Lecture Notes

On

POWER SYSTEM-II

A COURSE IN 6TH SEMESTER OF BACHELOR OF TECHNOLOGY PROGRAMME IN ELECTRICAL ENGINEERING

Department of Electrical Engineering

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UNIT I

Power system analysis

The evaluation of power system is called as power system analysis

Functions of power system analysis

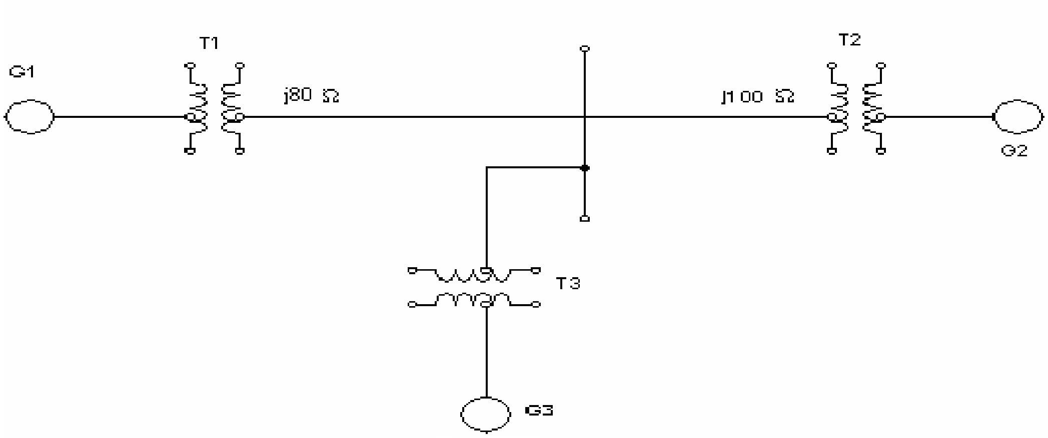
- To monitor the voltage at various buses, real and reactive power flow between buses.
- To design the circuit breakers.
- To plan future expansion of the existing system
- To analyze the system under different fault conditions
- To study the ability of the system for small and large disturbances (Stability studies)

COMPONENTS OF A POWER SYSTEM

1.Alternator
2.Power transformer
3.Transmission lines
4.Substation transformer
5.Distribution transformer
6.Loads

SINGLE LINE DIAGRAM

A single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and interconnection between them are shown by a straight line9eventhough the system is three phase system0.The ratings and the impedances of the components are also marked on the single line diagram.



Purpose of using single line diagram

The purpose of the single line diagram is to supply in concise form of the significant information about the system.

Per unit value.

The per unit value of any quantity is defined as the ratio of the actual value of the any quantity to the base value of the same quantity as a decimal.

per unit=actual value/base value

Need for base values

The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance rating of components of power system are expressed with reference to a common value called base value.

Advantages of per unit system

i. Per unit data representation yields valuable relative magnitude information.

ii. Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.

iii. The p.u systems are ideal for the computerized analysis and simulation of complex power system problems.

iv. Manufacturers usually specify the impedance values of equivalent in per unit of the equipments rating. If the any data is not available, it is easier to assume its per unit value than its numerical value.

v. The ohmic values of impedances are refereed to secondary is different from the value as referee to primary. However, if base values are selected properly, the p.u impedance is the same on the two sides of the transformer.
vi. The circuit laws are valid in p.u systems, and the power and voltages equations are simplified since the factors of √3 and 3 are eliminated.

Change the base impedance from one set of base values to another set

Let Z=Actual impedance ,Ω

Zb=Base impedance ,Ω

Per unit impedance of a circuit element= $\frac{Z}{Z_b} = \frac{Z}{\frac{(kVb)^2}{MVA_b}} = \frac{Z \times MVA_b}{(kVb)^2}$ (1)

The eqn 1 show that the per unit impedance is directly proportional to base megavoltampere and inversely proportional to the square of the base voltage.

Using Eqn 1 we can derive an expression to convert the p.u impedance expressed in one base value (old base) to another base (new base)

Let $kV_{b,old}$ and $MVA_{b,old}$ represents old base values and $kV_{b,new}$ and $MVA_{b,new}$ represent new base value

Let $Z_{p.u,old}$=p.u. impedance of a circuit element calculated on old base

$Z_{p.u,new}$=p.u. impedance of a circuit element calculated on new base

If old base values are used to compute the p.u.impedance of a circuit element ,with impedance Z then eqn 1 can be written as

$$Z_{p.u,old} = \frac{Z \times MVA_{b,old}}{(kV_{b,old})^2}$$

$$Z = Z_{p.u,old} \frac{(kV_{b,old})^2}{MVA_{b,old}} \qquad (2)$$

If the new base values are used to compute thep.u. impedance of a circuit element with impedance Z, then eqn 1 can be written as

$$Z_{p.u,new} = \frac{Z \times MVA_{b,new}}{(kV_{b,new})^2} \qquad (3)$$

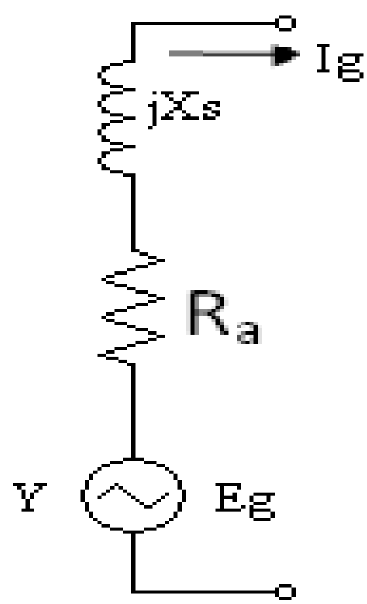
On substituting for Z from eqn 2 in eqn 3 we get

$$Z_{p.u,new} = Z_{p.u.old} \frac{(kV_{b,old})^2}{MVA_{b,old}} \times \frac{MVA_{b,new}}{(kV_{b,new})^2}$$

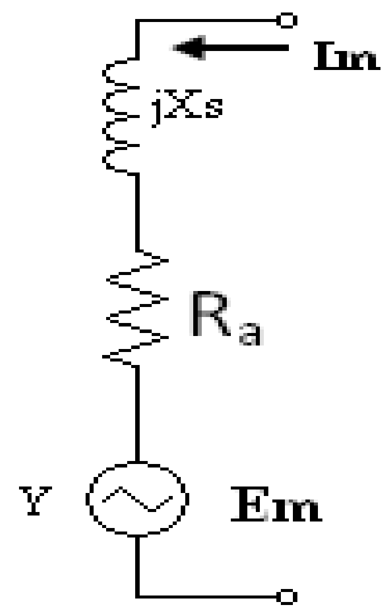
$$Z_{p.u,new} = Z_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) \qquad (4)$$

The eqn 4 is used to convert the p.u.impedance expressed on one base value to another base

MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR

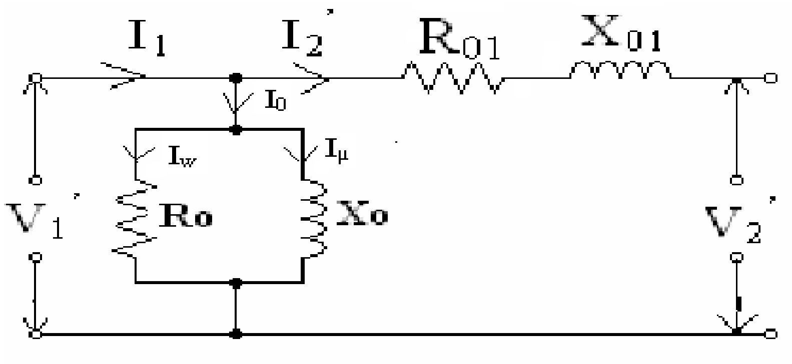


1Φ equivalent circuit of generator



1Φ equivalent circuit of synchronous motor

MODELLING OF TRANSFORMER

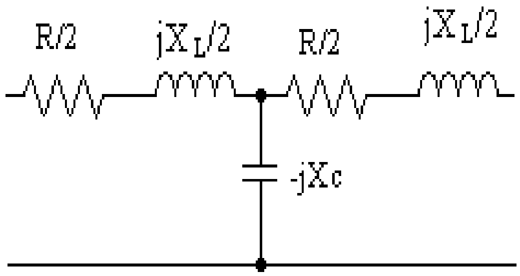


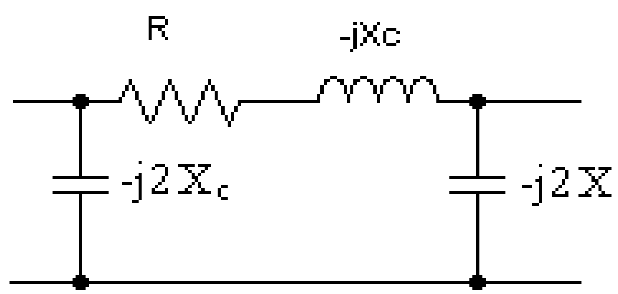
$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2^{'} = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1°

$$X_{01} = X_1 + X_2^{'} = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1°

MODELLING OF TRANSMISSION LINE





T type

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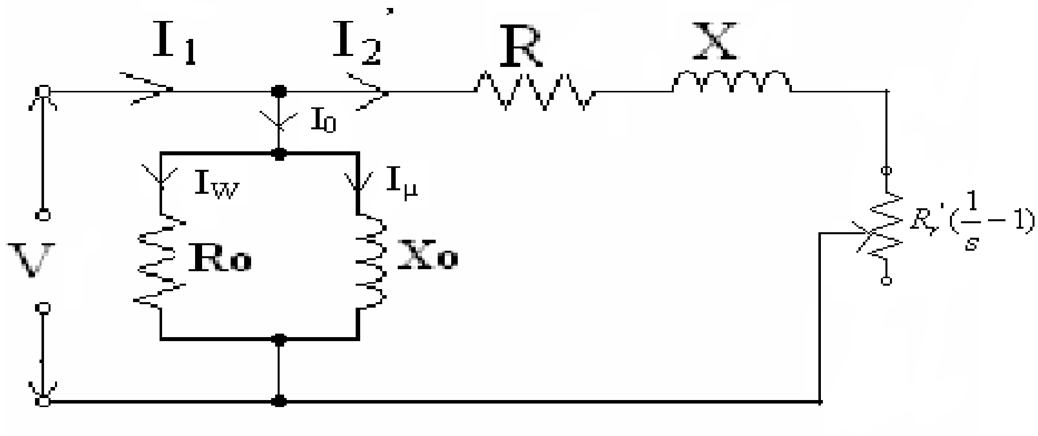
MODELLING OF INDUCTION MOTOR



$R_r'(\frac{1}{s}-1)$ =Resistance representing load

$R = R_s + R_r'$ =Equivalent resistance referred to stator

$X = X_s + X_r'$ =Equivalent reactance referred to stator

Impedance diagram & approximations made in impedance diagram

The impedance diagram is the equivalent circuit of power system in which the various components of power system are represented by their approximate or simplified equivalent circuits. The impedance diagram is used for load flow studies.

Approximation:

(i) The neutral reactances are neglected.

(ii) The shunt branches in equivalent circuit of transformers are neglected.

Reactance diagram & approximations made in reactance diagram

The reactance diagram is the simplified equivalent circuit of power system in which the various components of power system are represented by their reactances. The reactance diagram can be obtained from impedance diagram if all the resistive components are neglected. The reactance diagram is used for fault calculations.

Approximation:

(i) The neutral reactances are neglected.

(ii) The shunt branches in equivalent circuit of transformers are neglected.

(iii) The resistances are neglected.

(iv) All static loads are neglected.

(v) The capacitance of transmission lines are neglected.

PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE LINE DIAGRAM

1.Select a base power kVA_b or MVA_b

2.Select a base voltage kV_b

3. The voltage conversion is achieved by means of transformer kV_b on LT section= kV_b on HT section x LT voltage rating/HT voltage rating

4. When specified reactance of a component is in ohms

   p.u reactance=actual reactance/base reactance

   specified reactance of a component is in p.u

$$X_{p.u,new} = X_{p.u,old} * \frac{(kV_{b,old})^2}{(kV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

EXAMPLE

1. The single line diagram of an unloaded power system is shown in Fig 1.The generator transformer ratings are as follows.

G1=20 MVA, 11 kV, X''=25%

G2=30 MVA, 18 kV, X''=25%

G3=30 MVA, 20 kV, X''=21%

T1=25 MVA, 220/13.8 kV (Δ/Y), X=15%

T2=3 single phase units each rated 10 MVA, 127/18 kV(Y/Δ), X=15%

T3=15 MVA, 220/20 kV(Y/Δ), X=15%

Draw the reactance diagram using a base of 50 MVA and 11 kV on the generator1.

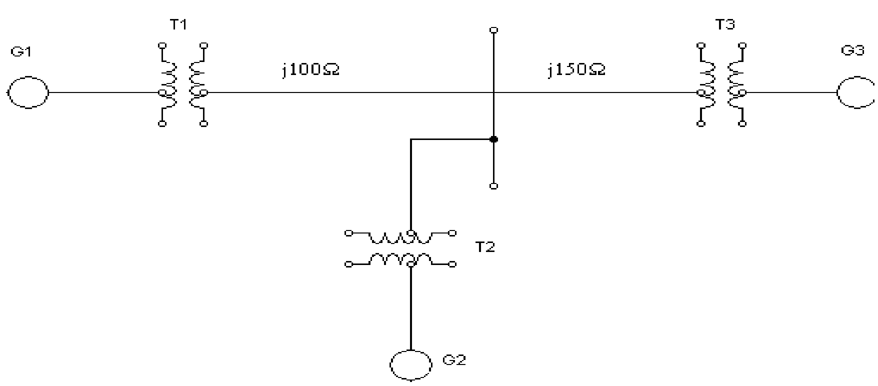


Fig 1

SOLUTION

Base megavoltampere,$MVA_{b,new}$=50 MVA

Base kilovolt $kV_{b,new}$=11 kV (generator side)

FORMULA

The new p.u. reactance $X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

Reactance of Generator G

$kV_{b,old}$=11 kV $\quad$ $kV_{b,new}$=11 kV

$MVA_{b,old}$= 20 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.25p.u

The new p.u. reactance of Generator G=$X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$$=0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{20}\right) = j0.625 p.u$$

Reactance of Transformer T1

$kV_{b,old}$=11 kV $\quad$ $kV_{b,new}$=11 kV

$MVA_{b,old}$= 25 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.15p.u

The new p.u. reactance of Transformer T1=$X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$$=0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{25}\right) = j0.3\ p.u$$

Reactance of Transmission Line

It is connected to the HT side of the Transformer T1

Base kV on HT side of transformer T 1 $= Base\ kV\ on\ LT\ side \times \frac{HT\ voltage\ rating}{LT\ voltage\ rating}$

$$=11 \times \frac{220}{11} = 220\ kV$$

Actual Impedance X_{actual}= 100ohm

Base impedance $X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{220^2}{50} = 968\ o\ \ m$

$p.u$ *reactance of 100 Ω transmission line* $= \frac{Actual\ Reactance\ ,ohm}{Base\ Reactance\ ,ohm} = \frac{100}{968} = j0.103\ p.u$

$p.u$ *reactance of 150 Ω transmission line* $= \frac{Actual\ Reactance\ ,ohm}{Base\ Reactance\ ,ohm} = \frac{150}{968} = j0.154\ p.u$

Reactance of Transformer T2

$kV_{b,old} = 127 * \sqrt{3}\ kV = 220\ kV$ $kV_{b,new} = 220\ kV$

$MVA_{b,old} = 10 * 3 = 30\ MVA$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.15 p.u$

$$\text{The new p.u. reactance of Transformer T2} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

Reactance of Generator G2

It is connected to the LT side of the Transformer T2

$$\text{Base kV on LT side of transformer T 2} = Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$$

$$= 220 \times \frac{18}{220} = 18\ kV$$

$kV_{b,old} = 18\ kV$ $kV_{b,new} = 18\ kV$

$MVA_{b,old} = 30\ MVA$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.25\ p.u$

$$\text{The new p.u. reactance of Generator G 2} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.25 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{30}\right) = j0.4167\ p.u$$

Reactance of Transformer T3

$kV_{b,old} = 20\ kV$ $kV_{b,new} = 20\ kV$

$MVA_{b,old} = 20\ MVA$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old}$=0.15p.u

$$\text{The new p.u. reactance of Transformer T3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

Reactance of Generator G3

It is connected to the LT side of the Transformer T3

$$\text{Base kV on LT side of transformer T 3} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{20}{220} = 20\ kV$$

$kV_{b,old}$=20 kV $\quad$ $kV_{b,new}$=20 kV

$MVA_{b,old}$= 30 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.21 p.u

$$\text{The new p.u. reactance of Generator G 3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.21 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.35\ p.u$$

2) Draw the reactance diagram for the power system shown in fig 4 .Use a base of 50MVA 230 kV in 30 Ω line. The ratings of the generator, motor and transformers are

Generator = 20 MVA, 20 kV, X=20%

Motor = 35 MVA, 13.2 kV, X=25%

T1 = 25 MVA, 18/230 kV (Y/Y), X=10%

T2 = 45 MVA, 230/13.8 kV (Y/Δ), X=15%

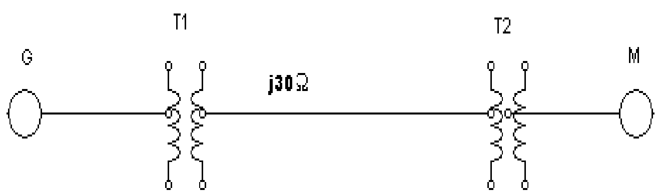


Fig 4

Solution

Base megavoltampere, $MVA_{b,new}$ *=50 MVA*

Base kilovolt $kV_{b,new}$ *=230 kV (Transmission line side)*

FORMULA

The new p.u. reactance $X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

Reactance of Generator G
It is connected to the LT side of the T1 transformer
Base kV on LT side of transformer T 1 $= Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$

$$=230 \times \frac{18}{230} = 18\ kV$$

$kV_{b,old}$ *=20 kV* $kV_{b,new}$ *=18 kV*

$MVA_{b,old}$ *= 20 MVA* $MVA_{b,new}$ *=50 MVA*

$X_{p.u,old}$ *=0.2p.u*

The new p.u. reactance of Generator G= $X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$$= 0.2 \times \left(\frac{20}{18}\right)^2 \times \left(\frac{50}{20}\right) = j0.617\ p.u$$

