$\rho AV \times V$$

But the mass flux *i.e.*, ρAV = constant ...By continuity equation

Thus the momentum equation is completely independent of the compressibility effects and for compressible fluids the momentum equation, say in X-direction, may be expressed as :

$$\Sigma F_x = (\rho A V V_x)_2 - (\rho A V V_x)_1$$
 ...(16.3)

16.2.3. Bernoulli's or Energy Equation

As the flow of compressible fluid is steady, the Euler equation is given as :

$$\frac{dp}{\rho} + VdV + gdz = 0$$
 ...(16.4)

Integrating both sides, we get

$$\int \frac{dp}{\rho} + \int VdV + \int gdz = \text{constant}$$

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant} \qquad \dots(16.5)$$

or

or

In compressible flow since ρ is not constant it cannot be taken outside the integration sign. In compressible fluids the pressure (p) changes with change of density (ρ) , depending on the type of process. Let us find out the Bernoulli's equation for isothermal and adiabatic processes.

(a) Bernoulli's or energy equation for isothermal process :

In case of an isothermal process,

$$pv = \text{constant or } \frac{p}{\rho} = \text{constant} = c_1 (say)$$

(where $v = \text{specific volume} = 1/\rho$)

Δ.

$$\rho = \frac{p}{c}$$

Hence

$$\int \frac{dp}{\rho} = \int \frac{dp}{p/c_1} = \int \frac{c_1 dp}{p} = c_1 \int \frac{dp}{p} = c_1 \log_e p = \frac{p}{\rho} \log_e p \left(\because c_1 = \frac{p}{\rho} \right)$$

Substituting the value of $\int \frac{dp}{p}$ in eqn. (16.5), we get $\frac{p}{p} \log_{e} p + \frac{V^{2}}{2} + gz = \text{constant}$ Dividing both sides by g, we get

$$\frac{p}{\rho g} \log_e p + \frac{V^2}{2g} + z = \text{constant} \qquad \dots (16.6)$$

Eqn. (16.6) is the Bernoulli's equation for compressible flow undergoing isothermal process. (b) Bernoulli's equation for adiabatic process :

In case of an adiabatic process,

Substituting the value of $\int \frac{dp}{p}$ in eqn. (16.6), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Dividing both sides by g, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$
 ...(16.7)

Eqn. (16.7) is the Bernoulli's equation for compressible flow undergoing adiabatic process.

PROPAGATION OF DISTURBANCES IN FLUID AND VELOCITY OF SOUND

The solids as well as fluids consist of molecules. Whereas the molecules in solids are close together, these are relatively apart in fluids. Consequently whenever there is a minor disturbance, it travels instantaneously in case of solids ; but in case of fluid the molecules change its position before the transmission or propagation of the disturbance. Thus the velocity of disturbance in case of fluids will be less than the velocity of the disturbance in solids. In case of fluid, the propagation of disturbance depends upon its elastic properties. The velocity of disturbance depends up on the changes in pressure and density of the fluid. The propagation of disturbance is similar to the propagation of sound through a media. The speed of propagation of sound in a media is known as acoustic or sonic velocity and depends upon the difference of pressure. In compressible flow, velocity of sound (sonic velocity) is of paramount importance.

Derivation of Sonic Velocity (velocity of sound)

Consider a one-dimensional flow through long straight rigid pipe of uniform crosssectional area filled with a frictionless piston at one end as shown in Fig. 16.1. The tube is filled with a compressible fluid initially at rest. If the piston is moved suddenly to the right with a velocity, a pressure wave would be propagated through the fluid with the velocity of sound wave.



Fig. 16.1. One dimensional pressure wave propagation.

Let A = cross-sectional area of the pipe,

V = piston velocity,

p = fluid pressure in the pipe before the piston movement,

ρ = fluid density before the piston movement,

dt = a small interval of time during which piston moves, and

C = velocity of pressure wave or sound wave (travelling in the fluid).

Before the movement of the piston the length dL has an initial density ρ , and its total mass = $\rho \times dL \times A$.

When the piston moves through a distance dx, the fluid density within the compressed region of length (dL - dx) will be increased and becomes $(\rho + d\rho)$ and subsequently the total mass of fluid in the compressed region = $(\rho + d\rho) (dL - dx) \times A$

	$\rho \times dL \times A = (\rho + d\rho) (dL - dx) \times A$ by principle of continuity.
But	dL = C dt and $dx = Vdt$; therefore, the above equation becomes
	$\rho C dt = (\rho + d\rho) (C - V) dt$
or,	$\rho C = (\rho + d\rho) (C - V) \text{ or } \rho C = \rho C - \rho V + d\rho \cdot C - d\rho \cdot V$

 $0 = -\rho V + d\rho \cdot C - d\rho \cdot V$

or,

Neglecting the term $d\rho V$ (V being much smaller than C), we get

$$d\rho \cdot C = \rho V \text{ or } C = \frac{\rho V}{d\rho}$$
 ...(16.8)

Further in the region of compressed fluid, the fluid particles have attained a velocity which is apparently equal to V (velocity of the piston), accompanied by an increase in pressure dp due to sudden motion of the piston. Applying inpulse-momentum equation for the fluid in the compressed region during dt, we get

 $dp \times A \times dt = \rho \times dL \times A (V - 0)$

(force on the fluid) (rate of change of momentum)

$$dp = \rho \frac{dL}{dt} V = \rho \times \frac{Cdt}{dt} \times V = \rho CV \qquad (\because dL = Cdt)$$
$$C = \frac{dp}{\rho V} \qquad \dots (16.9)$$

or,

or,

Multiplying eqns. (16.8) and (16.9), we get

$$C^{2} = \frac{\rho V}{d\rho} \times \frac{dp}{\rho V} = \frac{dp}{d\rho}$$

$$C = \sqrt{\frac{dp}{d\rho}} \qquad \dots (16.10)$$

16.3.2. Sonic Velocity in Terms of Bulk Modulus

The bulk modulus of elasticity of fluid (K) is defined as

$$K = \frac{dp}{\left(\frac{dv}{v}\right)} \qquad \dots (i)$$

where, dv = decrease in volume, and v = original volume (– ve sign indicates that volume decreases with increase in pressure)

Also, volume $v \propto \frac{1}{\rho}$, or $v\rho$ = constant Differentiating both sides, we get

...

$$vd\rho + \rho dv = 0$$
 or $-\frac{dv}{v} = \frac{d\rho}{\rho}$

Substituting the value of $-\frac{dv}{v}\left(=\frac{dp}{K}\right)$ from eqn. (i), we get

$$\frac{dp}{K} = \frac{d\rho}{\rho} \text{ or } \frac{dp}{d\rho} = \frac{K}{\rho}$$

Substituting this value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\frac{K}{\rho}} \qquad \dots (16.11)$$

Eqn. (16.11) is applicable for liquids and gases.

16.3.3. Sonic Velocity for Isothermal Process

For isothermal process : $\frac{p}{\rho}$ = constant Differentiating both sides, we get

$$\frac{\rho \cdot dp - p \cdot d\rho}{\rho^2} = 0 \quad \text{or} \quad \frac{dp}{\rho} - \frac{p \cdot d\rho}{\rho^2} = 0$$
$$\frac{dp}{\rho} = \frac{p \cdot d\rho}{\rho^2} \quad \text{or} \quad \frac{dp}{d\rho} = \frac{p}{\rho} = RT \quad \dots (16.12)$$

or,

 $\left(\frac{p}{\rho} = RT \quad \dots \text{ equation of state}\right)$

Substituting the value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\frac{p}{\rho}} = \sqrt{RT} \qquad \dots (16.13)$$

16.3.4. Sonic Velocity for Adiabatic Process

For isentropic (reversible adiabatic) process : $\frac{p}{\rho^{\gamma}} = \text{constant}$ or $p \cdot \rho^{-\gamma} = \text{constant}$ Differentiating both sides, we have $p(-\gamma) \cdot \rho^{-\gamma-1} d\rho + \rho^{-\gamma} dp = 0$ Dividing both sides by $\rho^{-\gamma}$, we get $-p \gamma \rho^{-1} d\rho + dp = 0$ or $dp = p \gamma \rho^{-1} d\rho$ or, $\frac{dp}{d\rho} = \frac{p}{\rho} \gamma = \gamma RT$ $\left(\because \frac{p}{\rho} = RT\right)$ Substituting the value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get $C = \sqrt{\gamma RT}$...(16.14)

The following points are worth noting :

(*i*) The process is assumed to be *adiabatic* when minor disturbances are to be propagated through air ; due to *very high velocity* of disturbances/pressure waves appreciable heat transfer does not take place.

(*ii*) For calculation of velocity of the sound/pressure waves, *isothermal process* is considered only when it is mentioned in the numerical problem (that the process is isothermal). When no process is mentioned in the problem, calculation are made assuming the process to be *adiabatic*.

MACH NUMBER

The mach number is an important parameter in dealing with the flow of compressible fluids, when elastic forces become important and predominant.

Mach number is defined as the *square root of the ratio of the inertia force of a fluid to the elastic force.*

$$\therefore \text{ Mach number}, \qquad M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho A V^2}{KA}}$$

$$= \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \qquad [\because \sqrt{K/\rho} = C \dots \text{Eqn. (16.11)}]$$

$$M = \frac{V}{C} \qquad \dots (16.15)$$
Thus,
$$M = \frac{\text{Velocity at a point in a fluid}}{\text{Velocity of sound at that point at a given instant of time}}$$

i.e.

Depending on the value of Mach number, the flow can be classified as follows :

1. Subsonic flow : Mach number is less than 1.0 (or M < 1); in this case V < C.

2. Sonic flow : Mach number is equal to 1.0 (or M = 1); in this case V = C.

3. Supersonic flow : Mach number is greater than 1.0 (or M > 1); in this case V > C.

When the Mach number in flow region is slightly less to slightly greater than 1.0, the flow is termed as *transonic flow*.