Reactance of Transformer T1

$kV_{b,old}$ *=18 kV* $kV_{b,new}$ *=18 kV*

$MVA_{b,old}$ *= 25 MVA* $MVA_{b,new}$ *=50 MVA*
$X_{p.u,old}$ *=0.1p.u*

The new p.u. reactance of Transformer T1= $X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$$= 0.1 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{25}\right) = j0.2\ p.u$$

Reactance of Transmission Line

It is connected to the HT side of the Transformer T1

Actual Impedance $X_{actual} = j30$ *ohm*

Base impedance $X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{230^2}{50} = 1058\ ohm$

p.u reactance of j30 Ω transmission line $= \frac{Actual\ Reactance,ohm}{Base\ Reactance,ohm} = \frac{j30}{1058} = j0.028\ p.u$

Reactance of Transformer T2

$kV_{b,old} = 230\ kV$ $kV_{b,new} = 230\ kV$

$MVA_{b,old} = 45\ MVA$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.15 p.u$

The new p.u. reactance of Transformer T2 $= X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$= 0.15 \times \left(\frac{230}{230}\right)^2 \times \left(\frac{50}{45}\right) = j0.166\ p.u$

Reactance of Motor M2

It is connected to the LT side of the Transformer T2

Base kV on LT side of transformer T 2 $= Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$

$= 230 \times \frac{13.8}{230} = 13.8\ kV$

$kV_{b,old} = 13.2\ kV$ $kV_{b,new} = 13.8\ kV$

$MVA_{b,old} = 35\ MVA$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.25\ p.u$

The new p.u. reactance of Generator G 2 $= X_{pu,old} \times \left(\frac{kVb,old}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$= 0.25 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{50}{35}\right) = j0.326\ p.u$

Unit II

Symmetrical Components

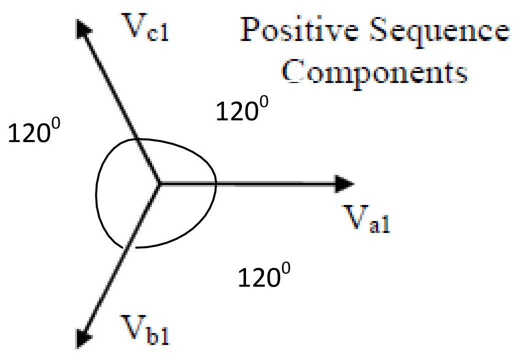
An unbalanced system of N related vectors can be resolved into N systems of balanced vectors. The N – sets of balanced vectors are called symmetrical components. Each set consists of N – vectors which are equal in length and having equal phase angles between adjacent vectors.

Sequence Impedance and Sequence Network

The sequence impedances are impedances offered by the devices or components for the like sequence component of the current .The single phase equivalent circuit of a power system consisting of impedances to the current of any one sequence only is called sequence network.

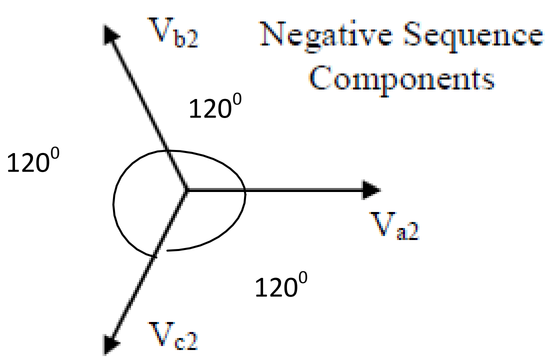
Positive Sequence Components

The positive sequence components are equal in magnitude and displayed from each other by 120^{o} with the same sequence as the original phases. The positive sequence currents and voltages follow the same cycle order of the original source. In the case of typical counter clockwise rotation electrical system, the positive sequence phasor are shown in Fig . The same case applies for the positive current phasors. This sequence is also called the “abc” sequence and usually denoted by the symbol “+” or “1”



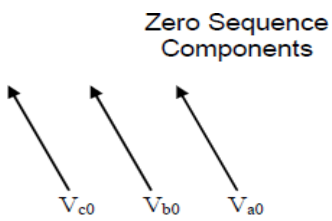
Negative Sequence Components

This sequence has components that are also equal in magnitude and displayed from each other by 120o similar to the positive sequence components. However, it has an opposite phase sequence from the original system. The negative sequence is identified as the “acb” sequence and usually denoted by the symbol “-” or “2” [9].The phasors of this sequence are shown in Fig where the phasors rotate anti- clockwise. This sequence occurs only in case of an unsymmetrical fault in addition to the positive sequence components,



Zero Sequence Components

In this sequence, its components consist of three phasors which are equal in magnitude as before but with a zero displacement. The phasor components are in phase with each other. This is illustrated in Fig . Under an asymmetrical fault condition, this sequence symbolizes the residual electricity in the system in terms of voltages and currents where a ground or a fourth wire exists. It happens when ground currents return to the power system through any grounding point in the electrical system. In this type of faults, the positive and the negative components are also present. This sequence is known by the symbol “0” .



EXAMPLE

1. The symmetrical components of a phase –a voltage in a 3-phase unbalanced system are $V_{a0} = 10\angle 180^0$ V, $V_{a1} = 50\angle 0^0$ V and $V_{a2} = 20\angle 90^0$ V. Determine the phase voltages V_a, V_b and V_c

The phase voltages of $V_a, V_b \, and \, V_c$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$$

$V_{a0} = 10\angle 180^0 = -10 + j0$ V

$V_{a1} = 50\angle 0^0 = 50 + j0$ V

$V_{a2} = 20\angle 90^0 = 0 + j20$ V

a=$1\angle 120^0$ a^2= $1\angle 240^0$

$a^2 V_{a1} = 1\angle 240^0 \times 50\angle 0^0 = 50\angle 240^0 = -25 - \mathrm{j}43.30$

$aV_{a1} = 1\angle 120^0 \times 50\angle 0^0 = 50\angle 120^0 = -25 + \mathrm{j}43.30$

$a^2 V_{a2} = 1\angle 240^0 \times 20\angle 90^0 = 20\angle 233 = 17.32 - \mathrm{j}10$

$aV_{a2} = 1\angle 120^0 \times 20\angle 90^0 = 20\angle 210^0 = -17.32 - \mathrm{j}10$

$$V_a = V_{a0} + V_{a1} + V_{a2} = (-10 + j0) + (50 + j0) + (0 + j20) = 40 + j20 = 44.72\angle 27^0 \, V$$

$V_b = V_{a0} + a^2V_{a1} + aV_{a2} = (-10 + j0) + (-25 - \text{j}43.30) + (-17.32 - \text{j}10) = -52.32 - j53.90$
$= 74.69\angle{-134^0}\ \text{V}$

$V_c = V_{a0} + aV_{a1} + a^2V_{a2} = (-25 - \text{j}43.30) + (-25 + \text{j}43.30) + 17.32 - \text{j}10$ =-17.68+j33.3

= 37.70 $\angle{-118^0}\ V$

THREE-SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

Positive sequence currents give rise to only positive sequence voltages, the negative sequence currents give rise to only negative sequence voltages and zero sequence currents give rise to only zero sequence voltages, hence each network can be regarded as flowing within in its own network through impedances of its own sequence only.

In any part of the circuit, the voltage drop caused by current of a certain sequence depends on the impedance of that part of the circuit to current of that sequence.

The impedance of any section of a balanced network to current of one sequence may be different from impedance to current of another sequence.

The impedance of a circuit when positive sequence currents are flowing is called impedance, When only negative sequence currents are flowing the impedance is termed as negative sequence impedance. With only zero sequence currents flowing the impedance is termed as zero sequence impedance.

The analysis of unsymmetrical faults in power systems is carried out by finding the symmetrical components of the unbalanced currents.

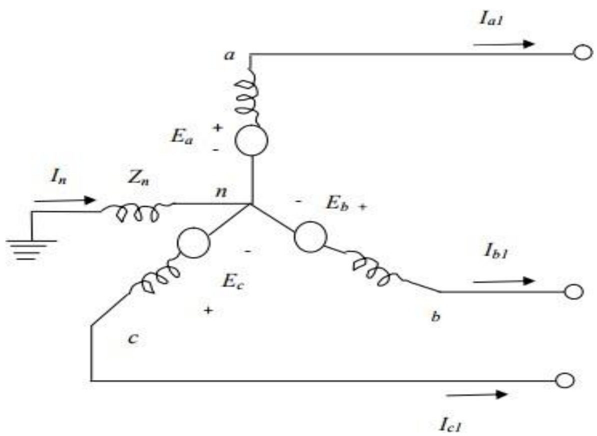
Since each sequence current causes a voltage drop of that sequence only, each sequence current can be considered to flow in an independent network composed of impedances to current of that sequence only.

The single phase equivalent circuit composed of the impedances to current of any one sequence only is

called the sequence network of that particular sequence. The sequence networks contain the generated emfs and impedances of like sequence. Therefore for every power system we can form three- sequence network s. These sequence networks, carrying current Ia1, Ia2 and Ia0 are then inter-connected to represent the different fault conditions.

SEQUENCE NETWORKS OF SYNCHRONOUS MACHINES

An unloaded synchronous machine having its neutral earthed through impedance, Zn, is shown in fig. below. A fault at its terminals causes currents I_a, I_b and I_c to flow in the lines. If fault involves earth, a current In flows into the neutral from the earth. This current flows through the neutral impedance Zn. Thus depending on the type of fault, one or more of the line currents may be zero. Thus depending on the type of fault, one or more of the line currents may be zero.



POSITIVE SEQUENCE NETWORK

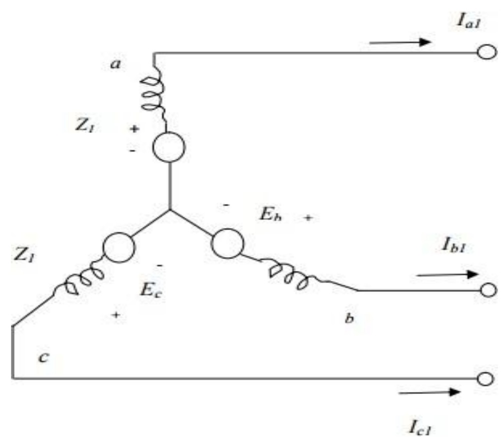
The generated voltages of a synchronous machine are of positive sequence only since the windings of a synchronous machine are symmetrical.

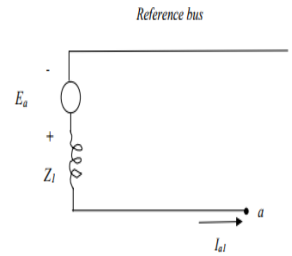
The positive sequence network consists of an emf equal to no load terminal voltages and is in series with the positive sequence impedance Z1 of the machine. Fig.2 (b) and fig.2(c) shows the paths for positive sequence currents and positive sequence network respectively on a single phase basis in the synchronous machine.

The neutral impedance Zn does not appear in the circuit because the phasor sum of I_{a1}, I_{b1} and I_{c1} is zero and no positive sequence current can flow through Zn. Since its a balanced circuit, the positive sequence N The reference bus for the positive sequence network is the neutral of the generator. The positive sequence impedance Z1 consists of winding resistance and direct axis reactance. The reactance is the sub-transient reactance X"d or transient reactance X'd or synchronous reactance Xd depending on whether sub-transient, transient or steady state conditions are being studied. From fig.2 (b) ,

the positive sequence voltage of terminal a with respect to the reference bus is given by:

$V_{a1}= E_a - Z_1 I_{a1}$





NEGATIVE SEQUENCE NETWORK

A synchronous machine does not generate any negative sequence voltage. The flow of negative sequence currents in the stator windings creates an mmf which rotates at synchronous speed in a direction opposite to the direction of rotor, i.e., at twice the synchronous speed with respect to rotor.

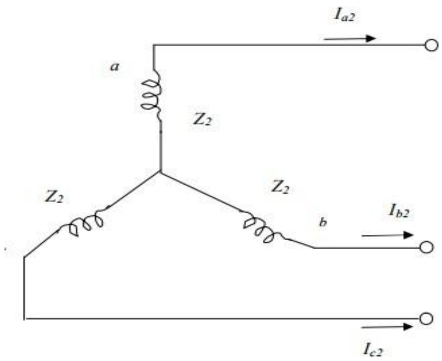
Thus the negative sequence mmf alternates past the direct and quadrature axis and sets up a varying armature reaction effect. Thus, the negative sequence reactance is taken as the average of direct axis and quadrature axis sub-transient reactance, i.e.,

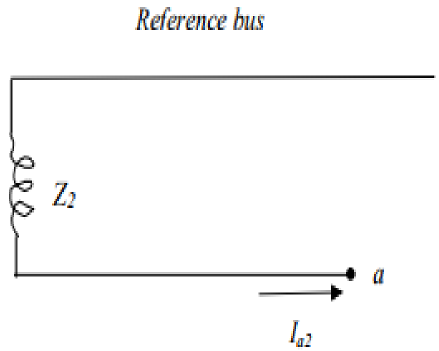
$X_2 = 0.5 (X''d + X''q)$.

It not necessary to consider any time variation of X2 during transient conditions because there is no normal constant armature reaction to be effected. For more accurate calculations, the negative sequence resistance should be considered to account for power dissipated in the rotor poles or damper winding by double supply frequency induced currents. The fig.below shows the negative sequence currents paths and the negative sequence network respectively on a single phase basis of a synchronous machine. The reference bus for the negative sequence network is the neutral of the machine.

Thus, the negative sequence voltage of terminal a with respect to the reference bus is given by:

$V_{a2}= -Z_2I_{a2}$





ZERO SEQUENCE NETWORK

No zero sequence voltage is induced in a synchronous machine. The flow of zero sequence currents in the stator windings produces three mmf which are in time phase. If each phase winding produced a sinusoidal space mmf, then with the rotor removed, the flux at a point on the axis of the stator due to zero sequence current would be zero at every instant.

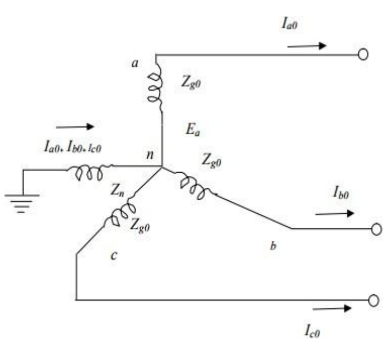
When the flux in the air gap or the leakage flux around slots or end connections is considered, no point in these regions is equidistant from all the three –phase windings of the stator.

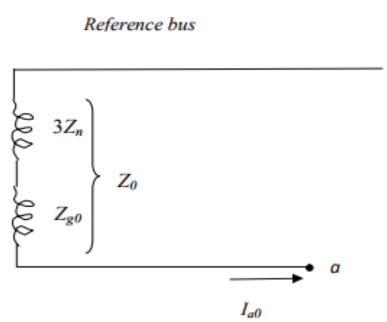
The mmf produced by a phase winding departs from a sine wave, by amounts which depend upon the arrangement of the winding.

The zero sequence currents flow through the neutral impedance Zn and the current flowing through this impedance is $3I_{a0}$

Fig.2(f) and fig.2(g) shows the zero sequence current paths and zero sequence network respectively, and as can be seen, the zero sequence voltage drop from point a to ground is $-3I_{a0}Z_n - I_{a0}Z_{g0}$ where Z_{g0} is the zero sequence impedance per phase of the generator.

Since the current in the zero sequence network is I_{a0} this network must have an impedance of 3Zn +Zg0. Thus, Z0 =3Zn +Zg0 The zero sequence voltage of terminal a with respect to the reference bus is thus: Va0 = -Ia0Z0





Symmetrical & Unsymmetrical Faults

Normally, a power system operates under balanced conditions. When the system becomes unbalanced due to the failures of insulation at any point or due to the contact of live wires, a short–circuit or fault, is said to occur in the line. Faults may occur in the power system due to the number of reasons like natural disturbances (lightning, high-speed winds, earthquakes), insulation breakdown, falling of a tree, bird shorting, etc.

Faults that occurs in transmission lines are broadly classified as

- Symmetrical faults
- Unsymmetrical faults

Symmetrical faults

In such types of faults, all the phases are short-circuited to each other and often to earth. Such fault is balanced in the sense that the systems remain symmetrical, or we can say the lines displaced by an equal angle (i.e. 120° in three phase line). It is the most severe type of fault involving largest current, but it occurs rarely. For this reason balanced short- circuit calculation is performed to determine these large currents.

Need for fault analysis

- To determine the magnitude of fault current throughout the power system after fault occurs.
- To select the ratings for fuses, breakers and switchgear.
- To check the MVA ratings of the existing circuit breakers when new generators are added into a system.

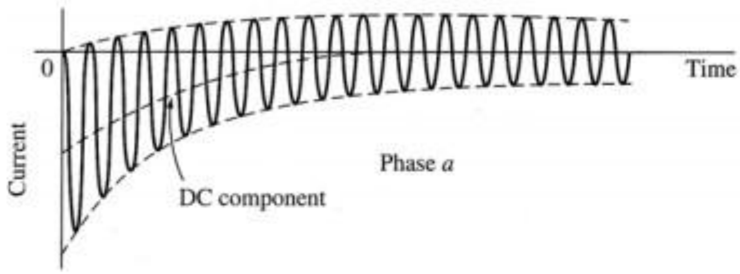
Common Power System Faults

Power system faults may be categorized as one of four types; in order of frequency of occurrence, they are: ·

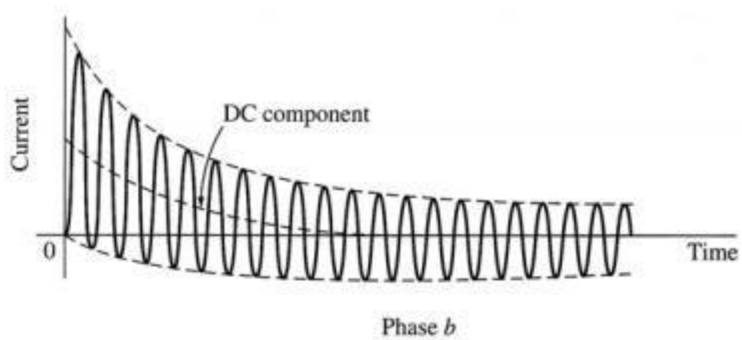
- Single line to ground fault
- Line to line fault
- Double line to ground fault
- Balanced three phase fault

3- Phase fault current transients in Phase fault current transients in synchronous generators synchronous generators

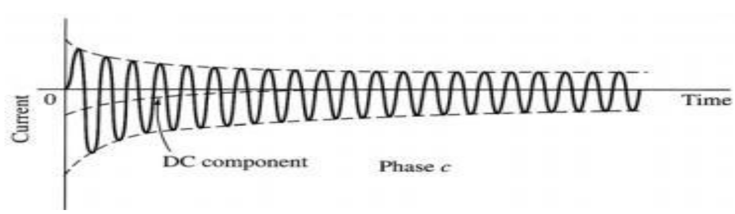
When a symmetrical 3-phase fault occurs at the terminals of a synchronous generator, the resulting current flow in the phases of the generator can appear as shown.



The current can be represented as a transient DC component added on top of a symmetrical AC component.

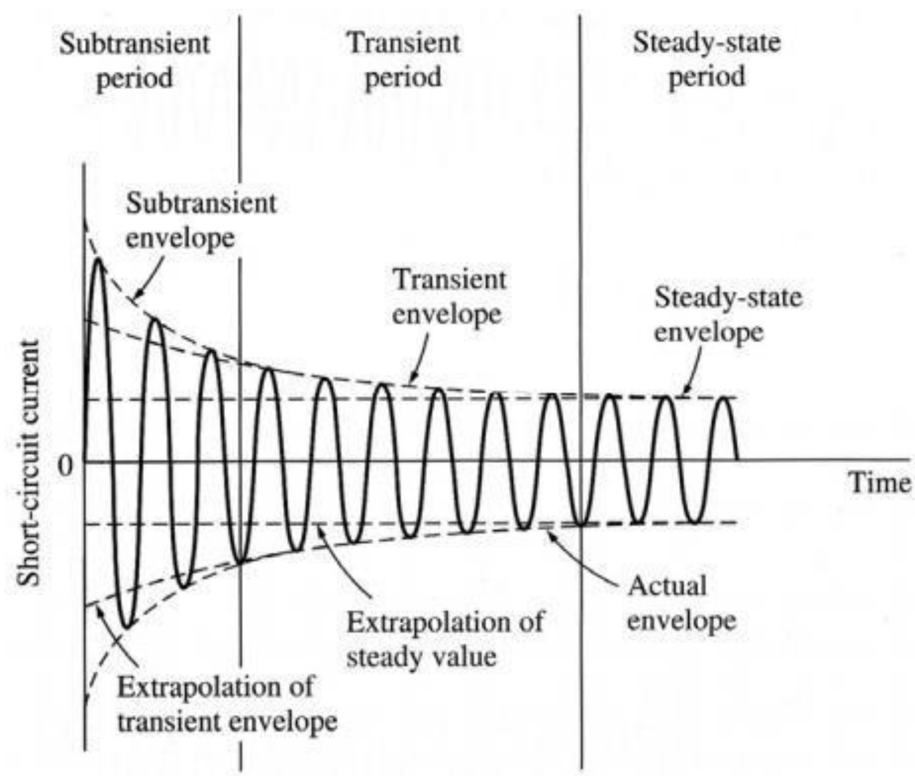


Therefore, while before the fault, only AC voltages and currents were present within the generator, immediately after the fault, both AC and DC currents are present.



Fault current transients in machines Fault current transients in machines

When the fault occurs, the AC component When the fault occurs, the AC component of current jumps to a very large value, but of current jumps to a very large value, but the total current cannot change instantly the total current cannot change instantly since the series inductance of the machine since the series inductance of the machine will prevent this from happening. will prevent this from happening.
The transient DC component of current is The transient DC component of current is just large enough such that the sum of the just large enough such that the sum of the AC and DC components just AC and DC components just after the fault the fault equals the AC current just equals the AC current just before the fault. the fault.Since the instantaneous values of current Since the instantaneous values of current at the moment of the fault are different in at the moment of the fault are different in each phase, the magnitude of DC each phase, the magnitude of DC components will be different in different components will be different in different phases.



There are three periods of time:

- ❖ Sub -transient period: first cycle or so after the fault transient period: first cycle or so after the fault – AC current is very AC current is very large and falls rapidly; large and falls rapidly;
- ❖ Transient period: current falls at a slower rate; Transient period: current falls at a slower rate;
- ❖ Steady -state period: current reaches its steady value. state period: current reaches its steady value.

It is possible to determine t It is possible to determine the time constants f he time constants for the sub or the sub -transient transient and transient periods . and transient periods

SHORT CIRCUIT CAPACITY

- ❖ It is the product of magnitudes of the prefault voltage and the post fault current.
- ❖ It is used to determine the dimension of a bus bar and the interrupting capacity of a circuit breaker.

Short Circuit Capacity (SCC)

$$|SCC| = |V^0||I_F|$$

$$|I_F| = \frac{|V_T|}{|Z_T|}$$

$$|SCC|_{1\phi} = \frac{|V_T|^2}{|Z_T|} = \frac{S_{b,1\phi}}{|Z_T|_{p.u}} MVA/\phi$$

$$|SCC|_{3\phi} = \frac{S_{b,3\phi}}{|Z_T|_{p.u}} MVA$$

$$I_f = \frac{|SCC|_{3\phi} * 10^6}{\sqrt{3} * V_{L,b} * 10^6}$$

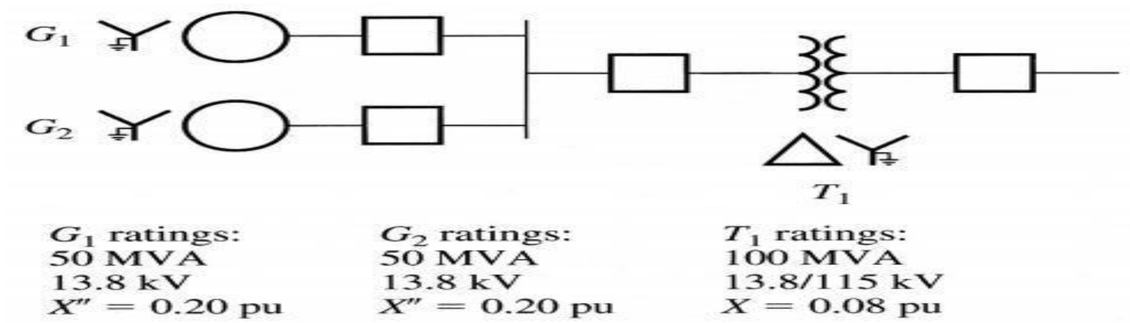
Procedure for calculating short circuit capacity and fault current

- ❖ Draw a single line diagram and select common base S_b MVA and kV
- ❖ Draw the reactance diagram and calculate the total p.u impedance from the fault point to source (Thevenin impedance Z_T)
- ❖ Determine SCC and I_f

EXAMPLE

Two generators are connected in parallel to the low voltage side of a transformer. Generators G1 and G 2 are each rated at 50 MVA, 13.8 kV, with a subtransient resistance of 0.2 pu. Transformer T1 is rated at 100 MVA, 13.8/115 kV with a series reactance of 0.08 pu and negligible resistance.

Assume that initially the voltage on the high side of the transformer is 120 kV, that the transformer is unloaded, and that there are no circulating currents between the generators. Calculate the subtransient fault current that will flow if a 3 phase fault occurs at the high-voltage side of transformer



Let choose the per-unit base values for this power system to be 100 MVA and 115 kV at the high-voltage side and 13.8 kV at the low-voltage side of the transformer. The subtransient reactance of the two generators to the system base is

$$X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$X_1'' = X_2'' = 0.2 \times \left(\frac{13{,}800}{13{,}800}\right)^2 \times \left(\frac{100{,}000}{50{,}000}\right) = j0.4 p.u$$

The reactance of the transformer is already given on the system base, it will not change

$$X_T = 0.08\, p.u$$

The per-unit voltage on the high-voltage side of the transformer is

$$V_{pu} = \frac{Actual\ value}{Base\ value} = \frac{120{,}000}{115{,}000} = j1.044\, p.u$$

Since there is no load on the system, the voltage at the terminals of each generator, and the internal generated voltage of each generator must also be 1.044 pu. The per-phase per-unit equivalent circuit of the system is We observe that the phases of internal generated voltages are arbitrarily chosen as 0 0. The phase angles of both voltages must be the same since the generators were working in parallel

To find the subtransient fault current, we need to solve for the voltage at the bus 1 of the system. To find this voltage, we must convert first the per-unit impedances to admittances, and the voltage sources to equivalent current sources. The Thevenin impedance of each generator is ZTh = j0.4, so the short-circuit current of each generator is

$$I_{su} = \frac{V_{oc}}{Z_t} = \frac{1.044\angle 0^0}{j0.4} = 2.61\angle 90^0$$

Then the node equation for voltage V1

$$V_1(-j2.5) + V_1(-j2.5) + V_1(-j12.5) = 2.61\angle -90^0 + 2.61\angle -90^0$$

$$V_1 = \frac{5.22\angle 90^0}{-j17.5} = 0.298\angle 0^0$$

Therefore, the subtransient current in the fault is

$$I_F = V_1(-j12.5) = 3.729\angle{-90^0}\ p.u$$

Since the base current at the high-voltage side of the transformer is

$$I^{base} = \frac{S_{3\Phi,base}}{\sqrt{3}V_{LL,base}} = \frac{100{,}000{,}000}{\sqrt{3}115{,}000} = 502\ A$$

the subtransient fault current will be

$I_F = I_{F,p.u}\ I_{base} = 3.729 \times 502 = 1872$ A

ALGORITHM FOR SHORT CIRCUIT ANALYSIS USING BUS IMPEDANCE MATRIX

- Consider a n bus network. Assume that three phase fault is applied at bus k through a fault impedance z_f

Prefault voltages at all the buses are

$$V_{bus}(0) = \begin{bmatrix} V_1(0) \\ V_2(0) \\ . \\ V_k(0) \\ . \\ V_n(0) \end{bmatrix}$$

- Draw the Thevenin equivalent circuit i.e Zeroing all voltage sources and add voltage source at faulted bus k and draw the reactance diagram
- The change in bus voltage due to fault is

$$\Delta V_{bus} = \begin{bmatrix} \Delta V_1 \\ . \\ . \\ \Delta V_k \\ . \\ \Delta V_n \end{bmatrix}$$

- The bus voltages during the fault is

$$V_{bus}(F) = V_{bus}(0) + \Delta V_{bus}$$

- The current entering into all the buses is zero.the current entering into faulted bus k is –ve of the current leaving the bus k

$$\Delta V_{bus} = Z_{bus} I_{bus}$$

$$\Delta V_{bus} = \begin{pmatrix} Z_{11} & . & Z_{1k} & . & Z_{1n} \\ . & . & . & . & . \\ Z_{k1} & . & Z_{kk} & . & Z_{kn} \\ . & . & . & . & . \\ Z_{n1} & . & Z_{nk} & . & Z_{nn} \end{pmatrix} \begin{bmatrix} 0 \\ . \\ -I_k(F) \\ . \\ 0 \end{bmatrix}$$

$$V_k(F) = V_k(0) - Z_{kk} I_k(F)$$

$$V_k(F) = Z_f I_k(F)$$

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f}$$

$$V_i(F) = V_i(0) - Z_{ik} I_k(F)$$

UNSYMMETRICAL FAULTS

- One or two phases are involved
- Voltages and currents become unbalanced and each phase is to be treated individually
- The various types of faults are

Shunt type faults

1. Line to Ground fault (LG)
2. Line to Line fault (LL)
3. Line to Line to Ground fault (LLG)

Series type faults

- Open conductor fault (one or two conductor open fault)

- Symmetrical components can be used to transform three phase unbalanced voltages and currents to balanced voltages and currents
- Three phase unbalanced phasors can be resolved into following three sequences

1. Positive sequence components
2. Negative sequence components
3. Zero sequence components

Single-Line-to-Ground Fault

Let a 1LG fault has occurred at node *k* of a network. The faulted segment is then as shown in Fig. 8where it is assumed that phase-a has touched the ground through an impedance Z_f . Since the system is unloaded before the occurrence of the fault we have

$$I_{fb} = I_{fc} = 0 \qquad (1)$$

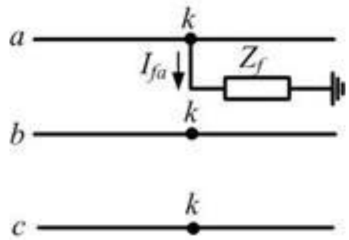


Fig. Representation of 1LG fault.

Also the phase-a voltage at the fault point is given by

$$V_{ka} = Z_f I_{fa} \tag{2}$$

From (1) we can write

$$I_{fa012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

Solving (.3) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_{fa}}{3} \tag{4}$$

his implies that the three sequence currents are in series for the 1LG fault. Let us denote the zero, positive and negative sequence Thevenin impedance at the faulted point as Z_{kk0} , Z_{kk1} and Z_{kk2} respectively.

$$\begin{aligned} V_{ka0} &= -Z_{kk0} I_{fa0} \\ V_{ka1} &= V_f - Z_{kk1} I_{fa1} \\ V_{ka2} &= -Z_{kk2} I_{fa2} \end{aligned} \tag{5}$$

Then from (4) and (5) we can write

$$\begin{aligned} V_{ka} &= V_{ka0} + V_{ka1} + V_{ka2} \\ &= V_f - \left(Z_{kk0} + Z_{kk1} + Z_{kk2}\right) I_{fa0} \end{aligned} \tag{6}$$

Again since

$$V_{ka} = Z_f I_{fa} = Z_f \left(I_{fa0} + I_{fa1} + I_{fa2}\right) = 3 Z_f I_{fa0} \tag{7}$$

The Thevenin equivalent of the sequence network is shown in Fig. 8.3.

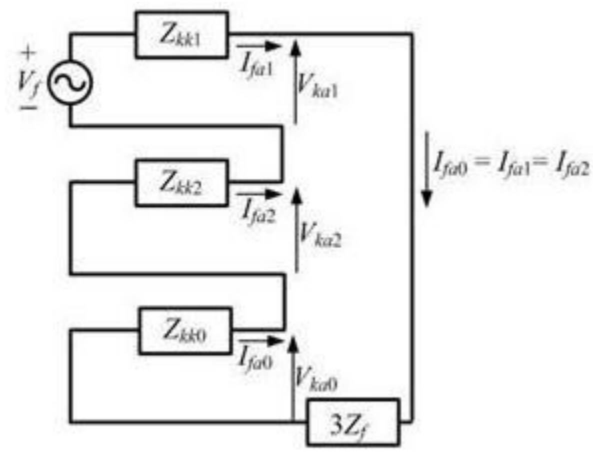
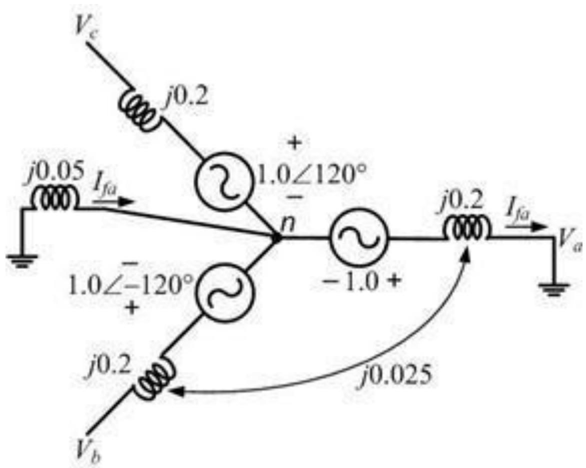


Fig. Thevenin equivalent of a 1LG fault.

Example 1

A three-phase Y-connected synchronous generator is running unloaded with rated voltage when a 1LG fault occurs at its terminals. The generator is rated 20 kV, 220 MVA, with subsynchronous reactance of 0.2 per unit. Assume that the subtransient mutual reactance between the windings is 0.025 per unit. The neutral of the generator is grounded through a 0.05 per unit reactance. The equivalent circuit of the generator is shown in Fig. We have to find out the negative and zero sequence reactances.



Since the generator is unloaded the internal emfs are

$$E_{an} = 1.0 \quad E_{bn} = 1.0\angle -120^0 \quad E_{cn} = 1.0\angle 120^0$$

Since no current flows in phases b and c, once the fault occurs, we have from Fig.

$$I_{fa} = \frac{1}{j(0.2 + 0.05)} = 2 - j4.0$$

Then we also have

$$V_n = -X_n I_{fa} = -0.2$$

From Fig. we get

$$V_a = 0$$
$$V_b = E_{bn} + V_n + j0.025 I_{fa} = -0.6 - j0.866 = 1.0536\angle -124.72^0$$
$$V_c = E_{cn} + V_n + j0.025 I_{fa} = -0.6 + j0.866 = 1.0536\angle 124.72^0$$

Therefore

$$V_{a012} = C \begin{bmatrix} 0 \\ 1.0536\angle -124.72^0 \\ 1.0536\angle 124.72^0 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.7 \\ -0.3 \end{bmatrix}$$

$$I_{fa1} = \frac{E_{an} - V_{a1}}{Z_1} = \frac{1 - 0.7}{j0.225} = -j1.333$$

$$I_{fa0} = I_{fa1} = I_{fa2}$$

$$Z_{go} = \frac{-V_{a0}}{I_{a0}} - 3X_n = j(0.3 - 0.15) = j0.15$$

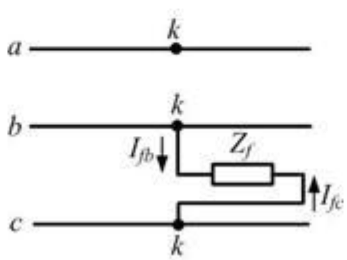
$$Z_2 = \frac{-V_{a2}}{I_{a2}} = j0.225$$

$$I_{fa0} = \frac{1}{j(0.225 + 0.225 + 0.15 + 3 \times 0.05)} = -j1.333$$

Line-to-Line Fault

The faulted segment for an L-L fault is shown in Fig. where it is assumed that the fault has occurred at node *k* of the network. In this the phases b and c got shorted through the impedance Z_f . Since the system is unloaded before the occurrence of the fault we have

$$I_{fa} = 0 \qquad (1)$$



Also since phases b and c are shorted we have

$$I_{fb} = -I_{fc} \tag{2}$$

Therefore from (1) and (2) we have

$$I_{fa012} = C\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (a-a^2)I_{fb} \\ (a^2-a)I_{fb} \end{bmatrix} \tag{3}$$

We can then summarize from (3)

$$\begin{aligned} I_{fa0} &= 0 \\ I_{fa1} &= -I_{fa2} \end{aligned} \tag{4}$$

herefore no zero sequence current is injected into the network at bus k and hence the zero sequence remains a dead network for an L-L fault. The positive and negative sequence currents are negative of each other.