The following points are worth noting:

(*i*) Mach number is important in those problems in which the flow velocity is comparable with the sonic velocity (velocity of sound). It may happen in case of airplanes travelling at very high speed, projectiles, bullets etc.

(*ii*) If for any flow system the Mach number is less than about 0.4, the effects of compressibility may be neglected (for that flow system).

STAGNATION PROPERTIES

The point on the immersed body where the velocity is zero is called **stagnation point.** At this point velocity head is converted into pressure head. The values of pressure (ps), temperature (Ts) and density (ρ s) at stagnation point are called stagnation properties.

16.6.1. Expression for Stagnation Pressure (ps) in Compressible Flow

Consider the flow of compressible fluid past an immersed body where the velocity becomes zero. Consider frictionless adiabatic (isentropic) condition.

Let us consider two points, O in the free stream and the stagnation point S as shown in Fig. 16.4.

Let, p0 = pressure of compressible fluid at point O,

V0 = velocity of fluid at O,

 $\rho 0 =$ density of fluid at O,

T0 = temperature of fluid at O,

and p_s , V_s , ρ_s and T_s are corresponding values of pressure, velocity density, and temperature at point S.

Applying Bernoulli's equation for adiabatic (frictionless) flow at points O and S, (given by eqn. 16.7), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0g} + \frac{V_0^2}{2g} + z_0 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_sg} + \frac{V_s^2}{2g} + z_0$$

But $z_0 = z_s$; the above equation reduces to

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0g} + \frac{V_0^2}{2g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_sg} + \frac{V_s^2}{2g}$$



Bod

Streamlines

Cancelling 'g' on both the sides, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0} + \frac{{V_0}^2}{2} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s} + \frac{{V_s}^2}{2}$$

At point S the velocity is zero, *i.e.*, $V_s = 0$; the above equation becomes

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0} - \frac{p_s}{\rho_s}\right) = -\frac{V_0^2}{2}$$
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1 - \frac{p_s}{\rho_s} \times \frac{\rho_0}{p_0}\right) = -\frac{V_0^2}{2}$$
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1 - \frac{p_s}{\rho_s} \times \frac{\rho_0}{p_0}\right) = V_0^2$$

or,

or,

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1-\frac{p_s}{p_0}\times\frac{\rho_0}{\rho_s}\right) = -\frac{V_0^2}{2} \qquad \dots (i)$$

For adiabatic process :
$$\frac{p_0}{\rho_0^{\gamma}} = \frac{p_s}{\rho_s^{\gamma}}$$
 or $\frac{p_0}{p_s} = \frac{\rho_0^{\gamma}}{\rho_s^{\gamma}}$ or $\frac{\rho_0}{\rho_s} = \left(\frac{p_0}{p_s}\right)^{\gamma}$...(ii)

Substituting the value of $\frac{\rho_0}{\rho_s}$ in eqn. (i), we get

$$\begin{split} \left(\frac{\gamma}{\gamma-1}\right) \frac{p_0}{\rho_0} \left[1 - \frac{p_s}{p_0} \times \left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}}\right] &= -\frac{V_0^2}{2} \\ \left(\frac{\gamma}{\gamma-1}\right) \frac{p_0}{\rho_0} \left\{1 - \left(\frac{p_s}{p_0}\right)^{1-\frac{1}{\gamma}}\right\} &= -\frac{V_0^2}{2} \\ \left[1 - \left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right] &= -\frac{V_0^2}{2} \left(\frac{\gamma-1}{\gamma}\right) \frac{\rho_0}{p_0} \\ 1 + \frac{V_0^2}{2} \left(\frac{\gamma-1}{\gamma}\right) \frac{\rho_0}{p_0} &= \left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \end{split} \qquad \dots (iii)$$

or,

or,

For adiabatic process, the sonic velocity is given by,

$$C = \sqrt{\gamma RT} = \sqrt{\gamma \frac{p}{\rho}} \qquad \qquad \left(\because \frac{p}{\rho} = RT \right)$$

For point O,

$$C_0 = \sqrt{\gamma \frac{p_0}{\rho_0}} \text{ or } C_0^2 = \gamma \frac{p_0}{\rho_0}$$

Substituting the value of $\frac{\gamma p_0}{\rho_0} = C_0^2$ in eqn. (iii), we get

$$1 + \frac{V_0^2}{2} (\gamma - 1) \times \frac{1}{C_0^2} = \left(\frac{p_s}{p_0}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$1 + \frac{V_0^2}{2C_0^2} (\gamma - 1) = \left(\frac{p_s}{p_0}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$1 + \frac{M_0^2}{2} (\gamma - 1) = \left(\frac{p_s}{p_0}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \left(\because \frac{V_0^2}{C_0^2} = M_0^2\right)$$

$$\left(\frac{p_s}{p_0}\right)^{\frac{\gamma - 1}{\gamma}} = \left[1 + \left(\frac{\gamma - 1}{2}\right)M_0^2\right]$$

or,

or,

or,

 $\frac{p_s}{p_0} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right]^{\frac{\gamma}{\gamma - 1}} \qquad \dots (iv)$

or,

$$p_{s} = p_{0} \left[1 + \left(\frac{\gamma - 1}{2}\right) M_{0}^{2} \right]^{\frac{\gamma}{\gamma - 1}} \qquad \dots (16.17)$$

Eqn. (16.17) gives the value of stagnation pressure.

Compressibility correction factor :

If the right hand side of eqn. (16.17) is expanded by the binomial theorem, we get

$$\begin{split} p_{5} &= p_{0} \left[1 + \frac{\gamma}{2} M_{0}^{2} + \frac{\gamma}{8} M_{0}^{4} + \frac{\gamma(2 - \gamma)}{48} M_{0}^{6} \right] \\ &= p_{0} \left[1 + \frac{\gamma M_{0}^{2}}{2} \left(1 + \frac{M^{2}}{4} + \frac{2 - \gamma}{24} M_{0}^{4} + \dots \right) \right] \\ p_{5} &= p_{0} + \frac{p_{0} \gamma M_{0}^{2}}{2} \left(1 + \frac{M_{0}^{2}}{4} + \frac{2 - \gamma}{24} M_{0}^{4} + \dots \right) \qquad \dots (16.18) \end{split}$$

or,

But,

$$M_{0}^{2} = \frac{V_{0}^{2}}{C_{0}^{2}} = \frac{V_{0}^{2}}{\left(\frac{\gamma p_{0}}{\rho_{0}}\right)} = \frac{V_{0}^{2} \rho_{0}}{\gamma p_{0}} \qquad \left(\because C_{0}^{2} = \frac{\gamma p_{0}}{\rho_{0}} \right)$$

Substituting the value of M_0^2 in eqn. (16.18), we get

$$p_{s} = p_{0} + \frac{p_{0}\gamma}{2} \times \frac{V_{0}^{2}\rho_{0}}{\gamma p_{0}} \left(1 + \frac{M_{0}^{2}}{4} + \frac{2-\gamma}{24} M_{0}^{4} + \dots \right)$$
$$p_{s} = p_{0} + \frac{\rho_{0}V_{0}^{2}}{2} \left(1 + \frac{M_{0}^{2}}{4} + \frac{2-\gamma}{24} M_{0}^{4} + \dots \right) \qquad \dots (16.19)$$

or,

Also,
$$p_s = p_0 + \frac{\rho_0 V_0^2}{2}$$
 (when compressibility effects are neglected) ...(16.20)

The comparison of eqns. (16.19) and (16.20) shows that the effects of compressibility are isolated in the bracketed quantity and that these effects *depend only* upon the *Mach number*. The

bracketed quantity $\left[i.e., \left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + ...\right)\right]$ may thus be considered as a compressibility

correction factor. It is worth noting that :

- For M < 0.2, the compressibility affects the pressure difference (p_s − p₀) by less than 1 per cent and the simple formula for flow at constant density is then sufficiently accurate.
- For larger value of M, as the terms of binomial expansion become significant, the compressibility effect must be taken into account.
- When the Mach number exceeds a value of about 0.3 the Pitot-static tube used for measuring aircraft speed needs calibration to take into account the compressibility effects.

16.6.2. Expression for Stagnation Density (ρ_s) From eqn. (ii), we have

$$\frac{\rho_0}{\rho_s} = \left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}} \text{ or } \frac{\rho_s}{\rho_0} = \left(\frac{p_s}{p_0}\right)^{\frac{1}{\gamma}} \text{ or } \rho_s = \rho_0 \left(\frac{p_s}{p_0}\right)^{\frac{1}{\gamma}}$$

Substituting the value of $\left(\frac{p_z}{p_0}\right)$ from eqn. (iv), we get

$$\begin{split} \rho_{s} &= \rho_{0} \left[\left\{ 1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right\}^{\frac{\gamma}{\gamma - 1}} \right]^{\frac{1}{\gamma}} \\ \rho_{s} &= \rho_{0} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right]^{\frac{1}{\gamma - 1}} \qquad \dots (16.21) \end{split}$$

or,

16.6.3. Expression for Stagnation Temperature (T_)

The equation of state is given by : $\frac{p}{\rho} = RT$

For stagnation point, the equation of state may be written as :

$$\frac{p_s}{\rho_s} = RT_s \text{ or } T_s = \frac{1}{R} \frac{p_s}{\rho_s}$$

Substituting the values of p_{ϵ} and ρ_{ϵ} from eqns. (16.17) and (16.18), we get

$$\begin{split} T_{s} &= \frac{1}{R} \frac{p_{0} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right]^{\frac{\gamma}{\gamma - 1}}}{\rho_{0} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right]^{\frac{\gamma}{\gamma - 1}}} \\ &= \frac{1}{R} \frac{p_{0}}{\rho_{0}} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right]^{\left(\frac{\gamma}{\gamma - 1} - \frac{1}{\gamma - 1} \right)} \\ &= \frac{1}{R} \frac{p_{0}}{\rho_{0}} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right]^{\left(\frac{\gamma}{\gamma - 1} - \frac{1}{\gamma - 1} \right)} \\ T_{s} &= T_{0} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right] \qquad \dots (16.22) \qquad \left(\because \frac{p_{0}}{\rho_{0}} = RT_{0} \right) \end{split}$$

Example 16.8. An aeroplane is flying at 1000 km/h through still air having a pressure of 78.5 kN/m^2 (abs.) and temperature – 8°C. Calculate on the stagnation point on the nose of the plane :

(i) Stagnation pressure, (ii) Stagnation temperature, and

(iii) Stagnation density.