Now from Fig. we get the following expression for the voltage at the faulted point

$$V_{kb} - V_{kc} = Z_f I_{fb} \tag{5}$$

Again

$$\begin{aligned} V_{kb} - V_{kc} &= V_{kb0} + V_{kb1} + V_{kb2} - V_{kc0} - V_{kc1} - V_{kc2} \\ &= (V_{kb1} - V_{kc1}) + (V_{kb2} - V_{kc2}) \\ &= (a^2 - a)V_{ka1} + (a - a^2)V_{ka2} \\ &= (a^2 - a)(V_{ka1} - V_{ka2}) \end{aligned} \tag{6}$$

Moreover since $I_{fa0} = I_{fb0} = 0$ and $I_{fa1} = - I_{fb2}$, we can write

$$I_{fb} = I_{fb1} + I_{fb2} = a^2 I_{fa1} + aI_{fb2} = (a^2 - a)I_{fa1} \tag{7}$$

Therefore combining (5) - (7) we get

$$V_{ka1} - V_{ka2} = Z_f I_{fa1} \tag{8}$$

Equations (5) and (8) indicate that the positive and negative sequence networks are in parallel. The sequence network is then as shown in Fig. From this network we get

$$I_{fa1} = -I_{fa2} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f}$$

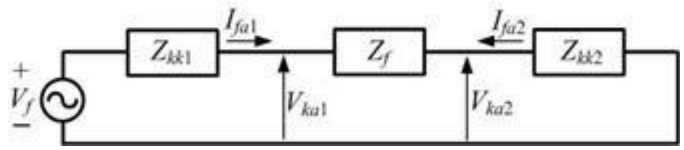


Fig. Thevenin equivalent of an LL fault.

Example 2

Let us consider the same generator as given in Example 1. Assume that the generator is unloaded when a bolted (Z_f = 0) short circuit occurs between phases b and c. Then we get from (2) $I_{fb} = -I_{fc}$. Also since the generator is unloaded, we have $I_{fa} = 0$.

$$V_{an} = E_{an} = 1.0$$
$$V_{bn} = E_{bn} - j0.225 I_{fb} = 1.\angle -120^0 - j0.225 I_{fb}$$

$$V_{cn} = E_{cn} - j0.225 I_{fc} = 1.\angle 120^0 + j0.225 I_{fb}$$

Also since V $_{bn}$ = V $_{cn}$, we can combine the above two equations to get

$$I_{fb} = -I_{fc} = \frac{1\angle -120^0 - 1\angle 120^0}{j0.45} = -3.849$$

Then

$$I_{fa012} = C \begin{bmatrix} 0 \\ -3.849 \\ 3.849 \end{bmatrix} = \begin{bmatrix} 0 \\ -j2.2222 \\ j2.2222 \end{bmatrix}$$

We can also obtain the above equation from (9) as

$$I_{fa1} = -I_{fb2} = \frac{1}{j0.225 + j0.225} = -j2.222$$

Also since the neutral current I n is zero, we can write V a = 1.0 and

$$V_b = V_c = V_{bn} = V_{bn} = -0.5$$

Hence the sequence components of the line voltages are

$$V_{a012} = C \begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Also note that

$$V_{a1} = 1.0 - j0.2251 I_{fa1}$$
$$V_{a2} = -j0.2251 I_{fa2} = 0.5$$

which are the same as obtained before.

Double- Line -to Ground Fault

The faulted segment for a 2LG fault is shown in Fig. where it is assumed that the fault has occurred at node *k* of the network. In this the phases b and c got shorted through the impedance Z_f to the ground.

$$I_{fa0} = \frac{1}{3}\left(I_{fa} + I_{fb} + I_{fc}\right) = \frac{1}{3}\left(I_{fb} + I_{fc}\right)$$

$$\Rightarrow 3I_{fa0} = I_{fb} + I_{fc} \tag{1}$$

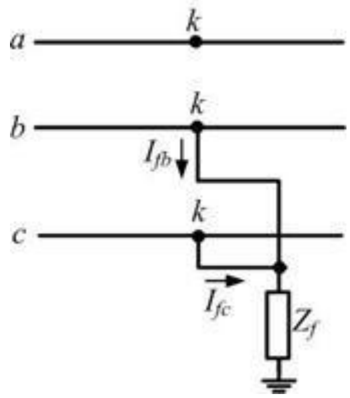


Fig. Representation of 2LG fault.

Also voltages of phases b and c are given by

$$V_{kb} = V_{kc} = Z_f\left(I_b + I_c\right) = 3Z_f I_{fa0} \tag{2}$$

Therefore

$$V_{ka012} = C\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \end{bmatrix} \tag{3}$$

We thus get the following two equations from (3)

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \tag{4}$$

$$V_{ka1} = V_{ka2}$$

$$3V_{ka0} = V_{ka} + 2V_{kb} = V_{ka0} + V_{ka1} + V_{ka2} + 2V_{kb} \tag{5}$$

Substituting (8.18) and (8.20) in (8.21) and rearranging we get

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \tag{6}$$

Also since $I_{fa} = 0$ we have

$$I_{fa0} + I_{fa1} + I_{fa2} = 0 \tag{7}$$

The Thevenin equivalent circuit for 2LG fault is shown in Fig. 8.8. From this figure we ge

$$I_{fa1} = \frac{V_f}{Z_{kk1} + Z_{kk2} \| (Z_{kk0} + 3Z_f)} = \frac{V_f}{Z_{kk1} + \dfrac{Z_{kk2}(Z_{kk0} + 3Z_f)}{Z_{kk2} + Z_{kk0} + 3Z_f}} \tag{8}$$

$$I_{fa0} = -I_{fa1}\left(\frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{9}$$

$$I_{fa2} = -I_{fa1}\left(\frac{Z_{kk0} + 3Z_f}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{10}$$

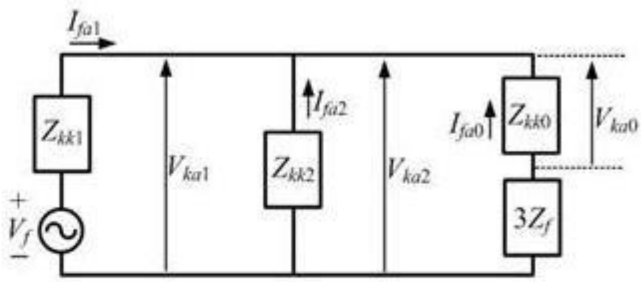


Fig. Thevenin equivalent of a 2LG fault.

Example 3

Let us consider the same generator as given in Examples 1 and 2. Let us assume that the generator is operating without any load when a bolted 2LG fault occurs in phases b and c. The equivalent circuit for this fault is shown in Fig. 8.9. From this figure we can write

$$E_{bn} + V_n = 1\angle -120° + V_n = j0.2I_{fb} - j0.025I_{fc}$$

$$E_{cn} + V_n = 1\angle 120° + V_n = j0.2I_{fc} - j0.025I_{fb}$$

$$V_n = -j0.05(I_{fb} + I_{fc})$$

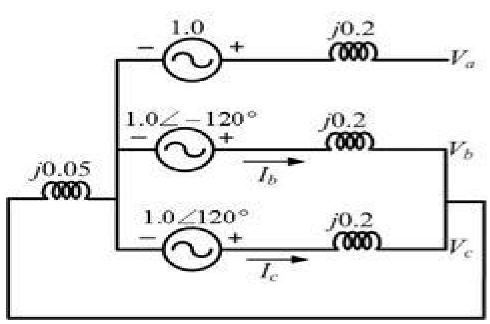


Fig. Equivalent circuit of the generator for a 2LG fault in phases b and c.

Combining the above three equations we can write the following vector-matrix form

$$j\begin{bmatrix} 0.25 & 0.025 \\ 0.025 & 0.25 \end{bmatrix}\begin{bmatrix} I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1\angle -120^\circ \\ 1\angle 120^\circ \end{bmatrix}$$

Solving the above equation we get

$I_{fb} = -3.849 + j1.8182$

$I_{fc} = 3.849 + j1.8182$

Hence

We can also obtain the above values using (8)-(10). Note from Example 1 that

$$Z_1 = Z_2 = j0.225, Z_0 = j(0.15 + 3 \times 0.05) = j0.3 \;\; and \; Z_f = 0$$

Then

$$I_{fa1} = \frac{1}{j0.225 + (\frac{j0.225 \times j0.3}{j0.525})} = -j2.8283$$

$$I_{fa0} = -I_{fa1}\frac{j0.225}{j0.525} = j1.2121$$

Now the sequence components of the voltages are

$$V_{a1} = 1.0 - j0.225 I_{fa1} = 0.3636$$

$$V_{a2} = j0.225 I_{fa2} = 0.3636$$

$$V_{a0} = -j0.3 I_{fa0} = 0.3636$$

Also note from above Fig. that

$$V_a = E_{an} + V_n + j0.0225(I_{fb} + I_{fc}) = 1.0909$$

and Vb = Vc = 0. Therefore

$$V_{a012} = C\begin{bmatrix} 1.0909 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.3636 \\ 0.3636 \\ 0.3636 \end{bmatrix}$$

which are the same as obtained before.

UNIT III

LOAD FLOW STUDIES

Load Flow Study (Or) Power Flow Study

The study of various methods of solution to power system network is referred to as load flow study. The solution provides the voltages at various buses, power flowing in various lines and line-losses.

The following work has to be performed for a load flow study.

(i) Representation of the system by single line diagrams.

(ii) Determining the impedance diagram using the information in single line diagram.

(iii) Formulation of network equations.

(iv) Solution of network equations.

Information's that are obtained from a load flow study

The information obtained from a load flow study is magnitude and phase angle of voltages, real and reactive power flowing in each line and the line losses. The load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied.

Need for load flow study

The load flow study of a power system is essential to decide the best operation of existing system and for planning the future expansion of the system. It is also essential foe designing a new power system.

Quantities associated with each bus in a system

Each bus in a power system is associated with four quantities and they are real power (P), reactive power (Q), magnitude of voltage (V), and phase angle of voltage (δ).

Different types of buses n a power system

Types of bus	**Known or specified quantities**	**Unknown quantities or quantities to be determined.**
Slack or Swing or Reference bus	V, δ	P,Q
Generator or Voltage control or PV bus	P, V	Q, δ
Load or PQ bus	P, Q	V, δ

Need for slack bus

The slack bus is needed to account for transmission line losses. In a power system the total power generated will be equal to sum of power consumed by loads and losses. In a power system only the generated power and load power are specified for buses. The slack bus is assumed to generate the power required for losses. Since the losses are unknown the real and reactive power are not specified for slack bus.

Iterative methods to solve load flow problems

The load flow equations are non linear algebraic equations and so explicit solution as not possible. The solution of non linear equations can be obtained only by iterative numerical techniques.

Mainly used for solution of load flow study

- The Gauss seidal method,
- Newton Raphson method
- Fast decouple methods.

Flat voltage start

In iterative method of load flow solution, the initial voltages of all buses except slack bus assumed as 1+j0 p.u. This is refereed to as flat voltage start

Effect of acceleration factor in load flow study

Acceleration factor is used in gauss seidal method of load flow solution to increase the rate of convergence. Best value of A.F=1.6

Generator buses are treated as load bus

If the reactive power constraints of a generator bus violates the specified limits then the generator is treated as load bus.

Advantages and disadvantages of Gauss serial method

Advantages: Calculations are simple and so the programming task is lessees. The memory requirement is less. Useful for small systems;

Disadvantages: Requires large no. of iterations to reach converge .Not suitable for large systems. Convergence time increases with size of the system

Advantages and disadvantages of N.R method

Advantages: Faster, more reliable and results are accurate, require less number of iterations;

Disadvantages: Program is more complex, memory is more complex.

Compare the Gauss seidel and Newton raphson methods of load flow study

S.No	G.S	N.R	FDLF
1	Require large number of iterations to reach convergence.	Require less number of iterations to reach convergence.	Require more number of iterations than N.R method.
2	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
3	It has linear convergence characteristics	It has quadratic convergence characteristics	------
4	The number of iterations required for convergence increases with size of the system	The number of iterations are independent of the size of the system	The number of iterations are does not dependent of the size of the system
5	Less memory requirements.	More memory requirements.	Less memory requirements than N.R.method.

Gauss-Seidel method

The step by step computational procedure for the Gauss-Seidel method of load flow studies

Algorithm when PV buses are present

1) Read the system data and formulate Y_{BUS} for the given power system network.

2) Assume a flat voltage profile (1+j0) for all the bus voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.

3) Assume a suitable value of **€** called convergence criterion. Here **€** is a specified change in the bus voltage that is used to compare the actual change in bus voltage between and iteration. K^{th} and $(k+1)^{th}$ iteration

4) Set iteration count k= 0

5) Set bus count p=1.

6) Check for slack bus. If it is a slack bus then go to step (13), otherwise go to next step.

7) Check for generator bus. If it is a generator bus go to next step, otherwise go to step (9)

8) Replace the value of the voltage magnitude of generator bus in that iteration by the specified value. Keep the phase angle same as in that iteration. Calculate Q for generator bus.

The reactive power of the generator bus can be calculated by using the following equation

$$Q_{p,cal}^{k+1} = (-1) I.P.of \left\{ (V_P^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

The calculated reactive power may be within specified limits or it may violate the limits. If the calculated reactive power violates the specified limit for the reactive power then treat this bus as the load bus. The magnitude of the reactive power at this bus will correspond to the limit it has violated

$$\text{i.e. if } Q_{p,cal}^{k+1} < Q_{p,\min} \quad \text{then} \quad Q_p = Q_{p,\min}$$

$$\text{or} \quad \text{if } Q_{p,cal}^{k+1} > Q_{p,\max} \text{ then } Q_p = Q_{p,\max}$$

Since the bus is treated as load bus, take actual value of Vp^k for $(k+1)^{th}$ iteration

i.e. | Vp^k | need not be replaced by $|V_p|_{sep}$ when the generator bus is treated as

load bus. Go to step (10).

9) For generator bus the magnitude of voltage does not change and so for all iterations the magnitude of bus voltage is the specified value only. The phase of the bus voltage can be calculated as shown below.

$$V_{p,temp}^{K+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p}^{n} Y_{pq} V_q^K \right]$$

$$\delta_p^{k+1} = \tan^{-1} \left[\frac{I.P. \, of \;\; V_{p,temp}^{k+1}}{R.P. \, of \;\; V_{p,temp}^{k+1}} \right]$$

Now the $(k+1)^{th}$ iteration voltage of the generator bus is given by

$$V_p^{k+1} = |V_p|_{spe} \, \delta_p^{k+1}$$

Where $|V_p|_{spe}$ is magnitude of specified voltage.

After calculating V_p^{k+1} for generator bus go to step (12)

10) For the load bus the (k+1)th iteration value of load bus-p voltage, V_p^{k+1} can be calculated with the following equation.

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

11) An acceleration factor ∝ can be used for faster convergence. If acceleration factor is specified then modify the (k+1)th iteration value of bus-p voltage using the following equation.

$$V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$$

Then set $V_p^{k+1} = V_{p,acc}^{k+1}$

12) Calculate the change in bus-p voltage, using the relation

$$\Delta V_p^k = V_p^{k+1} - V_p^k$$

Advance the bus count by 1 to evaluate other values of V_p^{k+1} and ΔV_p^k

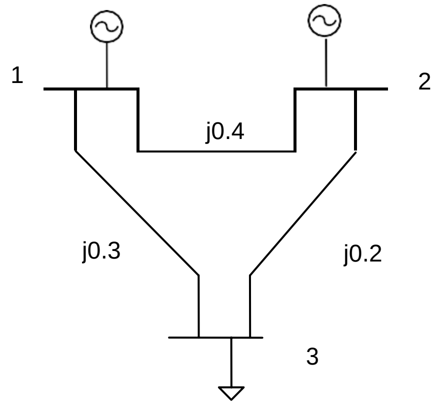
13) Check all the buses have been taken into account or not. If yes, go to the next step, Otherwise go back to step (6).

14) Determine the largest absolute value of change in voltage $|\Delta V|_{max}$

15) If $|\Delta V|_{max}$ is less than the pre specified tolerance €, then evaluate line flows and print the bus voltages and line flows. If not, advance the iteration count K= K+1 and go back to step (5).

EXAMPLE

1) Fig. shows a three bus power system.
Bus 1 : Slack bus, V= 1.05/00 p.u.
Bus 2 : PV bus, V = 1.0 p.u. Pg = 3 p.u.
Bus 3 : PQ bus, P_L = 4 p.u., Q_L = 2 p.u.
Carry out one iteration of load flow solution by Gauss Seidel method.



Neglect limits on reactive power generation.