Take for air : R = 287 J/kg K and $\gamma = 1.4$.

Sol. Speed of aeroplane, V = 1000 km/h = $\frac{1000 \times 1000}{60 \times 60} = 277.77$ m/s

For air : R = 287 J/kg K, $\gamma = 1.4$

The sonic velocity for adiabatic flow is given by,

$$C_0 = \sqrt{\gamma R T_0} = \sqrt{14 \times 287 \times 265} = 326.31 \text{ m/s}$$

 $M_0 = \frac{V_0}{C_0} = \frac{277.77}{326.31} = 0.851$

∴ Mach number,

(i) Stagnation pressure, p. :

The stagnation pressure (p_s) is given by the relation,

$$\begin{split} p_{s} &= p_{0} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right]^{\frac{\gamma}{\gamma - 1}} \qquad \dots [\mathbb{E}qn. \ (16.17)] \\ p_{s} &= 78.5 \left[1 + \left(\frac{14 - 1}{2} \right) \times 0.851^{2} \right]^{\frac{14}{14 - 1}} \\ &= 78.5 \ (1.145)^{3.5} = 126.1 \ \text{kN/m}^{2} \quad (\text{Ans.}) \end{split}$$

- (ii) Stagnation temperature, T_z:
- The stagnation temperature is given by,

$$T_{s} = T_{0} \left[1 + \left(\frac{\gamma - 1}{2} \right) M_{0}^{2} \right] \qquad \dots [Eqn. (16.22)]$$
$$= 265 \left[1 + \frac{14 - 1}{2} \times 0.851^{2} \right] = 303.4 \text{ K or } 30.4^{\circ}\text{C} \quad (\text{Ans.})$$

(iii) Stagnation density, ρ_e:

The stagnation density (ρ_s) is given by,

$$\frac{p_s}{\rho_s} = RT_s \text{ or } \rho_s = \frac{p_s}{RT_s}$$

$$\rho_s = \frac{126.1 \times 10^3}{287 \times 303.4} = 1.448 \text{ kg/m}^3 \text{ (Ans.)}$$

or,

Example 16.9. Air has a velocity of 1000 km/h at a pressure of 9.81 kN/m² in vacuum and a temperature of 47°C. Compute its stagnation properties and the local Mach number. Take atmospheric pressure = 98.1 kN/m², R = 287 J/kg K and $\gamma = 1.4$.

What would be the compressibility correction factor for a pitot-static tube to measure the velocity at a Mach number of 0.8.

Sol. Velocity of air,	$V_0 = 1000 \text{ km/h} = \frac{1000 \times 1000}{60 \times 60} = 277.78 \text{ m/s}$
Temperature of air,	$T_0 = 47 + 273 = 320 \text{ K}$
Atmospheric pressure,	$p_{atm} = 98.1 \text{ kN/m}^2$
Pressure of air (static),	$p_0 = 98.1 - 9.81 = 88.29 \text{ kN/m}^2$
	$R = 287 \text{ J/kg K}, \gamma = 1.4$
Sonic velocity,	$C_0=\sqrt{\gamma R T_0}=\sqrt{14\times 287\times 320}=358.6~{\rm m/s}$
\therefore Mach number,	$M_0 = \frac{V_0}{C_0} = \frac{277.78}{358.6} = 0.7746$

Stagnation pressure, p_z :

The stagnation pressure is given by,

$$\begin{split} p_s &= p_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2 \right]^{\frac{\gamma}{\gamma - 1}} & \dots [\text{Eqn. (16.17)}] \\ p_s &= 88.29 \left[1 + \frac{1.4 - 1}{2} \times 0.7746^2 \right]^{\frac{1.4}{1.4 - 1}} \\ &= 88.29 \ (1.12)^{3.5} = 131.27 \ \text{kN/m}^2 \quad (\text{Ans.}) \\ T_s : \end{split}$$

or,

$$T_{\rm s} = T_0 \left[1 + \left(\frac{\gamma - 1}{2} \right) M_0^2 \right] \qquad \dots [Eqn. (16.22)]$$

$$T_s = 320 \left[1 + \frac{14 - 1}{2} \times 0.7746^2 \right] = 358.4 \text{ K or } 85.4^{\circ}\text{C}$$
 (Ans.)

Stagnation density, ρ_z :

$$\rho_s = \frac{p_s}{RT_s} = \frac{131.27 \times 10^3}{287 \times 358.4} = 1.276 \text{ kg/m}^3 \text{ (Ans.)}$$

Compressibility factor at M = 0.8 :

Compressibility factor
$$= 1 + \frac{M_0^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \dots$$
$$= 1 + \frac{0.8^2}{4} + \frac{2 - 14}{24} \times 0.8^4 = 1.1702 \quad (Ans.)$$

Example 16.10. Air at a pressure of 220 kN/m^2 and temperature 27°C is moving at a velocity of 200 m/s. Calculate the stagnation pressure if

(i) Compressibility is neglected ;
 (ii) Compressibility is accounted for.

For air take R = 287 J/kg K, $\gamma = 1.4$.

Sol. Pressure of air, $p_0 = 200 \text{ kN/m}^2$

Temperature of air, $T_0 = 27 + 233 = 300$ K

Velocity of air, $V_0 = 200 \text{ m/s}$

Stagnation pressure, p_s :

(i) Compressibility is neglected :

$$p_s = p_0 + \frac{\rho_0 V_0^2}{2}$$

where $\rho_0 = \frac{p_0}{RT_0} = \frac{220 \times 10^3}{287 \times 300} = 2.555 \text{ kg/m}^3$

$$p_s = 220 + \frac{2.555 \times 200^2}{2} \times 10^{-3} (\text{kN/m}^2) = 271.1 \text{ kN/m}^2$$
. Ans

(ii) Compressibility is accounted for :

The stagnation pressure, when compressibility is accounted for, is given by,

$$p_{s} = p_{0} + \frac{\rho_{0}V_{0}^{2}}{2} \left(1 + \frac{M_{0}^{2}}{4} + \frac{2 - \gamma}{24} M_{0}^{4} + \dots \right) \qquad \dots [Eqn. (16.19)]$$

Mach number,

$$M_0 = \frac{V_0}{C_0} = \frac{200}{\sqrt{\gamma R T_0}} = \frac{200}{\sqrt{14 \times 287 \times 300}} = 0.576$$

Whence,

4

$$\begin{split} p_s &= 220 + \frac{2.555 \times 200^2}{2} \times 10^{-3} \left(1 + \frac{0.576^2}{4} + \frac{2 - 1.4}{24} \times 0.576^4 \right) \\ p_s &= 220 + 51.1 \; (1 + 0.0829 + 0.00275) = 275.47 \; \mathrm{kN/m^2} \quad (\mathrm{Ans.}) \end{split}$$

AREA-VELOCITY RELATIONSHIP AND EFFECT OF VARIATION OF AREA FORSUBSONIC, SONIC AND SUPERSONIC FLOWS

For an incompressible flow the continuity equation may be expressed as : AV = constant, which when differentiated gives

$$AdV + VdA = 0$$
 or $\frac{dA}{A} = -\frac{dV}{V}$...(16.23)

But in case of compressible flow, the continuity equation is given by,

 $\rho AV = \text{constant}$, which can be differentiated to give

$$\rho d(AV) + AVd\rho = 0$$
 or $\rho(AdV + VdA) + AVd\rho = 0$

or,

$$\rho A dV + \rho V dA + A V d\rho = 0$$

Dividing both sides by ρAV , we get

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \qquad ...(16.24)$$

or,

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho} \qquad \dots [16.24 \ (a)]$$

The Euler's equation for compressible fluid is given by,

$$\frac{dp}{\rho} + VdV + gdz = 0$$

Neglecting the z terms the above equation reduces to, $\frac{dp}{\rho} + VdV = 0$

This equation can also be expressed as :

$$\frac{dp}{\rho} \times \frac{d\rho}{d\rho} + VdV = 0 \quad \text{or} \quad \frac{dp}{d\rho} \times \frac{d\rho}{\rho} + VdV = 0$$
$$\frac{dp}{d\rho} = C^2 \qquad \dots [\text{Eqn. (16.10)}]$$

But

$$\therefore \qquad C^2 \times \frac{d\rho}{\rho} + VdV = 0 \quad \text{or} \quad C^2 \frac{d\rho}{\rho} = -VdV \quad \text{or} \quad \frac{d\rho}{\rho} = -\frac{VdV}{C^2}$$

Substituting the value of $\frac{d\rho}{\rho}$ in eqn. (16.24), we get

$$\frac{dV}{V} + \frac{dA}{A} - \frac{VdV}{C^2} = 0$$

$$\frac{dA}{A} = \frac{VdV}{C^2} - \frac{dV}{V} = \frac{dV}{V} \left(\frac{V^2}{C^2} - 1\right)$$

$$\frac{dA}{A} = \frac{dV}{V} (M^2 - 1) \qquad \left(\because M = \frac{V}{C}\right) \qquad \dots (16.25)$$

or,

٨.