Solution

Admittance of each line

$y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.4} = -j2.5\ p.u$

$y_{13} = \frac{1}{Z_{13}} = \frac{1}{j0.3} = -j3.333\ p.u$

$y_{23} = \frac{1}{Z_{23}} = \frac{1}{j0.2} = -j5\ p.u$

$$Y_{11} = y_{12} + y_{13} = -j2.5\ - j3.333 = -j5.833\ p.u$$

$$Y_{22} = y_{12} + y_{23} = -j2.5 - j5 = -j7.5\ p.u$$

$$Y_{33} = y_{13} + y_{23} = -j3.333 - j5 = -j8.333\ p.u$$

$$Y_{12} = Y_{21} = -y_{12} = -(-j2.5\ ) = j2.5\ p.u$$

$$Y_{13} = Y_{31} = -y_{13} = -(-j3.33\ ) = j3.33\ p.u$$

$$Y_{23} = Y_{32} = -y_{23} = -(-j5\ ) = j5\ p.u$$

The admittance matrix is given as

$$Y_{bus} = \begin{vmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{32} + y_{31} \end{vmatrix}$$

$$= \begin{vmatrix} -j5.833 & j2.5 & j3.33 \\ j2.5 & -j7.5 & j5 \\ j3.33 & j5 & -j8.333 \end{vmatrix}$$

Assume initial voltages to all buses

$V_1^{(0)} = 1.05\angle 0^0 = 1.05 + j0$ p.u

$V_2^{(0)} = 1.0 + j0$ p.u

$V_3^{(0)} = 1.0 + j0$ p.u

Bus 1 is a slack bus

$V_1^{(1)} = 1.05\angle 0^0 = 1.05 + j0$ p.u

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1} = (-1) \times Im\{(V_p^k)^* [\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k]\}$$

$$Q_{2cal}^1 = (-1) \times Im\{(V_2^0)^* [Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0]\}$$

$$=(-1) \times Im(1 - j0)[(j2.5)(1.05 + j0) + (-j7.5)(1 + j0) + (j5)(1 + j0)]\}$$

$Q_{2cal}^1 = -0.125$ p.u

The phase of bus -2 voltage in first iteration is given by phase of $V_{p,temp}^{K+1}$

When p=3 $Q_2^1 = -0.125$ p.u and k=0

$$V_{P,temp}^{k+1} = \frac{1}{Y_{pp}} [\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k]$$

$$V_{2,temp}^{0+1} = \frac{1}{Y_{22}} [\frac{P_2 - jQ_2}{(V_2^0)^*} - \sum_{q=1}^{2-1} Y_{2q} V_q^{0+1} - \sum_{q=2+1}^{3} Y_{2q} V_q^0]$$

$$V_{2,temp}^1 = \frac{1}{Y_{22}} [\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0]$$

$$= \frac{1}{-j7.5} [\frac{3 + j0.125}{1 - j0} - (j2.5)(1.05 + j0) - (j5)(1 + j0)]$$

$$V_2^1 = \frac{1}{-j7.5}[3 - j7.5] = 1.077\angle 21.8^0 \text{ V}$$

$\delta_2^1 = \angle V_{2,temp}^1 = 21.8^0$ V

$|V_2^1| = |V_2|_{spc}\angle\delta_2^1 = 1.0\angle 21.8^0$

$$|V_2^1| = \mathbf{0.928429 + j0.3713\ V}$$

Bus 3 Load Bus

The specified powers are load powers and so they considered as negative powers

$P_3 = -P_L = -4$

$Q_3 = -Q_L = -2$

$$V_P^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$= \frac{1}{-j8.333}\left[\frac{-4+j2}{1-j0} - (j3.33)(1.05 + j0) - (j5)(\mathbf{0.928429 + j0.37135})\right]$$

$$V_3^1 = 0.7806\angle -19.24^0$$

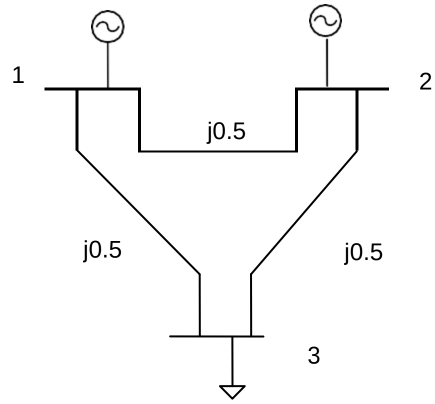
$$V_3^1 = 0.737046 - j0.25724\ p.u$$

2) Carry out one iteration of load flow analysis of the system given below by Gauss-Seidal method

Bus no	**Bus type**	**P**	**Q**	**V p.u**
1	**Slack**	**-**	**-**	**1.02**
2	**P-V**	**0.8**	**0.1 ≤ Q ≤ 1**	**1**
3	**P-Q**	**1.0**	**0.4**	**-**

Line reactance in p.u

Bus code	**Impedance**
1-2	**j0.5**
2-3	**j0.5**
3-1	**j0.5**



$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.5} = -j2\ p.u$$
$$y_{13} = \frac{1}{Z_{13}} = \frac{1}{j0.5} = -j2\ p.u$$
$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{j0.5} = -j2\ p.u$$

$$Y_{11} = y_{12} + y_{13} = -j2 - j2 = -j4\ p.u$$
$$Y_{22} = y_{12} + y_{23} = -j2 - j2 = -j4\ p.u$$
$$Y_{33} = y_{13} + y_{23} = -j2 - j2 = -j4\ p.u$$
$$Y_{12} = Y_{21} = -y_{12} = -(-j2) = j2.p.u$$
$$Y_{13} = Y_{31} = -y_{13} = -(-j2) = j2.p.u$$
$$Y_{23} = Y_{32} = -y_{23} = -(-j2) = j2.p.u$$

The admittance matrix is given as

$$Y_{bus} = \begin{vmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{32} + y_{31} \end{vmatrix}$$

$$= \begin{vmatrix} -j4 & j2 & j2 \\ j2 & -j4 & j2 \\ j2 & j2 & -j4 \end{vmatrix}$$

Assume initial voltages to all buses

$V_1^{(0)}$= 1.02∠0^0=1.02+j0 p.u

$V_2^{(0)}$=1.0+j0 p.u

$V_3^{(0)}$=1.0+j0 p.u

Bus 1 is a slack bus

$V_1^{(1)}$= 1.02∠0^0=1.02+j0 p.u

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1} = (-1) \times Im\{(V_p^k)^* [\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k]\}$$

$$Q_{2cal}^1 = (-1) \times Im\{(V_2^0)^* [Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0]\}$$

$$=(-1) \times Im(1 - j0)[(j2)(1.02 + j0) + (-j4)(1 + j0) + (j2)(1 + j0)]\}$$

$Q_{2cal}^1 = -0.04$p.u

This value is not with in the specified limit .so treat this bus as load bus
Q_2=0.1 P_2=0.3 and V_2^0=1.0+j0

$$V_P^{k+1} = \frac{1}{Y_{pp}}[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k]$$

$$= \frac{1}{Y_{22}}[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0]$$

$$\frac{1}{-j4}[\frac{0.8 - j0.1}{1 - j0} - (j2)(1.02 + j0) - (j2)(1 + j0)]$$

=

$$|\boldsymbol{V_2^1}| = \boldsymbol{1.035 + j0.2 = 1.054\angle 10.93^0\ V}$$

Bus 3 Load Bus

The specified powers are load powers and so they considered as negative powers

P_3= −P_L= −1

Q_3= −Q_L= −0.4

$$V_P^{k+1} = \frac{1}{Y_{pp}}[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k]$$

$$V_3^1 = \frac{1}{Y_{33}}[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1]$$

$$= \frac{1}{-j4}[\frac{-1+j0.4}{1-j0} - (j2)(1.02+j0) - (j2)(\mathbf{01.035 + j0.2})]$$

$$V_3^1 =$$

LOAD FLOW USING NEWTON-RAPHSON METHOD

Newton-Raphson (NR) method is more efficient and practical for large power systems. Main advantage of this method is that the number of iterations required to obtain a solution is independent of the size of the problem and computationally it is very fast. Here load flow problem is formulated in polar form.

Rewriting eqn. (7.15) and (7.16)

$$P_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\cos(\theta_{ik} - \delta_i + \delta_k) \qquad \text{...(7.50)}$$

$$Q_i = -\sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\sin(\theta_{ik} - \delta_i + \delta_k) \qquad \text{...(7.51)}$$

Equations (7.50) and (7.51) constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit and phase angles in radians, we can easily observe that two equations for each load bus given by eqn. (7.50) and (7.51) and one equation for each voltage controlled bus, given by eqn. (7.50). Expanding eqns. (7.50) and (7.51) in Taylor-series and neglecting higher-order terms. We obtain,

$$\therefore \begin{bmatrix} \Delta P_2^{(p)} \\ \vdots \\ \Delta P_n^{(p)} \\ \hline \Delta Q_2^{(p)} \\ \vdots \\ \Delta Q_n^{(p)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial P_2}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial P_2}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial P_2}{\partial |V_n|}\right)^{(p)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial P_n}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial P_n}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial P_n}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial P_n}{\partial |V_n|}\right)^{(p)} \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial Q_2}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial Q_2}{\partial |V_n|}\right)^{(p)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial Q_n}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial Q_n}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial Q_n}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial Q_n}{\partial |V_n|}\right)^{(p)} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(p)} \\ \vdots \\ \Delta \delta_n^{(p)} \\ \hline \Delta |V_2|^{(p)} \\ \vdots \\ \Delta |V_n|^{(p)} \end{bmatrix} \qquad \text{...(7.52)}$$

In the above equation, bus-1 is assumed to be the slack bus.

Eqn. (7.52) can be written is short form i.e.,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \text{...(7.53)}$$

The diagonal elements of J_1 described by eqn. (7.57) may be written as:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{k=1}^{n} |V_i|\,|V_k|\,|Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) - |V_i|^2\,|Y_{ii}| \sin\theta_{ii} \quad ...(7.65)$$

Using eqns. (7.65) and (7.51), we get

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2\,|Y_{ii}| \sin\theta_{ii}$$

$$\therefore \qquad \frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 B_{ii} \quad ...(7.66)$$

where $B_{ii} = |Y_{ii}| \sin\theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. In a practical power system, $B_{ii} >> Q_i$ and hence we may neglect Q_i. Further simplification is obtained by assuming $|V_i|^2 \approx |V_i|$, which gives,

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \quad ...(7.67)$$

Under normal operating conditions, $\delta_k - \delta_i$ is quite small. Therefore, $\theta_{ik} - \delta_i + \delta_k \approx \theta_{ik}$ and eqn. (7.58) reduces to

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i|\,|V_k| B_{ik}$$

Assuming $\quad |V_k| \approx 1.0$

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i| B_{ik} \quad ...(7.68)$$

Similarly, the diagonal elements of J_4 as given by eqn. (7.59) may be written as:

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,|Y_{ii}| \sin\theta_{ii} - \sum_{k=1}^{n} |V_i|\,|V_k|\,|Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \quad ...(7.69)$$

Using eqns. (7.69) and (7.51), we get,

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,|Y_{ii}| \sin\theta_{ii} + Q_i$$

$$\therefore \qquad \frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} + Q_i \quad ...(7.70)$$

Again $B_{ii} >> Q_i$, Q_i may be neglected.

$$\therefore \qquad \frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} \quad ...(7.71)$$

MODULE IV
ECONOMIC OPERATION OF POWER SYSTEM

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants.

The operation economics can again be subdivided into two parts.

i) Problem of economic dispatch, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.

ii) Problem of optimal power flow, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

During operation of the plant, a generator may be in one of the following states:

i) Base supply without regulation: the output is a constant.

ii) Base supply with regulation: output power is regulated based on system load.

iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes.

iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost. Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

PERFORMANCE CURVESINPUT-OUTPUT CURVE

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig.4.1

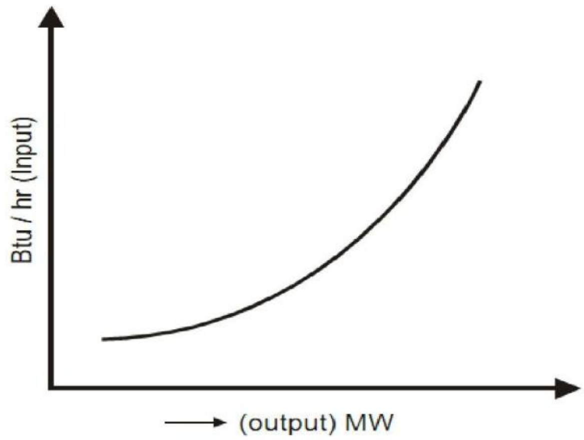


Fig.4.1: Input output curve

HEAT RATE CURVE

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel – efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .

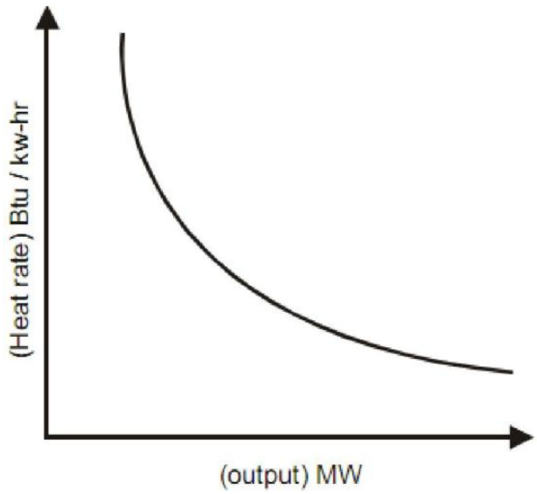


Fig.4.2: Heat Rate Curve

INCREMENTAL FUEL RATE CURVE

The incremental fuel rate is equal to a small change in input divided by the corresponding changein output.

Incremental fuel rate =ΔInput/ΔOutput

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig.4.3

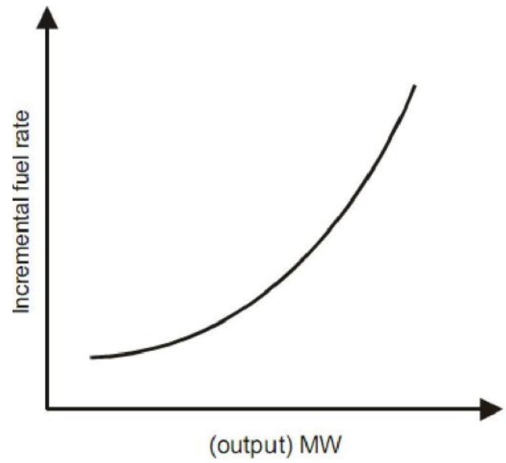


Fig 4.3: Incremental Fuel Rate Curve

Incremental cost curve

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $/Btu). The curve in shown in Fig.4.4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

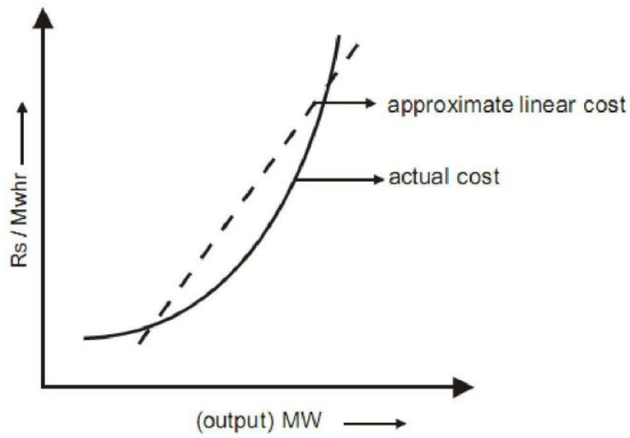


Fig. 4.4: Incremental Cost curve

In general, the fuel cost F_i for a plant, is approximated as a quadratic function of the

generated output P_{Gi}.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \text{ Rs/h}$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \text{Rs/MWh}$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between P_{Gmin}, the minimum loading limit, below which it is technically infeasible to operate a unit and P_{Gmax}, which is the maximum output limit.

ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

In an early attempt at economic operation it was decided to supply power from the most efficient plant at light load conditions. As the load increased, the power was supplied by this most efficient plant till the point of maximum efficiency of this plant was reached. With further increase in load, the next most efficient plant would supply power till its maximum efficiency is reached. In this way the power would be supplied by the most efficient to the least efficient plant to reach the peak demand. Unfortunately however, this method failed to minimize the total cost of electricity generation. We must therefore search for alternative method which takes into account the total cost generation of all the units of a plant that is supplying a load.

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand P_D.

Consider a system with n_g number of generating plants supplying the total demand P_D. If F_i is the cost of plant i in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Where F_T= total cost

P_{Gi}= generation of plant i

P_D= total demand

This is a constrained optimization problem, which can be solved by Lagrange's Method.

LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$L = F_T + \lambda(P_D - \sum_{i=1}^{n_g} P_{Gi})$$

The minimum is obtained when

$$\frac{\partial L}{\partial P_{Gi}} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0$$

$$\frac{\partial L}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant F_i depends only on its own output P_{Gi}, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial L}{dP_{Gi}} = \frac{dF_i}{dP_{Gi}} - \lambda = 0 \qquad i = 1, 2, \text{---------}, n_g$$

We can write $$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1, 2, \text{----------}, n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand.

ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR (NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi(min)} \leq P_{Gi} \leq P_{Gi(max)} \qquad i = 1, 2, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $\quad F_T = \sum_{i=1}^{n_g} F_i$

Such that $\quad \sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$

Where $\quad P_L$ is the total loss

The Lagrange function is now written as

$$L = F_T + \lambda(P_D - \sum_{i=1}^{n_g} P_{Gi} + P_L)$$

The minimum point is obtained when

$$\frac{\partial L}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda(1 - \frac{\partial P_L}{\partial P_{Gi}}) = 0 \qquad i=1,2,\ldots\ldots\ldots n_g$$

$$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} + P_L = 0 \qquad \text{(same as the constraint)}$$

Since $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$

$$\frac{dF_i}{dP_{Gi}} = \lambda\left(1 - \frac{dP_L}{dP_{Gi}}\right)$$

$$\therefore \lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1 - \frac{dP_L}{dP_{Gi}}}\right)$$

The term $\frac{1}{1 - \frac{dP_L}{dP_{Gi}}}$ is called the penalty factor of plant i, L_i. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i \qquad i=1,2,\ldots\ldots\ldots,n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered.

A rigorous general expression for the loss P_L is given by

$$P_L = \sum_m \sum_n P_{Gm} B_{mn} P_{Gn}$$

Where B_{mn} is called loss coefficient, depends on load composition.