This important equation is due to Hugoniot.

Eqns. (16.23) and (16.25) give variation of $\left(\frac{dA}{A}\right)$ for the flow of incompressible and com-

pressible fluids respectively. The ratios $\left(\frac{dA}{A}\right)$ and $\left(\frac{dV}{V}\right)$ are respectively fractional variations in the values of area and flux values in the flux pressure.

the values of area and flow velocity in the flow passage.

Further, in order to study the variation of pressure with the change in flow area, an expression similar to eqn. (16.25), as given below, can be obtained.

$$dp = \rho V^2 \left(\frac{1}{1-M^2}\right) \frac{dA}{A}$$
(16.26)

From eqns. (16.25) and (16.26), it is possible to formulate the following conclusions of practical significance.

(i) For subsonic flow (M < 1):

$$\frac{dV}{V} > 0$$
; $\frac{dA}{A} < 0$; $dp < 0$ (convergent nozzle)
 $\frac{dV}{V} < 0$; $\frac{dA}{A} > 0$; $dp > 0$ (divergent diffuser)



Fig. 16.5. Subsonic flow (M < 1).

(ii) For supersonic flow (M > 1):





(b) Convergent nozzle.

Fig. 16.6. Supersonic flow (M > 1).

(iii) For sonic flow (M = 1):

$$\frac{dA}{A} = 0$$
 (straight flow passage

since dA must be zero)

and dp = (zero/zero) *i.e.*, indeterminate, but when evaluated, the change of pressure p = 0, since dA = 0 and the flow is frictionless.



Fig. 16.7. Sonic flow (M = 1).

16.8. FLOW OF COMPRESSIBLE FLUID THROUGH A CONVERGENT NOZZLE

Fig. 16.8 shows a large tank/vessel fitted with a short convergent nozzle and containing a compressible fluid. Consider two points 1 and 2 inside the tank and exit of the nozzle respectively.

Let $p_1 =$ pressure of fluid at the point 1,

 $V_1 =$ velocity of fluid in the tank (= 0),

 $T_1 =$ temperature of fluid at point 1,

 ρ_1 = density of fluid at point 1, and p_2 , V_2 , T_2 and ρ_2 are corresponding values of pressure, velocity, temperature and density at point 2.

Assuming the flow to take place *adiabatically*, then by using Bernoulli's equation (for adiabatic flow), we have





$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2g} + \frac{V_2^2}{2g} + z_2 \qquad \text{[Eqn. (16.7)]}$$

But $z_1 = z_2$ and $V_1 = 0$

$$\therefore \qquad \frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_{1g}} = \left(\frac{\gamma}{\gamma - 1}\right) \frac{p_2}{\rho_{2g}} + \frac{V_2^2}{2g}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\left[\frac{p_1}{\rho_1g}-\frac{p_2}{\rho_2g}\right]=\frac{V_2^2}{2g}$$
 or $\frac{\gamma}{\gamma-1}\left[\frac{p_1}{\rho_1}-\frac{p_2}{\rho_2}\right]\frac{V_2^2}{2}$
$$\begin{split} V_2 &= \sqrt{\frac{2\gamma}{(\gamma - 1)} \left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right)} \\ V_2 &= \sqrt{\frac{2\gamma}{(\gamma - 1)} \frac{p_1}{\rho_1} \left(1 - \frac{p_2}{\rho_2} - \frac{\rho_1}{p_1}\right)} \qquad \dots (1) \end{split}$$

or

For adiabatic flow :
$$\frac{p_1}{\rho_1^{\gamma}} = \frac{p_2}{\rho_2^{\gamma}} \text{ or } \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \text{ or } \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \dots (i)$$

Substituting the value of $\frac{\rho_1}{\rho_2}$ in eqn. (1), we get

$$\begin{split} V_{2} &= \sqrt{\frac{2\gamma}{(\gamma-1)} \frac{p_{1}}{\rho_{1}} \left[1 - \frac{p_{2}}{p_{1}} \times \left(\frac{p_{1}}{p_{2}}\right)^{\frac{1}{\gamma}} \right]} = \sqrt{\frac{2\gamma}{(\gamma-1)} \frac{p_{1}}{\rho_{1}} \left[1 - \left(\frac{p_{2}}{p_{1}}\right)^{1 - \frac{1}{\gamma}} \right]} \\ V_{2} &= \sqrt{\frac{2\gamma}{(\gamma-1)} \frac{p_{1}}{\rho_{1}} \left[1 - \left(\frac{p_{2}}{p_{1}}\right)^{\frac{\gamma-1}{\gamma}} \right]} \qquad \dots (16.27) \end{split}$$

or

The mass rate of flow of the compressible fluid,

 $m=\rho_2 A_2 V_2, A_2$ being the area of the nozzle at the exit

 $=\rho_2 A_2 \sqrt{\frac{2\gamma}{(\gamma-1)} \frac{p_1}{\rho_1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}, \quad [\text{substituting } V_2 \text{ from eqn. (16.27)}]$

or

$$m = A_2 \sqrt{\frac{2\gamma}{(\gamma - 1)} \frac{p_1}{\rho_1} \times \rho_2^2} \left[1 - \left(\frac{p_2}{\rho_1}\right)^{\frac{\gamma - 1}{\gamma}} \right]$$

From eqn. (i), we have
$$\rho_2 = \frac{\rho_1}{(p_1/p_2)^{1/\gamma}} = \rho_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$
$$\therefore \qquad \qquad \rho_2^2 = \rho_1^2 \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}$$

Substituting this value in the above equation, we get

$$\begin{split} m &= A_2 \sqrt{\frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \times \rho_1^2 \left(\frac{p_2}{p_1}\right)^2 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]} \\ &= A_2 \sqrt{\frac{2\gamma}{\gamma - 1} p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma} + \frac{2}{\gamma}}\right]} \end{split}$$

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma - 1} p_1 \rho_1} \left[\left(\frac{p_2}{p_1} \right)^{2/\gamma} - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma + 1}{\gamma}} \right] \qquad \dots (16.28)$$

The mass rate of flow (m) depends on the value of $\frac{p_2}{p_1}$ (for the given values of p_1 and ρ_1 at point 1).

Value of $\frac{\mathbf{p}_2}{\mathbf{p}_1}$ for maximum value of mass rate of flow :

For maximum value of *m*, we have $\frac{d}{d\left(\frac{p_2}{p_1}\right)}(m) = 0$

As other quantities except the ratio $\frac{p_2}{p_1}$ are constant

$$\frac{d}{d\left(\frac{p_2}{p_1}\right)}(m) = \left[\left(\frac{p_2}{p_1}\right)^{2l\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right] = 0$$

or,

or.

....

$$\frac{2}{\gamma} \left(\frac{p_2}{p_1}\right)^{\gamma} - \left(\frac{\gamma+1}{\gamma}\right) \left(\frac{p_2}{p_1}\right)^{\gamma} = 0$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}-1} = \left(\frac{\gamma+1}{2}\right) \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \text{ or } \left(\frac{p_2}{p_1}\right)^{\frac{2-\gamma}{\gamma}} = \frac{\gamma+1}{2} \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{2-\gamma}{\gamma}} = \left(\frac{\gamma+1}{2}\right) \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \frac{\gamma+1}{2} \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

γ+1 ,

or.

or.

or,

$$\left(\frac{p_2}{p_1}\right)^{\gamma} = \left(\frac{\gamma+1}{2}\right)^{\gamma} \left(\frac{p_2}{p_1}\right)$$
$$\left(\frac{p_2}{p_1}\right)^{2-\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{\gamma} \text{ or } \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{\gamma+1}{2}\right)^{\gamma}$$

2,

or, $\left(\frac{p_2}{p_1}\right)^{\gamma-1} = \left(\frac{2}{\gamma+1}\right)^{\gamma}$

Eqn. (16.29) is the condition for maximum mass flow rate through the nozzle.

It may be pointed out that a convergent nozzle is employed when the exit pressure is equal to or more than the critical pressure, and a convergent-divergent nozzle is used when the discharge pressure is less than the critical pressure.

For air with $\gamma = 1.4$, the critical pressure ratio,

$$\frac{p_2}{p_1} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} = 0.528 \qquad \dots (16.30)$$

Relevant relations for critical density and temperature are :

$$\frac{\rho_2}{\rho_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \dots [16.30 \ (a)]$$
$$\frac{T_2}{T_1} = \frac{2}{\gamma+1} \dots [16.30 \ (b)]$$

Value of V₂ for maximum rate of flow of fluid :

Substituting the value of
$$\frac{p_2}{p_1}$$
 from eqn. (16.29) in eqn. (16.27), we get

$$V_2 = \sqrt{\frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left[1 - \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1} \times \frac{\gamma - 1}{\gamma}} \right]} = \sqrt{\frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left(1 - \frac{2}{\gamma + 1}\right)}$$

$$= \sqrt{\frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left(\frac{\gamma + 1 - 2}{\gamma + 1}\right)} = \sqrt{\frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left(\frac{\gamma - 1}{\gamma + 1}\right)}$$

$$V_2 = \sqrt{\frac{2\gamma}{\gamma + 1} \frac{p_1}{\rho_1}} (= C_2) \qquad \dots (16.31)$$

or

Maximum rate of flow of fluid through nozzle, m_{nax} :

Substituting the value of
$$\frac{p_2}{p_1}$$
 from eqn. (16.30) in eqn. (16.28), we get

$$m_{\max} = A_2 \sqrt{\left(\frac{2\gamma}{\gamma-1}\right) p_1 p_1} \left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1} \times \frac{2}{\gamma}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{\gamma+1}{\gamma}} \right]$$

$$= A_2 \sqrt{\left(\frac{2\gamma}{\gamma-1}\right) p_1 p_1} \left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \right]$$

For air, $\gamma = 1.4$,

$$\therefore \qquad m_{\max} = A_2 \sqrt{\frac{2 \times 1.4}{(1.4 - 1)} p_1 \rho_1} \left[\left(\frac{2}{1.4 + 1} \right)^{\frac{2}{1.4 - 1}} - \left(\frac{2}{1.4 + 1} \right)^{\frac{1.4 + 1}{1.4 - 1}} \right]$$

$$= A_2 \sqrt{7 p_1 \rho_1 (0.4018 - 0.3348)}$$

$$m_{\max} = 0.685 A_2 \sqrt{p_1 \rho_1} \qquad \dots (16.32)$$

or

Variation of mass flow rate of compressible fluid with pressure ratio $\left(\frac{\mathbf{p}_2}{\mathbf{p}_1}\right)$:

A passage in which the sonic velocity has been reached and thus in which the mass flow rate is maximum, is often said to be choked or in choking conditions. It is evident from eqn. (16.28) that for a fixed value of inlet pressure the mass flow depends on nozzle exit pressure.

Fig. 16.9. depicts the variation of actual and

theoretical mass flow rate versus $\frac{p_2}{p_1}$. Following points are *worthnoting* :

(i) The flow rate increases with a decrease in the pressure ratio $\frac{p_2}{p_1}$ and attains the maximum value of the critical pressure ratio $\frac{p_2}{p_1} = 0.528$ for air.

(ii) With further decrease in exit pressure below the critical value, the theoretical mass flow rate decreases. This is contrary to the actual results where the mass flow rate remains constant after attaining the maximum value. This may be explained as follows :



Fig. 16.9. Mass flow rate through a convergent nozzle.

At critical pressure ratio, the velocity V_2 at the throat is equal to the sonic speed (derived below).

For an accelerating flow of a compressible fluid in a convergent nozzle the velocity of flow within the nozzle is subsonic with a maximum velocity equal to the sonic velocity at the throat : Thus once the velocity V_2 at the throat has attained the sonic speed at the critical pressure ratio, it

remains at the same value for all the values of $\left(\frac{p_2}{p_1}\right)$ less than critical pressure ratio, since the flow

in the nozzle is being continuously accelerated with the reduction in the throat pressure below the critical values and hence the velocity cannot reduce. Thus, the mass flow rate for all values of

 $\left(\frac{p_2}{p_1}\right)$ less than critical pressure ratio remains constant at the maximum value (indicated by the

solid horizontal line in Fig. 16.9). This fact has been verified experimentally too.

Velocity at outlet of nozzle for maximum flow rate :

The velocity at outlet of nozzle for maximum flow rate is given by,

$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right)\frac{p_1}{\rho_1}}$	[Eqn. (16.31)]
$\frac{p_2}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$	

Now pressure ratio,

$$p_1 = \frac{p_2}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} = p_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

For adiabatic flow :

.....

$$\frac{p_1}{\rho_1^{\gamma}} = \frac{p_2}{\rho_2^{\gamma}} \quad \text{or} \quad \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \text{ or } \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}}$$

$$\rho_1 = \rho_2 \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}} \text{ or } \rho_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{1}{\gamma}} \text{ or } \rho_2 \left(\frac{2}{\gamma+1}\right)^{\frac{1}{1-\gamma}}$$

Substituting the values of p_1 and ρ_1 in the above eqn. (16.31), we get

4

$$\begin{split} V_2 &= \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times p_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{1-\gamma}} \times \left\{\frac{1}{\rho_2} \times \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\right\}} \\ &= \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{1-\gamma}+\frac{1}{\gamma-1}}} = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \left(\frac{2}{\gamma+1}\right)^{-1}} \\ \text{or} & V_2 &= \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \left(\frac{\gamma+1}{2}\right)} = \sqrt{\frac{\gamma p_2}{\rho_2}} = C_2 \\ i.e., & V_2 &= C_2 \qquad \dots (16.33) \end{split}$$

Hence the velocity at the outlet of nozzle for maximum flow rate equals sonic velocity.

FLOW THROUGH LAVAL NOZZLE (CONVERGENT-DIVERGENT NOZZLE)

Laval nozzle is a convergent-divergent nozzle (named after de Laval, the swedish scientist who invented it) in which subsonic flow prevails in the converging section, critical or transonic conditions in the throat and supersonic flow in the diverging section. Let p2 (= pc) = pressure in the throat when the flow is sonic for given pressure p1. When the pressure in the receiver, p3 = p1, there will be no flow through the nozzle, this is shown by line *a* in Fig.



Fig. 16.10. (*a*) Laval nozzle (convergent-divergent nozzle) ; (*b*) Pressure distribution through a convergent-divergent nozzle with flow of compressible fluid.

- When the receiver pressure is reduced, flow will occur through the nozzle. As long as the value of p3 is such that throat pressure p2 is greater than the critical pressure 0.528 p1, the flow in the converging and diverging sections will be subsonic. This condition is shown by line 'b'.
- With further reduction in p3, a stage is reached when p2 is equal to critical pressure pc = 0.528 p1, at this line M = 1 in the throat. This condition is shown by line 'c'. Flow is subsonic on the upstream as well the downstream of the throat. The flow is also *isentropic*.
- If p3 is further reduced, it does not effect the flow in convergent section. The flow in throat is sonic, downstream it is supersonic. Somewhere in the diverging section a shock wave occurs and flow changes to subsonic (curve d). The flow across the shock is non-isentropic. Downstream of the shock wave the flow is subsonic and decelerates.
- If the value of p3 is further reduced, the shock wave forms somewhat downstream (curve e).
- For p3 equal to pj, the shock wave will occur just at the exit of divergent section.
- > If the value of p3 lies before pf and pj oblique waves are formed at the exit.

SHOCK WAVES

Whenever a supersonic flow (compressible) abruptly changes to subsonic flow, a shock wave (analogous to hydraulic jump in an open channel) is produced, resulting in a sudden rise in pressure, density, temperature and entropy. This occurs due to pressure differentials and when the Mach number of the approaching flow M1 > 1. A shock wave is a pressure wave of finite thickness, of the order of 10–2 to 10–4 mm in the atmospheric pressure. A shock wave takes place in the diverging section of a nozzle, in a diffuser, throat of a supersonic wind tunnel, in front of sharp nosed bodies.

Shock waves are of two types :

- 1. Normal shocks which are almost perpendicular to the flow.
- 2. Oblique shocks which are inclined to the flow direction.

16.11.1. Normal Shock Wave

Consider a duct having a compressible sonic flow (see Fig. 16.11).

Let p1, p1, T1, and V1 be the pressure, density, temperature and velocity of the flow (M1 > 1) and p2, p2, T2 and V2 the corresponding values of pressure, density, temperature and velocity after a shock wave takes place (M2 < 1).



Fig. 16.11. Normal shock wave.

For analysing a normal shock wave, use will be made of the continuity, momentum and energy equations.

Assume unit area cross-section, $A_1 = A_2 = 1$. Continuity equation : $m = \rho_1 V_1 = \rho_2 V_2$...(i) Momentum equation : $\Sigma F_x = p_1 A_1 - p_2 A_2 = m (V_2 - V_1) = \rho_2 A_2 V_2^2 - \rho_1 A_1 V_1^2$ for $A_1 = A_2 = 1$, the pressure drop across the shock wave,

$$p_1 - p_2 = \rho_2 V_2^2 - \rho_1 V_1^2 \qquad ...(ii)$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

Consider the flow across the shock wave as adiabatic.

Energy equation :
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1} + \frac{V_1^2}{2} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2} + \frac{V_2^2}{2}$$
 ...[Eqn. (16.7)]

 $(z_1 = z_2, \text{ duct being in horizontal position})$

$$\frac{\gamma}{\gamma - 1} \left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) = \frac{V_1^2 - V_2^2}{2} \qquad \dots (iii)$$

or,

Combining continuity and momentum equations [refer eqns. (i) and (ii)], we get

$$p_1 + \frac{(\rho_1 V_1)^2}{\rho_1} = p_2 + \frac{(\rho_2 V_2)^2}{\rho_2}$$
 ...(16.42)

This equation is known as Rankine Line Equation.

Now combining continuity and energy equations [refer eqns. (i) and (iii)], we get

$$\frac{\gamma}{\gamma - 1} \left(\frac{p_1}{\rho_1}\right) + \frac{(\rho_1 V_1)^2}{2\rho_1^2} = \frac{\gamma}{\gamma - 1} \left(\frac{p_2}{\rho_2}\right) + \frac{(\rho_2 V_2)^2}{2\rho_2^2} \qquad \dots (16.43)$$

This equation is called Fanno Line Equation.

Further combining eqns. (i), (ii) and (iii) and solving for $\frac{p_2}{p_1}$, we get

$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_2}{\rho_1}} \dots (16.44)$$

Solving for density ratio $\frac{\rho_2}{\rho_1}$, the same equations yield

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{1 + \left(\frac{\gamma + 1}{\gamma - 1}\right) \frac{p_2}{p_1}}{\left(\frac{\gamma + 1}{\gamma - 1}\right) + \frac{p_2}{p_1}} \dots (16.45)$$

The eqns. (16.44) and (16.45) are called Ranking-Hugoniot equations.

One can also express $\frac{p_2}{p_1}$, $\frac{V_2}{V_1}$, $\frac{\rho_2}{\rho_1}$ and $\frac{T_2}{T_1}$ in terms of Mach number as follows :

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)} \qquad \dots (16.46)$$

$$\frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2} \qquad \dots (16.47)$$

$$\frac{T_2}{T_1} = \frac{\left[(\gamma - 1)M_1^2 + 2\right]\left[2\gamma M_1^2 - (\gamma - 1)\right]}{(\gamma + 1)^2 M_1^2} \qquad \dots (16.48)$$

By algebraic manipulation the following equation between M_1 and M_2 can be obtained.

$$M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)} \qquad \dots (16.49)$$

16.11.2. Oblique Shock Wave

As shown in Fig. 16.12, when a supersonic flow undergoes a sudden turn through a small angle α (positive), an oblique wave is established at the corner. In comparison with normal shock waves, the oblique shock waves, being weaker, are preferred.