For a two plant system

$$P_L = B_{11}P_{G1} + 2P_{G1}B_{12}P_{G2} + B_{22}P_{G2} \qquad \text{as } B_{12}=B_{21}$$

AUTOMATIC LOAD DISPATCH

Economic load dispatching is that aspect of power system operation wherein it is required to distribute the load among the generating units actually paralleled with the system in such a manner as to minimize the cost of supplying the minute to minute requirements of the system. In a large interconnected system it is humanly impossible to calculate and adjust such generations and hence the help of digital computer system along with analogue devices is sought and the whole process is carried out automatically; hence called automatic load dispatch. The objective of automatic load dispatch is to minimise the cost of supplying electricity to the load points while ensuring security of supply against loss of generation and transmission capacity and also maintaining the voltage and frequency of the system within specified limits. Since the interconnection is growing bigger and bigger in size with time, the control engineer has to make adjustments to various parameters in the system. Hence automatic control of load dispatch problem is required. The chosen control system is invariably based on a digital computer working on-line.

The components for automatic load dispatching are

Computer-The computer predicts the load and suggests economic loading. It transmits information to machine controller.

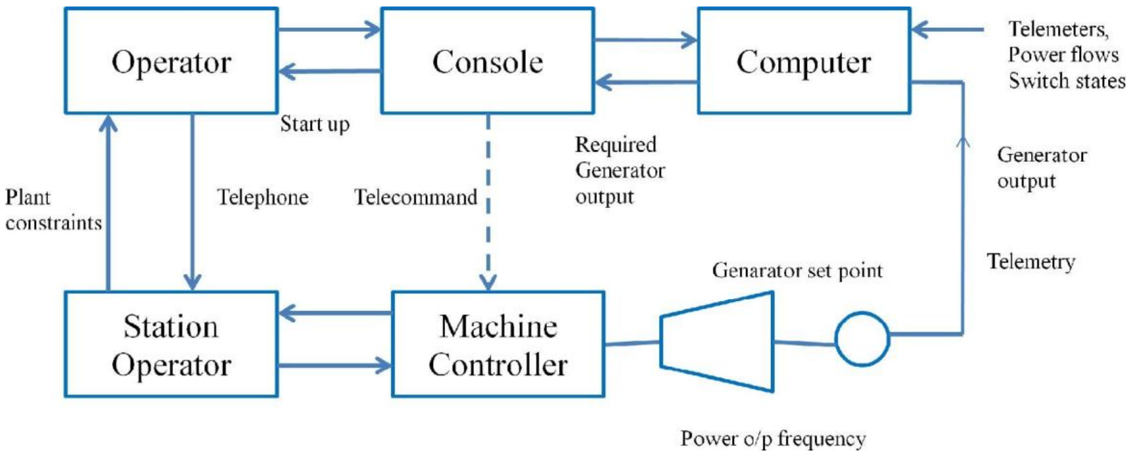


Fig.4.5: Schematic diagram of automatic load dispatching components

Data Input: The computer receives a lot of data from the telemetering system and from the paper tape. Telemetering data comes to the computer either as analog signals representing line power flows, plant outputs or as signal bits indicating switch or isolator positions. Paper tape stores all the basic data required e.g. the system parameters, load predictions, security constraints, etc.

Console: It is the component through which the operator can converse with the computer. He can obtain certain information required for some action to be taken under emergency condition or he can put data into it if needed. The console has the facilities of security checking and load flows for the network calculations.

Machine Controller: The computer sends information regarding the optimal generation to the machine controller at regular intervals which in turn implements them. Control on each machine is applied by a closed loop system which uses a measure of actual power generated and which operates through a conventional speeder motor. These are referred to as controller power loops. In the power frequency loop an error signal proportional to the difference between the derived and actual frequency and power is developed. A summed error signal is formed from these two components and is converted in the motor controller to a train of pulses that are applied to a speed governor reference setting motor called the speeder motor. The duration and amplitude of these pulses are fixed but the pulse rate is made proportional to the summed error signal. The pulses are applied as raise or lower command to the speeder motor in accordance with the error signal and thus the output of the generator is increased or decreased accordingly.

HYDROTHERMAL SCHEDULING LONG AND SHORT TERMS-

Long-Range Hydro-Scheduling:

The long-range hydro-scheduling problem involves the long-range forecasting of water availability and the scheduling of reservoir water releases (i.e., "drawdown") for an interval of time that depends on the reservoir capacities. Typical long-range scheduling goes anywhere from 1 week to 1 yr or several years. For hydro schemes with a capacity of impounding water over several seasons, the long-range problem involves meteorological and statistical analyses.

Short-Range Hydro-Scheduling

Short-range hydro-scheduling (1 day to 1 wk) involves the hour-by-hour scheduling of all generation on a system to achieve minimum production cost for the given time period. In such a scheduling problem, the load, hydraulic inflows, and unit availabilities are assumed known. A set of starting conditions (e.g., reservoir levels) is given, and the optimal hourly schedule that minimizes a desired objective, while meeting hydraulic steam, and electric system constraints, is sought. Hydrothermal systems where the hydroelectric system is by far the largest component may be scheduled by economically scheduling the system to produce the minimum cost for the thermal system. The schedules are usually developed to minimize thermal generation production costs, recognizing all the diverse hydraulic constraints that may exist.

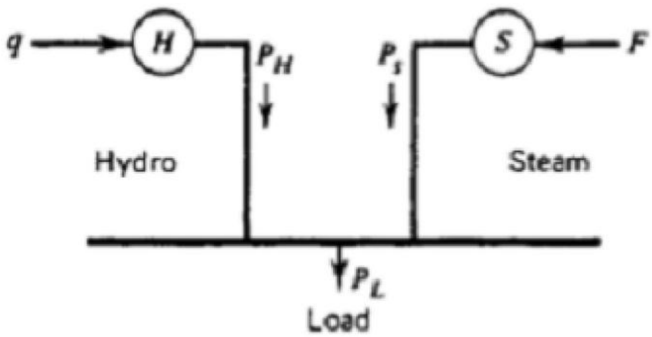


Fig. 4.6: Hydro Scheduling

The hydroplant can supply the load by itself for a limited time. That is, for any time period j,

$$P_{Hj}^{max} \geq P_{loadj} \qquad j=1,2,\ldots\ldots j_{max}$$

The energy available from the hydroplant is insufficient to meet the load.

$$\sum_{j=1}^{j_{max}} P_{Hj} n_j < \sum_{j=1}^{j_{max}} P_{loadj} n_j \qquad n_j \text{ is the no of hours in period j}$$

$$\sum_{j=1}^{j_{max}} n_j = T_{max} = \text{Total Interval}$$

Steam plant energy required is

$$\sum_{j=1}^{j_{max}} P_{loadj} n_j - \sum_{j=1}^{j_{max}} P_{Hj} n_j = E$$

$$\text{Where } E = \sum_{j=1}^{N_s} P_{sj} n_j \qquad N_s \text{ is the no of periods the steam plant is on}$$

$$\sum_{j=1}^{N_s} n_j \leq T_{max}$$

So the scheduling problem and the constraint are

$$\text{Min } F_T = \sum_{j=1}^{N_s} F(P_{sj}) n_j$$

$$\text{Subject to } \sum_{j=1}^{N_s} P_{sj} n_j - E = 0$$

Lagrange function is $$L = \sum_{j=1}^{N_s} F(P_{sj}) n_j + \lambda \left[E - \sum_{j=1}^{N_s} P_{sj} n_j \right]$$

$$\frac{\partial L}{\partial P_{sj}} = \frac{dF(P_{sj})}{dP_{sj}} n_j - \lambda n_j = 0 \qquad \text{for } j=1,2,\ldots\ldots\ldots\ldots N_s$$

$$\frac{dF(P_{sj})}{dP_{sj}} = \lambda$$

So steam plant should be run at constant incremental cost for the entire period it is on. Let this optimum value of steam-generated power be P_s^*, which is the same for all time intervalsthe steam unit is on.

The total cost over the interval is

$$F_T = \sum_{j=1}^{N_s} F(P_s^*) n_j = F(P_s^*) \sum_{j=1}^{N_s} n_j = F(P_s^*) T_s$$

T_s is the total run time for the steam plant

The total cost $$F_T = (a + bP_s^* + cP_s^{*2}) T_s$$

Also $$\sum_{j=1}^{N_s} P_{sj} n_j = \sum_{j=1}^{N_s} P_s^* n_j = P_s^* T_s = E$$

So $$T = \frac{E}{\;}$$

$$P_s \quad {}_s^*$$

$$F_T = (a + bP_s^* + cP_s^{*2})\left(\frac{E}{P_s^*}\right)$$

$$\text{Minimizing } F_T \text{, we get } P_s^* = \sqrt{\frac{a}{c}}$$

So the unit should be operated at its maximum efficiency point (P_s^*) long enough to supply the energy needed, E. Optimal hydrothermal schedule is as shown below:

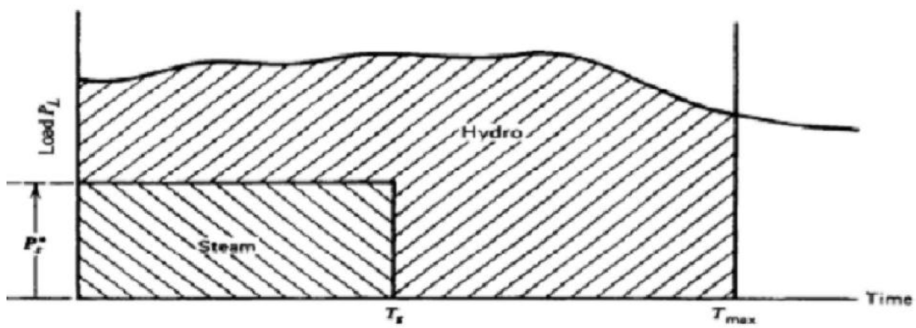


Fig.4.7: Optimal Hydrothermal Scheduling

UNIT VI

STABILITY ANALYSIS

MODULE-IV

Stability of power system is its ability to return to normal or stable operating condition after been subjected to some of disturbance. Instability means a condition representing loss of synchronism or fall out of step.

The instability of power system is divided into two parts

1. Steady state stability
2. Transient stability

Increase in load is a kind of disturbance to power system. If the increase in load takes place gradually and slowly in small steps and the system withstand this change in load and operates satisfactorily then this system phenomena is said to be STEADY STATE STABILITY.

Cause of transient disturbances

1. Sudden change of load.
2. Switching operation.
3. Loss of generation.
4. Fault.

Due to the following sudden disturbances in the power system, rotor angular difference, rotor speed and power transfer undergo fast changes whose magnitude are dependent upon the severity of disturbances.

If the disturbance is so large that the angular difference increases so much which can cause the machine out of synchronism. This kind of instability is denoted as transient instability. It is a very fast phenomenon it occurs within one second for the generating unit closer to the disturbance.

Dynamics Of A Synchronous Machine

The kinetic energy of the rotor at synchronous machine is

$$KE = \frac{1}{2} J\omega_{sm}^2 \times 10^{-6}\, MJ \tag{4.1}$$

J =rotor moment of inertia in kg-m^2

ω_{sm} =synchronous speed in rad (mech)/s

$\omega_s = \left(\frac{P}{2}\right)\omega_{sm}$ =rotor speed in rad(elect/s)

P =number of machine poles

$$KE = \frac{1}{2}\left[J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right]\omega_s = \frac{1}{2} M\omega_s \qquad (4.2)$$

Where

$$M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6} = \text{Moment of inertia MJ-s/elect. rad}$$

now inertia constant *h* be written as

$$GH = KE = \frac{1}{2} M\omega_s \text{ mj} \qquad (4.3)$$

g =machine rating(base)in mva(3-phase)

h =inertia constant in mj/mva or mw-s/mva

so,

$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{MJ-s/elect.rad} \qquad (4.4)$$

$$= \frac{GH}{180 f} \text{ MJ-s/elect.rad} \qquad (4.5)$$

Taking *G* as base, the inertia constant in pu is

$$M = \frac{H}{\pi f} \text{ s}^2\text{/elect.rad} \qquad (4.6)$$

$$M = \frac{H}{180 f} \text{ s}^2\text{/elect.degree} \qquad (4.7)$$

<u>Swing Equation</u>

The differential equation that relates the angular momentum *M*, acceleration power P_a and the rotor angle δ is known as swing equation. Solution of swing equation shows how the rotor angle changes with respect time following a disturbance. The plot δ Vs *t* is known as swing curve. The differential equation governing the rotor dynamics can then be written as.

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \qquad (4.8)$$

where,

J = rotor moment of inertia in kg-m^2, θ_m = angle in radian (mech.)

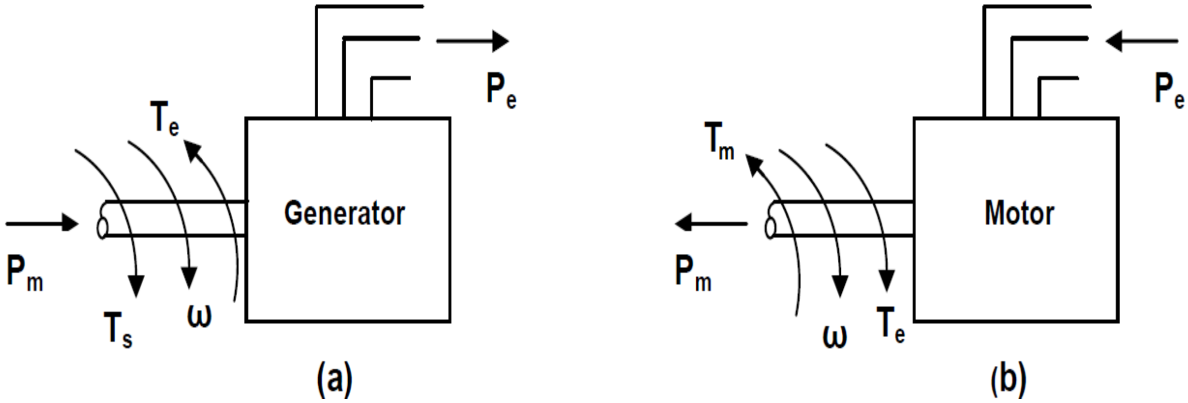


Fig. 4.1 Electrical and mechanical power flow in motor

While the rotor undergoes dynamics as per Equation (9), the rotor speed changes by insignificant magnitude for the time period of interest (1s)

Equation (4.8) can therefore be converted into its more convenient power form by assuming the rotor .speed (ω_{sm}). Multiplying both sides of Equation (4.8) by ω_{sm} we can write

$$J\omega_{sm}\frac{d^2\theta_m}{dt^2}\times 10^{-6} = P_m - P_e \,\text{MW} \tag{4.9}$$

Where,

P_m= mechanical power input in MW

P_e=electrical power output in MW; stator copper loss is assumed neglected.

Rewriting Equation (4.9)

$$\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right)\frac{d^2\theta_e}{dt^2} = \left(P_m - P_e\right)MW \tag{4.11}$$

$$M\frac{d^2\theta_e}{dt^2} = P_m - P_e \, MW \tag{4.12}$$

Where

Θ_e =angle in rad.(elect.)

As it is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference.

Let us assume,

$$\delta = \theta_e - \omega_s t \tag{4.13}$$

δ is rotor angular displacement from synchronously rotating reference frame, called ***Torque Angle/Power Angle.***

From Equation (4.9)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \tag{4.14}$$

Hence Equation (4.11) can be written in terms of *δ* as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e MW \tag{4.15}$$

Using Equation (4.11) we can also write

$$\left(\frac{GH}{\pi f}\right)\frac{d^2\delta}{dt^2} = P_m - P_e MW \tag{4.16}$$

Dividing throught by *G*, the MVA rating of the machine

$$M(pu)\frac{d^2\delta}{dt^2} = P_m - P_e \tag{4.17}$$

Where

$$M(pu) = \frac{H}{\pi f}, \quad \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{pu}$$

Equation (4.17) is called as swing equation and it describes the rotor dynamics for a synchronous machine (generating/motoring). It is a second-order differential equation where the damping term (proportional to $d\delta/dt$) is absent because of the assumption of a loss less machine and the fact that the torque of damper winding has been ignored. Since the electrical power P_e depends upon the sine of angle δ the swing equation is a non-linear second-order differential equation.

Multi-Machine System

In a multi-machine system a common system base must be chosen

Let

G_{mach}=machine rating (base)

G_{system}=system base

Equation(18) can then be written as

$$\frac{G_{mach}}{G_{system}}\left(\frac{H_{mach}}{f}\frac{d^2\delta}{dt^2}\right)=(P_m-P_e)\frac{G_{mach}}{G_{system}}$$

Or $\dfrac{H_{system}}{\pi f}\dfrac{d^2\delta}{dt^2}=P_m-P_e$ pu in system base. (4.18)

Where

$$H_{system}=H_{mach}\left(\frac{G_{mach}}{G_{system}}\right) \tag{4.19}$$

Consider the swing equations of two machines or a common system base.

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2}=P_{m1}-P_{e1} \text{ pu} \tag{4.20}$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2}=P_{m2}-P_{e2} \text{ pu} \tag{4.21}$$

Since the machine rotors swings together (coherently or in unison)

$\delta_1=\delta_2=\delta$

Adding Equation (4.20) and (4.21)

$$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2}=P_m-P_e \tag{4.22}$$

Where

$$P_m=P_{m1}+P_{m2}$$

$$P_e=P_{e1}+P_{e2}$$

$$H_{eq} = H_1 + H_2$$

The two machines swinging coherently are thus reduced to a single machine as in Equation (4. 22), the equivalent inertia in (4.22) can be written as

$$H_{eq} = H_{1mach} \frac{G_{1mach}}{G}\bigg|_{system} + H_{2mach} \frac{G_{2mach}}{G}\bigg|_{system} \tag{4.23}$$

The above results are easily extendable to any number of machines swinging coherently. To solving the swing equation (Equation (4.23), certain simplifying assumptions are usually made. These are:

1. Mechanical power input to the machine (P_m) remains constant during the period of electromechanical transient of interest. In other words, it means that the effect of the turbine governing loop is ignored being much slower than the speed of the transient. This assumption leads to pessimistic result-governing loop helps to stabilize the system.