The shock waves should be avoided or made as weak as possible, since during a shock wave conversion of mechanical energy into heat energy takes place.

Supersonic flow

16.11.3. Shock Strength



The strength of shock is defined as the ratio of pressure rise across the shock to the upstream pressure.

$$\begin{aligned} i.e. \quad \text{Strength of shock} &= \frac{p_2 - p_1}{p_1} = \frac{p_2}{p_1} - 1 \\ &= \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} - 1 = \frac{2\gamma M_1^2 - (\gamma - 1) - (\gamma + 1)}{\gamma + 1} \\ &= \frac{2\gamma M_1^2 - \gamma + 1 - \gamma - 1}{\gamma + 1} = \frac{2\gamma M_1^2 - 2\gamma}{\gamma + 1} = \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \\ \text{Hence, strength of shock} &= \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \\ \end{aligned}$$

MODUE 5

RECIPROCATING COMPRESSOR

What is Compressor ?

Compressor is a device which is used to increase the pressure of air from low pressure to high pressure by using some external energy

- For filling the air in tube of vehicles
- In automobile service station to clean vehicles.
- For spray painting in paint industries.
- In vehicle to operate air brakes.
- For cleaning workshop machines.
- For supercharging of an IC engines.
- For operation of pneumatic tools i.e. rock drills, vibrators etc.





Centrifugal compressor is widely used in chemical and petroleum refinery industry for specifies services.

Starter motor (not fitted) mounting

Fuel lines to injectors inside combustion chambers

16 Combustion chambers (inside flame cans) arranged around engine

centrifugal compressor and other parts

Definitions of Compressor

Compression ratio:- It is defined as the ratio of volume of air before compression to the volume of air after compression.

Compressor capacity:- It is the quantity of air actually delivered by a compressor in m³/minute or m³/sec.

Free air Delivered(FAD):- It is the volume of air delivered by compressor under the compressor intake conditions (i.e. temperature and pressure).

Swept Volume:- The volume displaced or swept by piston when it moves between top dead center and bottom dead center.

Clearance volume:- it is the difference between the total **volume** and the swept **volume**, basically the gap that remains between the piston head and the cylinder head when at top dead center.

Efficiencies:

Volumetric efficiency:-

It is the ratio of actual volume of the FAD at standard atmospheric condition in one delivery stroke (Actual air intake) to the swept volume (theoretical air intake) by the piston during the stroke.

Isothermal efficiency:-

It is defined as the ratio of isothermal power (P_{iso}) (i.e. required input power at isothermal process) done to the indicated power (IP) or actual work done.

Mechanical efficiency:-

It is the ratio of indicated power (IP) to the shaft(Brake) Power (P_{shaft}).

Overall efficiency:-

It is the ratio of isothermal power (P_{iso}) to the shaft(Brake) Power (P_{shaft}).

The Analysis Objectives

- To calculate air pressure and temperature before and after compression process
- To calculate indicated work/power
- To obtain compressor efficiency
- To calculate free air delivery rate (kg/s or m³/s)

The primary components of a typical reciprocating compressor system can be seen in Figures. The compression cylinders, also known as stages, of which a particular design may have from one to six or more, provide confinement for the process gas during compression. A piston is driven in a reciprocating action to compress the gas. Arrangements may be of single-

or dual-acting design. (In the dual-acting design, compression occurs on both sides of the piston during both the advancing and retreating stroke.) Some dual-acting cylinders in highpressure applications will have a piston rod on both sides of the piston to provide equal surface area and balance loads.



Tandem cylinder arrangements help minimize dynamic loads by locating cylinders in pairs, connected to a common crankshaft, so that the movements of the pistons oppose each other. Gas pressure is sealed and wear of expensive components is minimized through the use of disposable piston rings and rider bands respectively. These are formed from comparatively soft metals relative to piston and cylinder/liner metallurgy or materials such as polytetrafluoroethylene (PTFE).



In a reciprocating compressor, a volume of air is drawn into a cylinder, it is trapped, and compressed by piston and then discharged into the discharge line. The cylinder valves control the flow of air through the cylinder; these valves act as check valves.

Single – Acting compressor

It is a compressor that has one discharge per revolution of crankshaft.

Double – Acting Compressor

It is a compressor that completes two discharge strokes per revolutions of crankshaft. Most heavyduty compressors are double acting.

Multi-staging :Reduction in power required to drive the compressor.

- Better mechanical balance of the whole unit and uniform torque.
- Increase in volumetric efficiency.
- Reduced leakage loss.
- Less difficulty in lubrication due to low working temperature.
- Lighter cylinders can be used.
- Cheaper materials can be used for construction as the operating temperature is lower.





Water outlet Drain Cock Motor (without grouting) to produce opposite torque for mechanical dynamometer. Unloader valve: to keep the starting compressor operation in ambient pressure.

Assumptions

The working fluid is assumed as a perfect gas and *P*-*v*-*T* can be calculated by using simple equation of state. Usually, these assumptions are used to calculate estimate pressure, *P*, volume, *V*, and temperature, *T*, of the working fluid.

PV = mRT $PV/T = K_1$ $PV^n = K_2$

- Compressor without clearance volume
- Compressor with clearance volume
- Multistage compressors

Compressor without clearance volume

The Cycle of Operation

• The cycle of operation of a reciprocating air-compressor is best shown on a pressure-volume (*p-V*) diagram.

- It is known as an *indicator diagram for* the compressor.
- The cycle comprises of three processes:
- d a: An induction stroke
- a b: A compression stroke
- b c: A delivery stroke



Description of the Processes

d – a: The induction stroke

Intake valve opens, while exhaust valve closed. Atmospheric air is drawn into the cylinder at constant pressure *p1 and* temperature *T1. Ideally, there is no heat loss* to the surrounding from the air.

a – b: The compression stroke

Both intake and exhaust valves closed. The air is compressed according to a polytropic law $pV^n =$ constant. Its pressure is increased from p1 to p2. The temperature is also increased from T1 to T2.

b – c: The delivery stroke

The intake valve closed while the exhaust valve opens. The compressed air is pushed out of the cylinder at constant pressure *p2* and temperature *T2*. There is no heat loss from the air to the surroundings.



During compression, due to its excess temperature above the compressor surrounding, the air will lose some heat. Thus neglecting the internal effect of friction the index is less than γ (i.e. <1.4), the adiabatic index. If n= γ =1.4, area under curve is biggest, i.e. reversible adiabatic or entropy constant. Since work must be put into an air compressor to run it, every effort is made to reduce this amount of work input. It is observed that if compression is along isothermal, work done is less (though in practical it is not possible). Isothermal is attempted by cooling the compressor either by adding cooling fan of water jacket.

Compressor without clearance volume

Analysis of Cycle

Indicated work per cycle

The area under the *p*-*V* diagram represents the net or indicated work done on the air per cycle. Indicated work / cycle = area a-b-c-d = area 1-2-3-4-1 = area under 1-2+area under 2-3 – area under 4-1

$$= \left(\frac{p_2 V_2 - p_1 V_1}{n - 1}\right) + p_2 V_2 - p_1 V_1$$
$$W = \left(\frac{n}{n - 1}\right) \left(p_2 V_2 - p_1 V_1\right)$$

This work must be done on compressor

Assuming the air as a perfect gas,

 $p_1 V_1 = mRT_1 \qquad p_2 V_2 = mRT_2$

where *m* is the mass of air induced and delivered per cycle, *R* is the universal gas constant, where R = 0.287 kJ/kgK.

Substituting,

Indicated work / cycle $W = \left(\frac{n}{n-1}\right)mR(T_2 - T_1)$



Compressor without clearance volume

Analysis of Cycle

Other form of the equation _for indicated work/ cycle is

$$W = \left(\frac{n}{n-1}\right) m R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$
$$W = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

Indicated Power (IP):

The *indicated power (IP) is the* **work done on the air per unit time**. The mass flow per unit time **m** is often used to compute the work done/time or indicated power.

$$IP = \left(\frac{n}{n-1}\right)\dot{m}R\left(T_2 - T_1\right)$$

Mechanical efficiency:-

It is the ratio of indicated power to the shaft Power. Shaft power = indicated power + friction power

Shaft power is the power supplied by the electric motor to compressor.

Motor efficiency:-

It is the ratio of shaft power to the input Power. Input power is the electric power supplied to the electric motor



Compressor without clearance volume

Isothermal efficiency:-

It is the ratio of isothermal power (P_{iso}) to indicated power (IP)

Isothermal Power:

 $P_{iso} = \dot{m}RT_1 \ln(p_2 / p_1)$

$$n=1$$
 $PV = C$ process $W = \int_{1}^{2} \frac{C}{V} dV = C \ln t$



$$W = \int_{I}^{2} \frac{C}{V} dV = C \ln \frac{V_2}{V_1}$$

$$W = P_1 V_1 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{P_1}{P_2}$$

For ideal gas: $PV = mRT = C \implies T = const$

$$W = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

Compressor with clearance volume

Analysis of Cycle

It is a spacing between the top of the piston and the valve's heads when the piston is at the end of the delivery stroke. Good quality machines has a clearance volume of about 6%. But compressors with clearance of 30 – 35% are also common.

Processes

d – a: Induction process

The inlet valve opens. Fresh atmospheric air is induced into the cylinder at constant pressure p1and temperature T1. The volume of air induced is (Va – Vd). Ideally, there is no heat transfer from the air to the surroundings.

a – b: Compression process

Both valves closed. The induced air is compressed according to the polytropic law of pV^n = const., until the pressure and temperature increases to p2 and T2, respectively. Ideally, there is no heat transfer from the air to the surroundings of cylinder.

Clearance volume:

- Give a mechanical freedom to the moving parts
- Reduce noise and vibration during operation
- Prevent damage to moving components





Compressor with clearance volume

Analysis of Cycle

It is a spacing between the top of the piston and the valve's heads when the piston is at the end of the delivery stroke. Good quality machines has a clearance volume of about 6%. But compressors with clearance of 30 -35% are also common. Because of presence of clearance

to 85 %.

Processes

b – c: *Delivery process*

The exhaust valve opens. The compressed air is delivered out of the cylinder at constant pressure *p2 and temperature T2. Ideally, there is* no heat transfer from the air to the surroundings.

c – d: Expansion process

The piston begins the induction stroke. The compressed air occupying the clearance volume Vc expands according to the polytropic law of $pV^n = const.$, until the pressure and temperature fall to *p1* and *T1*, respectively. Ideally, there is no heat transfer from the air to the surroundings.

Note: At the end of the delivery stroke, the clearance volume Vc is filled with compressed air at pressure p2 and temperature T2.

Volumetric efficiency= $\frac{v_1 - v_4}{V_1 - V_2}$

It is the ratio between FAD at standard atmospheric condition in one delivery stroke (Actual air intake) to the swept volume (theoretical air intake)



Compressor with clearance volume

Assuming polytropic index to be same for both compression and clearance expansion **Indicated work / cycle =**

$$W = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] - \left(\frac{n}{n-1}\right) p_4 V_4 \left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} \right]$$

But $p_4=p_1$, $p_3=p_2$ therefore

$$W = \left(\frac{n}{n-1}\right) p_1 \left(V_1 - V_4\right) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \left(\frac{n}{n-1}\right) \left(\dot{m}_a - \dot{m}_d\right) R \left[T_2 - T_1\right]$$

Volumetric efficiency =

$$= \frac{V_1 - V_4}{V_1 - V_3} = \frac{V_1 - V_4}{V_s}$$
$$= 1 + k - k(V_1 / V_2)$$
$$= 1 + k - k(p_2 / p_1)^{1/n}$$
$$= 1 + k - k(p_3 / p_4)^{1/n}$$

Where, k=clearance ratio = $V_3/(V_1-V_3)=V_2/V_3$ Ratio has a value 4% - 10 %



The greater is the clearance ratio through a reciprocating compressor, the greater will be the effect of the clearance volume since the clearance air will expand through greater volume before intake condition is reached.

p!

P4

 p_3

P2

p1

Vc

d d' d"

Actual p-V (indicator diagram for single stage compressor

At point 4, the clearance air actually reduced to atmospheric pressure. The inlet valve in practice will not open. Reason : (i) inlet valve inertia (ii) there must be a pressure difference across the inlet valve in order to move it. Thus pressure drop away until the valve is forced off its seat. Some *valve bounce* will then set in (wavy line) Therefore intake will become near enough steady at some pressure *below atmospheric pressure*. The negative pressure difference, i.e. *intake depression* settles naturally.

Similar situation occurs at point 2. There is a constant pressure rise, followed by *valve bounce* and the pressure then settles at some pressure *above external delivery pressure* (i.e. Receiver tank pressure).

Other small effects at inlet and delivery would be *gas inertia* and *turbulence*. So, practical effects are responsible for the addition of the two small ripple h, negative work areas shown in figure.



There are certain disadvantages to increase the delivery pressure to a high value. When the delivery pressure is increased to p3, the volume of the *fresh air induced* is reduced from (Va - Vd) to (Va - Vd'), and so on, whereas swept volume Vs is remains constant. Since the volumetric efficiency is given by

 $\eta_{vol} = (Va - Vd)/Vs$

a

the volumetric efficiency decreases with increasing delivery pressure.

This situation can be improved by performing multistage compression process.

Multistage compression

Observation

After the first stage compression, the air is passed into a smaller cylinder, in which it is further compressed to desired final pressure. The cycle assumes that the delivery process of the first stage and the induction process of the second stage take place at the same pressure *pi*. *Advantage*

• Each cylinder works with lower pressure ratio. Thus the operational safety of the compressor is improved.

• The overall volumetric efficiency, η_v increases.

• *Mass flow rate* is increased , as clearance air expansion is reduced and effective swept volume of this cylinder is increased.



(i) Single stage Compressor, for delivery pressure upto 5 bar.
(ii) Two stage Compressor, for delivery pressure between 5 to 35 bar
(iii) Three stage Compressor, for delivery pressure between 35 to 85 bar.
(iv) Four stage compressor, for delivery pressure more than 85 bar

Multistage compression

Indicated power for stage 1.

$$W = \left(\frac{n}{n-1}\right) m R T_a \left[\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

Indicated power for stage 2.

$$W = \left(\frac{n}{n-1}\right) m R T_{a^{n}} \left[\left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}} - 1 \right]$$

- With multistage compression, the air can be cooled as it is being transferred from one cylinder to the next, by passing it through an *intercooler*.
- The process of cooling the air is called the intercooling process.
- With intercooling process, temperature is reduced, therefore internal energy of delivered air reduced. Since energy must have come from the input energy required to drive the machine, this results in a decrease in input work requirement for a given mass of delivered air. Thus the power supplied to the compressor can be reduced.

Complexity of machine limits the number of stages.





p_

With perfect intercooling: W= $\left(\frac{n}{n-1}\right)p_{1}V_{1}\left[\left(\frac{p_{4}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]+\left(\frac{n}{n-1}\right)p_{2}V_{2}\left[\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-1\right]$ Deliver temperature, $T_{3} = T_{2}\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}} = T_{1}\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}$, where $T_{2} = T_{1}$ if $T_{2} = T_{1}$, $p_{2}V_{2} = p_{1}V_{1}$, $p_{4} = p_{2}$ $W = \left(\frac{n}{n-1}\right)p_{1}V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}+\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-2\right]$

2453 = Work savings occurs

Perfect intercooling:

Reciprocating Compressor Multistage compression (Without Vc) Condition for minimum work done:

It is observed that an intermediate pressure p2 -->p1, then area 2453 --> 0. Also p2-->p3 , then area 2453 --> 0. this means, therefore there exists an intermediate pressure p2 which makes area 2453 maximum and W minimum.



Heat Transferred in intercooler = $mc_p(T_4 - T_2) = mc_p(T_4 - T_1)$

intercooling

MODULE 6 STEAM TURBINES

Steam Turbine

- Steam turbine convert a part of the energy of the steam evidenced by high temperature and pressure into mechanical power-in turn electrical power
- The steam from the boiler is expanded in a nozzle, resulting in the emission of a high velocity jet. This jet of steam impinges on the moving vanes or blades, mounted on a shaft. Here it undergoes a change of direction of motion which gives rise to a change in momentum and therefore a force.
- The motive power in a steam turbine is obtained by the rate of change in momentum of a high velocity jet of steam impinging on a curved blade which is free to rotate.
- The conversion of energy in the blades takes place by impulse, reaction or impulse reaction principle.
- Steam turbines are available in a few kW(as prime mover) to 1500 MW Impulse turbine are used for capacity up to

Steam, Gas and Hydraulic Turbines

The working substance differs for different types of turbines.

- Steam turbines are axial flow machines (radial steam turbines are rarely used) whereas gas turbines and hydraulic turbines of both axial and radial flow type are used based on applications.
- The pressure of working medium used in steam turbines is very high, whereas the temperature of working medium used is gas turbine is high comparatively.
- The pressure and temperature of working medium in hydraulic turbines is lower than steam turbines.
- Steam turbines of 1300 MW single units are available whereas largest gas turbines unit is 530 MWand 815 MW

Merits and Demerits of Steam Turbine

Merits:

- · Ability to utilize high pressure and high temperature steam.
- High component efficiency.
- High rotational speed.
- High capacity/weight ratio.
- Smooth, nearly vibration-free operation.
- No internal lubrication.
- Oil free exhaust steam.
- Can be built in small or very large units (up to 1200 MW).

Demerits:

- For slow speed application reduction gears are required.
- The steam turbine cannot be made reversible.
- · The efficiency of small simple steam turbines is poor

Steam Turbine Classification

Steam turbines can be classified in several different ways:

- 1. By details of stage design
 - Impulse or reaction.
- 2. By steam supply and exhaust conditions
 - · Condensing, or Non-condensing (back pressure),
 - Automatic or controlled extraction,
 - Mixed pressure
 - Reheat
- 3. By casing or shaft arrangement
 - Single casing, Tandem compound or Cross compound
- 4. By number of exhaust stages in parallel:
 - Two flow, Four flow or Six flow.
- 5. By direction of steam flow:
- · Axial flow, Radial flow or Tangential flow
- 6. Single or multi-stage
- 7. By steam supply

Types of Steam Turbine

Impulse turbine

Reaction turbine



Impulse turbine

 Process of complete expansion of steam takes place in stationary nozzle and the velocity energy is converted into mechanical work on the turbine work turbine blades.



De Laval Impulse Turbine.