2. Rotor speed changes are insignificant-these have already been ignored in formulating the swing equation.

3. Effect of voltage regulating loop during the transient is ignored, as a consequence the generated machine emf remains constant. This assumption also leads to pessimistic results-voltage regulator helps to stabilize the system.

Before the swing equation can be solved, it is necessary to determine the dependence of the electrical power output (P_e) upon the rotor angle.

Simplified Machine Model

For a non-salient pole machine, the per-phase induced emf-terminal voltage equation under steady conditions is.

$$E = V + jX_d I_d + jX_q I_q \,;\, X_d > X_q \tag{4.24}$$

Where

$$I = I_d + I_g$$

Under transient condition

$$X_d \rightarrow X_d' < X_d$$

$X_q' = X_d$ Since the main field is on the d-axis

$X_d' < X_q$;

Equation (4.24) during the transient modifies to.

$$E' = V + jX_d' I_d + jX_q I_q$$

$$= V + jX_q (I - I_d) + jX_d' I_d \tag{4.25}$$

$$= (V + jX_d I) + j(X_d' - X_q) I_d \tag{4.26}$$

The phasor diagram corresponding to Equation (4.25) and (4.26) is drawn in Fig. 4.2. Since under transient condition, $X_d' < X_d$ but X_d remains almost unaffected, it is fairly valid to assume that

$X_d' \approx X_d$

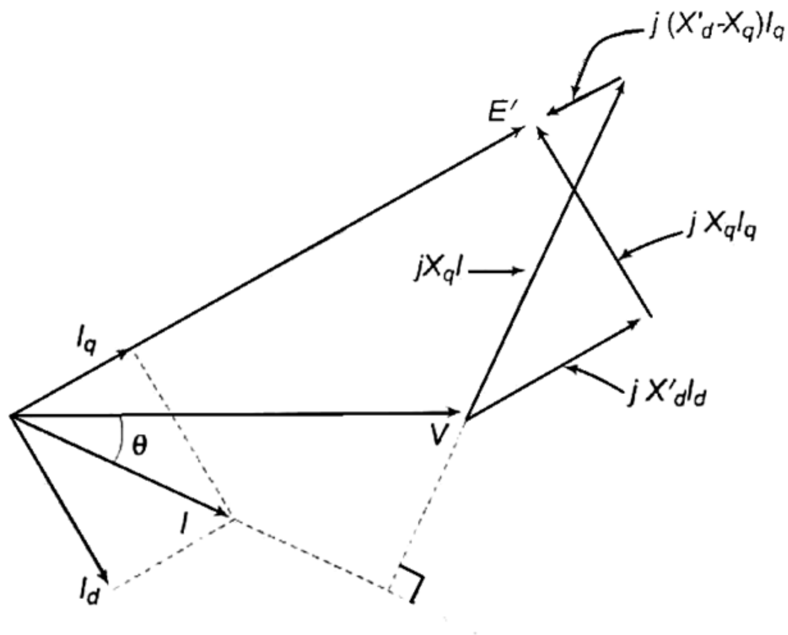


Fig. 4.2 Phasor diagram of a salient pole machine

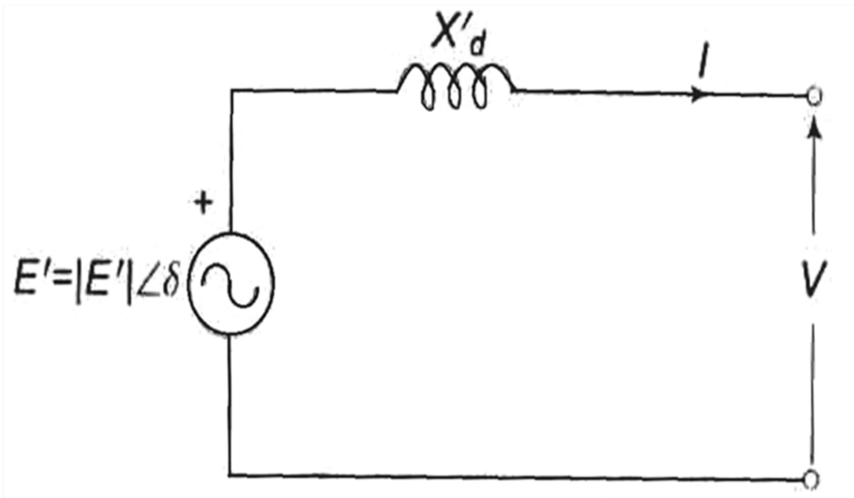


Fig.4.3 Simplified machine model.

The machine model corresponding to Eq. (4.26) is drawn in Fig. (4.3) which also applies to a cylindrical rotor machine where $X_d^{/} = X_q^{/} = X_s^{/}$ (transient synchronous reactance).

Power Angle Curve

For the purposes of stability studies $|E'|$, transient emf of generator motor remains constant or is the independent variable determined by the voltage regulating loop but *V*, the generator determined terminal voltage is a dependent variable. Therefore, the nodes (buses) of the stability study network to the ernf terminal in the machine model as shown in Fig.4.4, while the machine reactance (X_d') is absorbed in the system network as different from a load flow study. Further, the loads (other than large synchronous motor) will be replaced by equivalent static admittances (connected in shunt between transmission network buses and the reference bus).

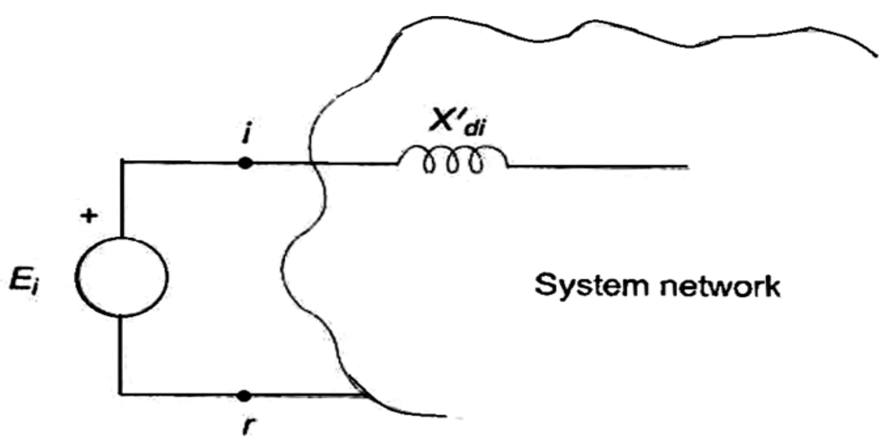


Fig. 4.4 Simplified Machine studied Network

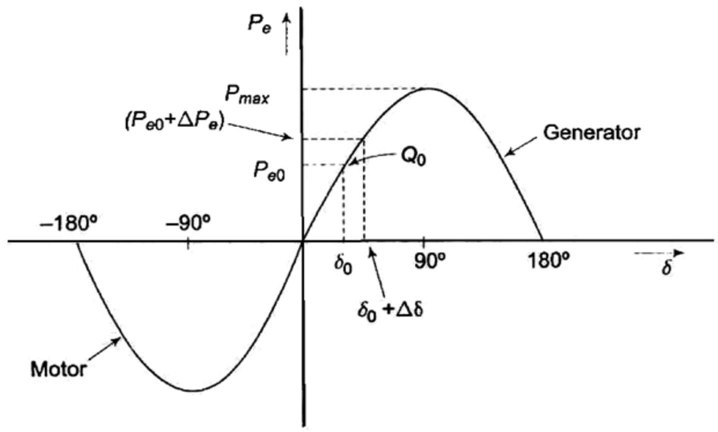


Fig 4.5 Power Angle Curve

This is so because load voltages vary during a stability study (in a load flow study, these remain constant within a narrow band). The simplified power angle equation is

$$P_e = P_{\max} \sin\delta \tag{4.27}$$

Where

$$P_{\max} = \frac{|E_1'||E_2'|}{X} \tag{4.28}$$

The graphical representation of power angle equation (4.28) is shown in Fig. 4.5. The swing equation (4.27) can now be written as

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{pu} \tag{4.29}$$

It is a non linear second-order differential equation with no damping.

Machine Connected to Infinite Bus

Figure 4.6 is the circuit model of a single machine connected to infinite bus through a line of reactance X_e. In this simple case

$$X'_{t\,ransfer} = X'_d + X_e$$

From Eq (4.30) we get

$$P_e = \frac{|E'||V|}{X_{transfer}}\sin\delta = P_{\max}\sin\delta \tag{4.30}$$

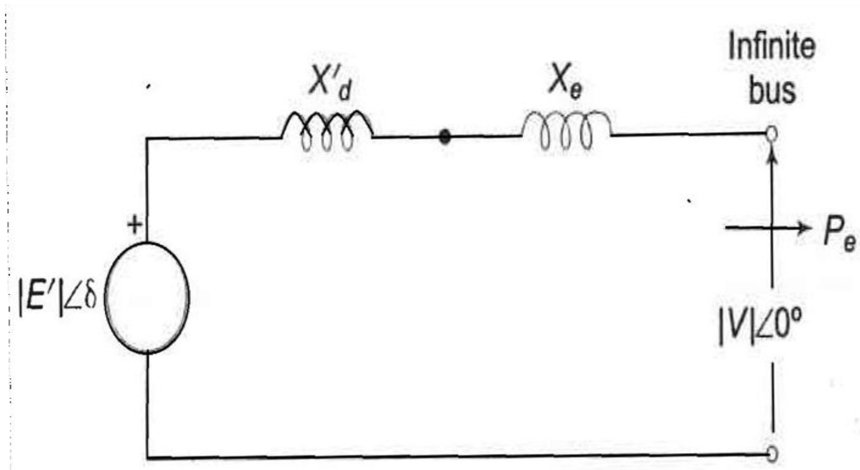


Fig. 4.6 Machine connected to infinite bus bar

The dynamics of this system are described in Eq. (4.15) as

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \tag{4.31}$$

Two Machine Systems

The case of two finite machines connected through a line (X_e) is illustrated in Fig. 5 where one of the machines must be generating and the other must be motoring. Under steady condition, before the system goes into dynamics and the mechanical input/output of the two machines is assumed to remain constant at these values throughout the dynamics (governor action assumed slow).During steady state or in dynamic condition, the electrical power output of the generator must be absorbed by the motor (network being lossless).

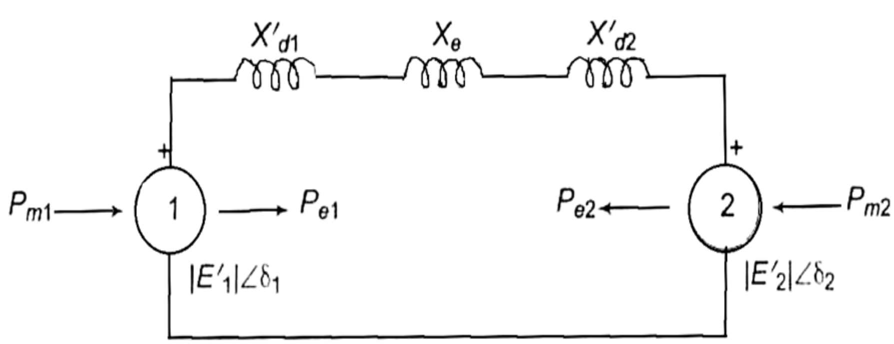


Fig. 4.7 Two machine system

Thus at all time

$$P_{m1} = -P_{m2} = P_m \qquad (4.32)$$

$$P_{e1} = -P_{em2} = P_e \qquad (4.33)$$

The swing equations for the two machines can now be written as

$$\frac{d^2\delta_1}{dt^2} = \pi f\left(\frac{P_{m1} - P_{e1}}{H_1}\right) = \pi f\left(\frac{P_m - P_e}{H_1}\right) \qquad (4.34)$$

And $$\frac{d^2\delta_2}{dt^2} = \pi f\left(\frac{P_{m2} - P_{e2}}{H_1}\right) = \pi f\left(\frac{P_e - P_m}{H_1}\right) \qquad (4.35)$$

Subtracting Eq. (36) from Eq. (35)

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \pi f\left(\frac{H_1 + H_2}{H_1 H_2}\right)(P_m - P_e) \qquad (4.36)$$

Or $$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad (4.37)$$

Where $\delta = \delta_1 - \delta_2$

$$H_{eq} = \left(\frac{H_1 H_2}{H_1 + H_2}\right) \qquad (4.38)$$

The electrical power interchange is given by expression.

$$P_e = \frac{|E_1'||E_2'|}{X_{d1}' + X_e + X_{d2}'}\sin\delta \qquad (4.39)$$

The swing equation Eq. (4.35) and the power angle equation Eq. (4.39) have the same form as for a single machine connected to infinite bus. Thus a two-machine system is equivalent to a single machine connected to infinite bus. Because of this, the single-machine (connected to infinite bus) system would be studied here.

Steady State Stability

The steady state stability limit of a particular circuit of a power system is defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Consider the simple system of Fig. 4.7 whose dynamics is described by equations

$$M\frac{d^2\delta_e}{dt^2} = P_m - P_e \, MW \tag{4.40}$$

$$M = \frac{H}{\pi f} \text{ in pu system} \tag{4.41}$$

And,

$$P_e = \frac{|E||V|}{X_d}\sin\delta = P_{\max}\sin\delta \tag{4.42}$$

For determination of steady state stability, the direct axis reactance (X_d) and, voltage behind X_d are used in the above equations. Let the system be operating with steady power transfer of $P_{e0}=P_m$ with torque angle δ_0 as indicated in the figure. Assume a small increment ΔP in the electric power with the input from the prime mover remaining fixed at P_m (governor response is slow compared to the speed of energy dynamics), causing the torque angle to change to $(\delta_0 + \Delta\delta)$. Linearizing about the operating point Q_0 (P_{e0}, δ_0) we can written as.

$$\Delta P_e = \left(\frac{\partial P_e}{\partial \delta}\right)_0 \Delta\delta$$

The excursions of $\Delta\delta$ are then described by

$$M\frac{d^2\Delta\delta}{dt^2} = P_m - (P_{e0} + \Delta P_e) = -\Delta P_e$$

or

$$M\frac{d^2\Delta\delta}{dt^2} + \left[\frac{\partial P_e}{\partial\delta}\right]_0 \Delta\delta = 0 \qquad (4.43)$$

or

$$Mp^2 + \left(\frac{\partial P_e}{\partial\delta}\right)_0 \Delta\delta = 0$$

Where

$$p = \frac{d}{dt}$$

The system stability to small change is determined from the characteristic equation.

$$Mp^2 + \left[\frac{\partial p_e}{\partial\delta}\right]_0 = 0$$

Its two roots are

$$p = \pm\left[\frac{-\partial p_e / \partial\delta}{M}\right]^{\frac{1}{2}}$$

As long as $(\partial p_e / \partial\delta)_0$ it positive, the roots are purely imaginary and conjugate and the system behaviour is oscillatory about δ_0 . Line resistance and damper windings of machine, which have been ignored in the above modelling, cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as

$$\left(\partial p_e / \partial\delta\right)_0 > 0 \qquad (4.44)$$

When $(\partial p_e / \partial\delta)_0$, is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment (disturbance) and the synchronism is soon lost. The system is therefore unstable for

$$\left(\partial p_e \middle/ \partial\delta\right)_0 < 0 \tag{4.45}$$

$\left(\partial p_e / \partial\delta\right)_0$ is known as synchronizing coefficient. This is also called stiffness (electrical) of synchronous machine.

Assuming $|E|$ and $|V|$ to remain constant, the system is unstable, if

$$\frac{|E||V|}{X}\cos\delta_0 < 0$$

$$\delta_0 > 90^\circ \tag{4.46}$$

The maximum power that can be transmitted without loss of stability (steady state) occurs for

$$\delta_0 = 90^\circ \tag{4.47}$$

$$P_{\max} = \frac{|E||V|}{X} \tag{4.48}$$

If the system is operating below the limit of steady stability condition (Eq.4.48), it may continue to oscillate for a long time if the damping is low. Persistent oscillations are a threat to system security. The study of system damping is the study of dynamical stability.

The above procedure is also applicable for complex systems wherein governor action and excitation control are also accounted for. The describing differential equation is linerized about the operating point. Condition for steady state stability is then determined from the corresponding characteristic equation (which now is of order higher than two).

It was assumed in the above account that the internal machine voltage $|E|$ remains constant (i.e., excitation is held constant). The result is that as loading increases, the terminal voltage $|V_t|$ dips heavily which cannot be tolerated in practice. Therefore, we must consider the steady state stability limit by assuming that excitation is adjusted for every load increase to keep $|V_t|$ constant. This is how the system will be operated practically. It may be understood that we are still not considering the effect of automatic excitation control.

Some Comment on Steady State Stability

Knowledge of steady state stability limit is important for various reasons. A system can be operated above its transient stability limit but not above its steady state limit. Now, with increased fault clearing speeds, it is possible to make the transient limit closely approach the steady state limit.

As is clear from Eq. (4.50), the methods of improving steady state stability limit of a system are to reduce *X* and increase either or both |*E*| and |*V*|. If the transmission lines are of sufficiently high reactance, the stability limit can be raised by using two parallel lines which incidentally also increases the reliability of the system. Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the stability limit by decreasing the line reactance. Higher excitation voltages and quick excitation system are also employed to improve the stability limit.

<u>Transient Stability</u>

The dynamics of a single synchronous machine connected to infinite bus bars is governed by the nonlinear differential equation

$$\begin{aligned}
&M\frac{d^2\delta}{dt^2} = P_m - P_e \\
&where \\
&P_e = P_{\max}\sin\delta \\
&or \\
&M\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta
\end{aligned} \qquad (4.49)$$

As said earlier, this equation is known as the swing equation. No closed form solution exists for swing equation except for the simple case $P_m = 0$ (not a practical case) which involves elliptical integrals. For small disturbance (say, gradual loading), the equation can be linearised leading to the concept of steady state stability where a unique criterion of stability $\left(\partial p_e / \partial\delta > 0\right)$ could be established. No generalized criteria are available for determining system stability with large disturbances (called transient stability). The practical approach to the transient stability problem is therefore to list all important severe disturbances along with their possible locations to which the system is likely to be subjected according to the experience and judgement of the power system analyst. Numerical solution of the swing equation (or equations for a multi-machine case) is then obtained in the presence of such disturbances giving a plot of δ *Vs t* called the swing curve. If δ starts to decrease after reaching a maximum value, it is normally assumed that the system is stable and the oscillation of δ around the equilibrium point will decay and finally die out. As already pointed out in the introduction, important severe disturbances are a short circuit or a sudden loss of load.