Reaction turbine

 Pressure drop with expansion and generation of kinetic energy takes place in the moving blades



Impulse Reaction Turbine

- Modern turbines are neither purely impulse or reaction but a combination of both.
- Pressure drop is effected partly in nozzles and partly in moving blades which are so designed that expansion of steam takes place in them.
- High velocity jet from nozzles produce an impulse on the moving blade and jet coming out from still higher velocity from moving blades produces a reaction.
- Impulse turbine began employing reaction of upto 20% at the root of the moving blades in order to counteract the poor efficiency incurred from zero or even negative reaction.
- Reaction at the root of reaction turbines has come down to as little as 30% to 40% resulting in the reduction of the number of stages required and the sustaining of 50% reaction at mid point.
- It may be more accurate to describe the two design as Disc and diaphragm turbine using low reaction blading Drum rotor turbine using high reaction blading

Flow Through Steam Turbine Stage

Distance through turbine(impuls turbine)

Distance through turbine(reaction turbine)




Compounding of Steam Turbines

- This is done to reduce the rotational speed of the impulse turbine to Compounding of Steam Turbines practical limits.
- Compounding is achieved by using more than one set of nozzles, blades rotors in a series keyed to a common shaft; so that either the steam pressure or the jet velocity is absorbed by the turbine in stages.

Three main types of compounded impulse turbines are:

- a. Pressure compounded
- b. Velocity compounded
- c. Pressure and velocity compounded impulse turbines.

Pressure compoundind

same shaft.

Involves splitting up of the whole pressure drop into a series of smaller pressure drops across several stages of impulse turbine. The nozzles are fitted into a diaphragm locked in separates one wheel chamber from the casing that another. All rotors are mounted on the

Pressure Compounding

continue

Pressure velocity compounding gives the advantage of

producing a shortened rotor compared to pure velocity compounding.

In this design steam velocity at exit to the nozzles is kept

reasonable and thus the blade speed (hence rotor rpm)

reduced.

Velocity drop is achieved through many moving rows of blades instead of a single row of moving blades. It consists of a nozzle or a set of nozzles and rows of moving blades attached to the rotor or the wheel and rows of fixed blades attached to the casing



Comparison between Impulse & Reaction Turbine

Impulse turbine

- An impulse turbine has fixed nozzles that orient the steam flow into high speed jets.
- Blade profile is symmetrical as no pressure drop takes place in the rotor blades
- Suitable for efficiently absorbing the high velocity and high pressure
- Steam pressure is constant across the blades and therefore fine tip clearances are not necessary
- Efficiency is not maintained in the lower pressure stages (high velocity cannot be achieved in steam for the lower pressure stages)

Reaction turbine

- Reaction turbine makes use of the reaction force produced as the steam accelerates through the nozzles formed by the rotor
- Blades have aerofoil profile (convergent p drop occurs partly in the rotor Blades passage) since pressure
- Efficient at the lower pressure stages
- Fine blade tip clearances are necessary due to the pressure leakages
- Inefficient at the high pressure stages due to the pressure leakages around the blade tips
- Fine tip clearances can cause damage to the tips of the blades

IMPULSE TURBINE



Blade efficiency or Diagram efficiency or

Utilization factor is given by

 $\eta_B = \frac{\dot{m} \cdot U \cdot \Delta V_W}{m(V_1^2/2)} = -\frac{Warkdone}{KE - supplied}$

$$\eta_{\mathcal{B}} = \frac{2U\Delta V_{W}}{V_{1}^{2}}$$

stage efficiency =
$$\eta_s = \frac{Work \text{ done by the rotor}}{Isentropic enthalpy drop}$$

 $\eta_s = \frac{\dot{m}U\Delta V_w}{\dot{m}(\Delta H)_{isen}} = \frac{\dot{m}U\Delta V_w}{\dot{m}\left(\frac{V_1^2}{2}\right)} \cdot \frac{\dot{m}(V_1^2/2)}{\dot{m}(\Delta H)_{isen}}$
or.
or.
 $\eta_s = \eta_b \times \eta_n$ [$\eta_n = Nozzle efficiency$]

The maximum value of blade efficiency $(\eta_b)_{max} = 2(\rho \cos \alpha_1 - \rho^2)(1 + \hbar c)$

 $=\frac{\cos^2 a_1}{(1+kr)}$

For equiangular blades

$$(\eta_{k})_{max} = \frac{\cos^{2}\alpha_{1}}{2}(1+k)$$

If the friction over blade surface is neglected

 $(\eta_b)_{max} = \cos^2 \alpha_1$

Compounding in Impulse Turbine

- If high velocity of steam is allowed to flow through one row of moving blades, it produces a rotor speed of about 30000 rpm which is too high for practical use.
- It is therefore essential to incorporate some improvements for practical use and also to achieve high performance. This is possible by making use of more than one set of nozzles, and rotors, in a series, keyed to the shaft so that either the steam pressure or the jet velocity is absorbed by the turbine in stages. This is called compounding. Two types of compounding can be accomplished: (a) velocity compounding and (b) pressure compounding



Velocity And Pressure Compounding



Stage Efficiency and Reheat factor

The Thermodynamic effect on the turbine efficiency can be understood by considering a number of stages between two stages as shown in Figure



Velocity Triangles

- The three velocity vectors namely, blade speed, absolute velocity and relative velocity in relation to the rotor are used to form a triangle called velocity triangle.
- Velocity triangles are used to illustrate the flow in the bladings of turbo machinery.
- Changes in the flow direction and velocity are easy to understand with the help of the velocity triangles.
- Note that the velocity triangles are drawn for the inlet and outlet of the rotor at certain radii.

Velocity triangle

Inlet Velocity Triangles

Outlet Velocity Triangles



Combined velocity triangle



Work Done – Impulse Steam Turbine

If the blade is symmetrical then $\beta 1 = \beta 2$ and neglecting frictional effects of the blades on the steam, W1 = W2. In actual case, the relative velocity is reduced by friction and expressed by a blade velocity coefficient k. Thus k = W2/W1From Euler's equation, work done by the steam is given by; $Wt = U(Vw1 \pm Vw2)$ (1)Since Vw2 is in the negative r direction, the work done per unit mass flow is given by, Wt = U(Vw1+Vw2)(2)If Va1 \neq Va2, there will an axial thrust in the flow direction. Assume that Va is constant then, Wt = UVa $(\tan \alpha 1 + \tan \alpha 2)$ (3) $W UV (t \beta + t \beta)$ (4)

Wt = UVa $tan\beta 1 + tan\beta 2$) Equation (4) is often referred to as the diagram work per unit mass flow and hence the diagram efficiency is defined as

 $\eta_{d} = \frac{\text{Rate of work performed per unit mass flow}}{\text{Energy supplied per unit mass of steam}}$ $\eta_{d} = (U\Delta V_{w}) x \frac{2}{V_{1}^{2}} = \frac{2U\Delta V_{w}}{V_{1}^{2}}$

maximum diagram efficiency

$$= \frac{4\cos\alpha_1}{2} \left(\cos\alpha_1 - \frac{\cos\alpha_1}{2}\right)$$

or $\eta_d = 4\cos^2\alpha_1$

Work Done – Impulse Steam Turbine

$$\begin{split} & \Pi_{d} = \frac{\text{Rate of work performed per unit mass flow}}{\text{Energy supplied per unit mass of steam}} \\ & \Pi_{d} = (U\Delta V_{w})x \frac{2}{V_{1}^{2}} = \frac{2U\Delta V_{w}}{V_{1}^{2}} \end{aligned} \tag{10}$$
Using the blade velocity coefficient (k=W₂/W₁) and symmetrical blades ($\beta_{1} = \beta_{2}$), then; $\Delta V_{w} = 2V_{1} \cos \alpha_{1} - U$ Hence $\Delta V_{w} = 2(V_{1} \cos \alpha_{1} - U)U$ (11)
And the rate of work performed per unit mass $= 2(V_{1} \cos \alpha_{1} - U)U$ Therefore; $\eta_{d} = 2(V_{1} \cos \alpha_{1} - U)U \times \frac{2}{V_{1}^{2}}$ $\eta_{d} = \frac{4(V_{1} \cos \alpha_{1} - U)U}{V_{1}^{2}} = \frac{4U}{V_{1}} \left(\cos \alpha_{1} \frac{U}{V_{1}}\right)$ where $\frac{U}{V_{1}}$ is called the blade speed ratio (12)

Degree of reaction

- Degree of reaction is a parameter that describes the relation between the energy transfer due to the static pressure change and the energy transfer due to dynamic pressure change.
- Degree of reaction is defined as the ratio of static pressure drop in the rotor to the static pressure drop in the stage. It is also defined as the ratio of static enthalpy drop in the rotor to the static enthalpy drop in the stage

Degree of reaction

Zero reaction stage

Let us first discuss the special case of zero reaction. According to the definition

of reaction, When $\Lambda = 0$, equation (upper) reveals that h1 = h2 and equation (lower) that

$$\beta 1 = \beta 2$$

Fifty percent reaction stage

From equation (16) for $\Lambda = 0.5 \alpha I$ = $\beta 2$ and the velocity diagram is symmetrical Because of symmetrical. symmetry, it is also clear that $\alpha 2 = \beta I$. For $\Lambda = 1/2$, the enthalpy drop in the nozzle

row equals the enthalpy drop in the rotor.

h0 - h1 = h1 - h2

$$\Lambda = \frac{(h_1 - h_2)}{\left(h_{00} - \frac{V_0^2}{2C_p}\right) - \left(h_{02} - \frac{V_2^2}{2C_p}\right)}$$

$$\Lambda = \frac{1}{2} + \frac{V_a}{2U} (\tan \beta_2 + \tan \alpha_2)$$

Putting $\Lambda = 0$ in equation we get

$$(\beta_2 = \beta_1)$$
 And $V_1 = V_2$ and for $\Lambda = 0.5$, $(\beta_2 = \alpha_1)$

Blade Height in Axial Flow turbine

The continuity equation $m = \rho AV$ may be used to find the blade height 'h'. The annular area of flow = π Dh. Thus the mass flow rate through an axial flow turbine is

$$m = \rho \pi D h V_a$$
$$h = \frac{m}{\rho \pi D V_a}$$

Blade height will increase in the direction of flow in a turbine and decrease in the direction of flow in a compressor.