For ease of analysis certain assumptions and simplifications are always made (some of these have already been made in arriving at the swing equation (Eq. 4.49). All the assumptions are listed, below along with their justification and consequences upon accuracy of results.

1. Transmission line as well as synchronous machine resistance is ignored. This leads to pessimistic result as resistance introduces damping term in the swing equation which helps stability.
2. Damping term contributed by synchronous machine damper windings is ignored. This also leads to pessimistic results for the transient stability limit.
3. Rotor speed is assumed to be synchronous. In fact it varies insignificantly during the course of the stability transient.
4. Mechanical input to machine is assumed to remain constant during the transient, i.e., regulating action of the generator loop is ignored. This leads to pessimistic results.
5. Voltage behind transient reactance is assumed to remain constant, i.e., action of voltage regulating loop is ignored. It also leads to pessimistic results.
6. Shunt capacitances are not difficult to account for in a stability study. Where ignored, no greatly significant error is caused.
7. Loads are modelled as constant admittances. This is a reasonably accurate representation.

Note: Since rotor speed and hence frequency vary insignificantly, the network parameters remain fixed during a stability study.

A digital computer programme to compute the transient following sudden disturbance can be suitably modified to include the effect of governor action and excitation control.

Preset day power system are so large that even after lumping of machines (Eq.(24)), the system remains a multi-machine one. Even then, a simple two machine system greatly aids the understanding of the transient stability problem. It has been shown in that an equivalent single machine infinite bus system can be found for a two- machine system (Eq. 4.45) to (Eq. 4.49)

Upon occurrence of a severe disturbance, say a short circuit, the power transfer between machines is greatly reduced, causing the machine torque angles to swing relatively. The circuit breakers near the fault disconnect the unhealthy part of the system so that power transfer can be partly restored, improving the chances of the system remain stable. The shorter the time to breaker operating, called *clearing time*, the higher is the probability of the system being stable. Most of the line faults are transient in nature and get cleared on opening the line. Therefore, it is common practice now to employ *auto-reclose breakers* which automatically close rapidly after each of the two sequential openings. If the fault still persists, the circuit breakers open and lock permanently till cleared manually. Since in the majority of faults the first *reclosure* will be successful, the chances of system stability are greatly enhanced by using *autoreclose* breakers.

The procedure of determining the stability of a system upon occurrence of a disturbance followed by various switching off and switching on action called a *stability study*. Steps to be followed in stability study are outlined below for single- machine infinite bus bar system shown in fig. 6. The fault is assumed to be transient one which is cleared by the time of first reclosure. In the case of a permanent fault, this system completely falls apart. This will not be the case in a multi-machine system. The steps listed, in fact, apply to a system of any size.

1. From prefault loading, determine the voltage behind transient reactance and the torque angle δ_0 of the machine with reference to the infinite bus.
2. For the specified fault, determine the power transfer equation $P_e(\delta)$ during fault. In this system $P_e = 0$ for a three-phase fault.
3. From the swing equation starting with δ_0 as obtained in step 1, calculate δ as a function of time using a numerical technique of solving the nonlinear differential equation.
4. After clearance of the fault, once again determine $P_e(\delta)$ and solve further for $\delta(t)$. In this case, $P_e(\delta) = 0$ as when the fault is cleared, the system gets disconnected.
5. After the transmission line is switched on, again find $P_e(\delta)$ and continue to calculate $\delta(t)$.

6. If $\delta(t)$ goes through a maximum value and starts to reduce, the system is regarded as stable. It is unstable if $\delta(t)$ continues to increase. Calculation is increased after a suitable length of time.

Equal Area Criteria for Stability

In a system where one machine is swinging with respect to an infinite bus, it is possible to study transient stability by means of a simple criterion, without resorting to the numerical solution of a swing equation.

Consider the equation

$$M\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{4.50}$$

P_a =accelerating power

lf the system is unstable δ continues to increase indefinitely with time and the machine loses synchronism. On the other hand, if the system is stable, $\delta(t)$ performs oscillations (nonsinusoidal) whose amplitude decreases in actual practice because of damping terms (not included in the swing equation).These two situations are shown in fig. 6. Since the system is no-linear, the nature of its response1 [$\delta(t)$] is not unique and it may exhibit instability in a fashion different from that indicated in Fig. 6, depending upon the nature and severity of disturbance. However, experience indicates that the response $\delta(t)$ in a power system generally falls in the two broad categories as shown in the figure. It can easily be visualized now (this has also been stated earlier) that for a stables system, indication of stability will be given by observation of the first swing where δ will go to a maximum and will start to reduce.

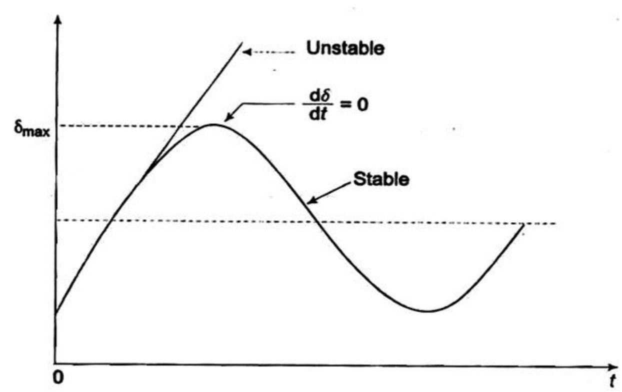


Fig. 4.8 Plot of δ vs t for stable and unstable system.

This fact can be stated as a stability criterion, that the system is stable if at some time

$$\frac{d\delta}{dt} = 0 \tag{4.51}$$

And is unstable, if

$$\frac{d\delta}{dt} > 0 \tag{4.52}$$

The stability criterion for power systems stated above can be converted into a simple and easily applicable form for a single machine infinite bus system. Multiplying both sides of the swing equation by $\left(2\frac{d\delta}{dt}\right)$, we get

$$2\frac{d\delta}{dt} \bullet \frac{d^2\delta}{dt^2} = 2\frac{P_a}{M}\frac{d\delta}{dt}$$

Integrating, both sides we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta$$

$$or$$

$$\frac{d\delta}{dt} = \left(\frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta\right)^{\frac{1}{2}} \tag{4.53}$$

Where δ_0 is the initial rotor angle and it begins to swing due to disturbances in the system. From Eqs. (4.53) and (4.54), the condition for stability can be written as

$$\left(\frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta\right)^{\frac{1}{2}} = 0$$

$$or \tag{4.54}$$

$$\int_{\delta_0}^{\delta} P_a d\delta = 0$$

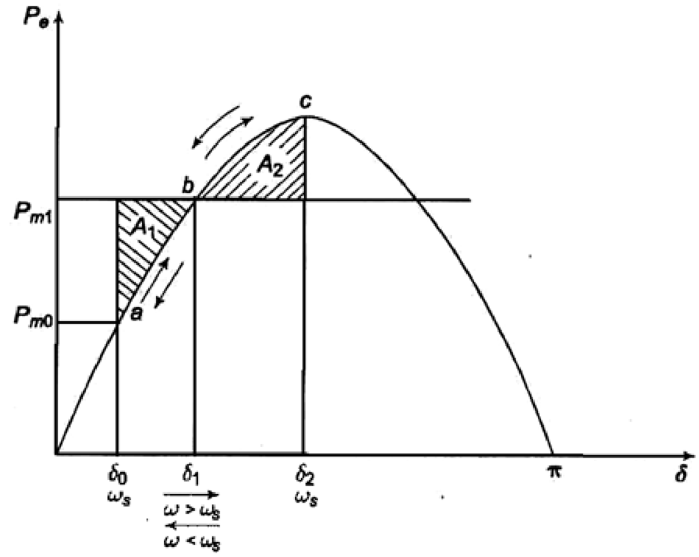


Fig.4.9 P_e- δ diagram for sudden increase in mechanical input

The condition of stability can therefore be stated as: the system is stable if the area under P_a (accelerating power) - δ curve reduces to zero at some value of δ . In other words, the positive (accelerating) area under P_a - δ curve must equal the negative (decelerating) area and hence the name „equal area" criterion of stability. To illustrate the equal area criterion of stability, we now consider several types of disturbances that may occur in a single machine infinite bus bar system. Figure 4.9 shows the transient model of a single machine tied to infinite bus-bar. The electrical power transmitted is given by

$$P_e = \frac{|E'||V|}{X^d} \sin\delta = P_{\max} \sin\delta$$

Under steady operating condition

$$P_{m0} = P_{e0} = P_{\max} \sin \delta_0$$

This is indicated by the point a in the P_e - δ diagram of Fig. 4.8.

Let the mechanical input to the rotor be suddenly increased to P_{m1} (by opening the steam valve). The accelerating power $P_a = P_{m1} - P_e$ causes the rotor speed to increase $(\omega > \omega_s)$ and so does the rotor angle. At angle δ_1, $P_a = P_{m1} - P_e = P_{\max} \sin\delta_1 = 0$ (state point at b) but the rotor angle continues to increase as $(\omega > \omega_s)$ .P_a now becomes negative (decelerating), the rotor speed begins to reduce but the angle continues to increase till at angle $\delta_2, (\omega > \omega_s)$ once again (state point at *c*. At *c*), the-decelerating area A_2 equals the accelerating area A_1, (areas are shaded), i.e,

$$\int_{\delta_0}^{\delta} P_a d\delta = 0$$

Since the rotor is decelerating, the speed reduces below ω_s and the rotor angle begins to reduce. The state point now traverses the $P_e \, Vs\delta$ curve in the opposite direction as indicated by arrows in Fig. 8.It is easily seen that the system oscillates about the new steady state point *b* $(\delta = \delta_1)$ with angle excursion up to δ_0 and δ_2 on the two sides. These oscillations are similar to the simple harmonic motion of an inertia-spring system except that these are not sinusoidal.

As the oscillations decay out because of inherent system damping (not modelled), the system settles to the new steady state where

$$P_{m1} = P_e = P_{\max} \sin \delta_1$$

From Fig. 12.20, areas A_1=A_2 are given by

$$A_1 = \int_{\delta_0}^{\delta_0} (P_{m1} - P_e) d\delta$$

or

$$A_1 = \int_{\delta_0}^{\delta_0} (P_e - P_{m1}) d\delta$$

For the system to be stable, it should be possible to find angle δ_2 such that A_1=A_2. As P_{m1} is increased, a limiting condition is finally reached when A_1 equals the area above the P_{m1} line as shown in Fig 4.10.Under this condition, δ_2 acquires the maximum value such that

$$\delta_2 = \delta_{\max} = \pi - \delta_1 = \pi - \sin^{-1} \frac{P_{m1}}{P_{\max}} \qquad (4.55)$$

Any further increase in P_{m1}, means that the area available for A_2 is less than A_1, so that the excess kinetic energy causes δ to increase beyond point *c* and the decelerating power changes over to accelerating power, with the system consequently becoming unstable.

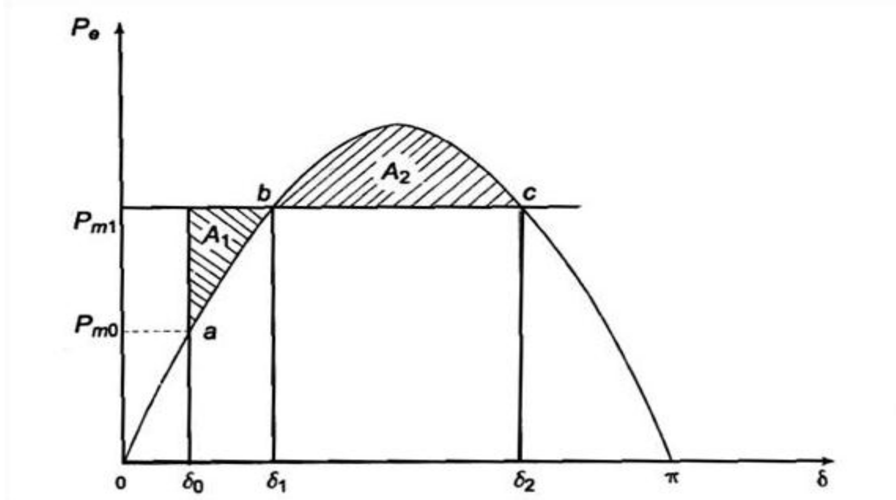


Fig. 4.10 Limiting case of transient stability with mechanical input suddenly increased

It has thus been shown by use of the equal area criterion that there-is an upper limit to sudden increase in mechanical input ($P_{m1} - P_{m0}$), for the system in question to remain stable'

It may be noted from Fig. 9 that the system will remain stable even though the rotor may oscillate beyond $\delta = 90^{\circ}$, so long as the equal area criteria is met. The condition of $\delta = 90^{\circ}$ is meant for use in steady state stability only and does not apply to the transient stability case.

Effect of Clearing Time on Stability

Let the system of Fig. 4.9 be operating with mechanical input P_m at a steady angle of δ (P_m=P_e) as shown by the point *a* on the P_e *Vs* δ diagram of Fig. 4.10. If a 3-phase fault occurs at the point *P* of the outgoing radial line, the electrical output of the generator instantly reduces to zero, i.e., $P_e = 0$ and the state point drops to *b*. The acceleration area A_1 begins to increase and so does the rotor angle while the state point moves along *bc*. At time t_c corresponding to angle δ_c, the faulted line is cleared by the opening of the line circuit breaker. The values of t_c and δ_c are respectively known as *clearing time* and, *clearing angle*. The system once again becomes healthy and transmits $P_e = P_{\max} \sin\delta$ i.e. the state point shifts to *d* on the original P_e *Vs* δ curve. The rotor now decelerates and the decelerating area A_2, begins while the state point moves along *de*. If an angle δ_1 can be found such that A_2=A_1, the system is found to be stable. The system finally settles down to the steady operating point a in an oscillatory manner because of inherent damping.

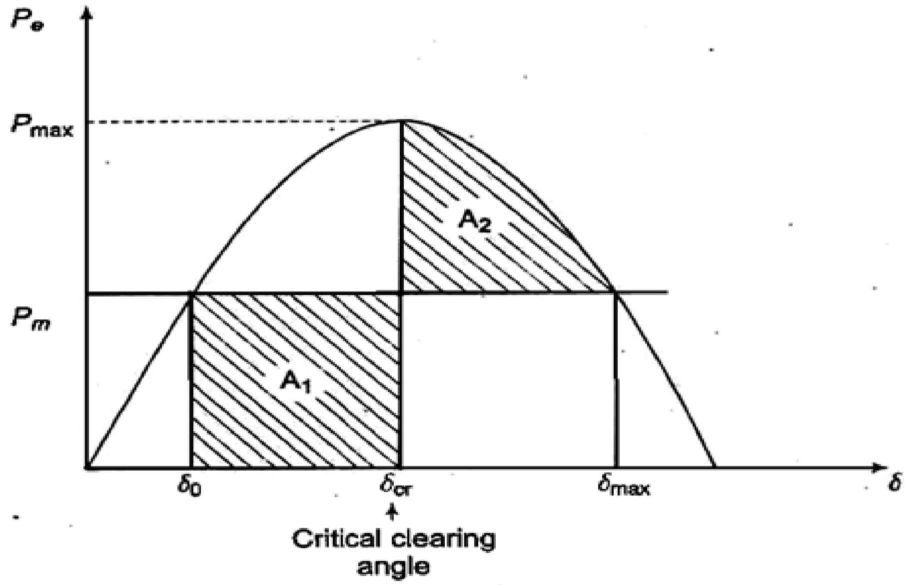


Fig. 4.10 Limiting case of transient stability with critical angle

The value of clearing time corresponding to a clearing angle can be established only by numerical integration except in this simple case. The equal area criterion therefore gives only qualitative answer to system stability as the time when the breaker should be opened is hard to establish.

As the clearing of the faulty line is delayed, A_1 increases and so does δ_1 , to find A_2=A_1 till $\delta_1 = \delta_{max}$ as shown in Fig. 4.10. For a clearing time (or angle) larger than this value, the system would be unstable as A_2<A. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as *critical clearing time and angle.*

For this simple case (P_e=0 during fault), explicit relationships for δ_c (critical) and t_c (critical) are established below. All angles are in radians.

It is easily seen from Fig.4.10

$$\delta_{max} = \pi - \delta_0 \tag{4.56}$$

and

$$P_m = P_{max} \sin\delta_0 \tag{4.57}$$

Now

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)d\delta = P_m(\delta_{cr} - \delta_0)$$

and

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max}\sin\delta - P_m)d\delta$$

$$= P_{max}(\cos\delta_{cr} - \cos\delta_m) - P_m(\delta_{max} - \delta_{cr})$$

For the system to be stable, $A_2 = A_1$ which gives

$$\cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max} \tag{4.58}$$

Where

δ_{cr} =critical clearing angle.

Substituting Eq. (58) and (59) in Eq.(60), we get

$$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0] \tag{4.59}$$

During the period the fault is persisting, the swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m; \qquad \text{where } P_e = 0 \tag{4.60}$$

Integrating twice

$$\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0$$

$$\text{Or} \quad \delta_{cr} = \frac{\pi f}{2H} P_m t^2_{cr} + \delta_0 \tag{4.61}$$

Where

t_{cr} =critical clearing time.

δ_{cr} =critical clearing angle

From Eq. (4.61)

$$\delta_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi . f . P_m}} \tag{4.62}$$

Whereδ_{cr} , is given by the expression of Eq. (4.62)

An explicit relationship for determining t_{cr} is possible in this case as during the faulted condition P_e =0 and so the swing equation can be integrated in closed form. This will not be the case in most other situations.

Consider now a single machine tied to infinite bus through two parallel lines as in Fig. 4.11a circuit model of the system is given in Fig. 4.11b.

Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady road. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X'_d + X_1 X_2} \sin\delta = P_{\max I} \sin\delta$$

Immediately on switching off line 2, power angle curve is given by

$$P_{eII} = \frac{|E'||V|}{X'_d + X_1 X_2} \sin\delta = P_{\max II} \sin\delta$$

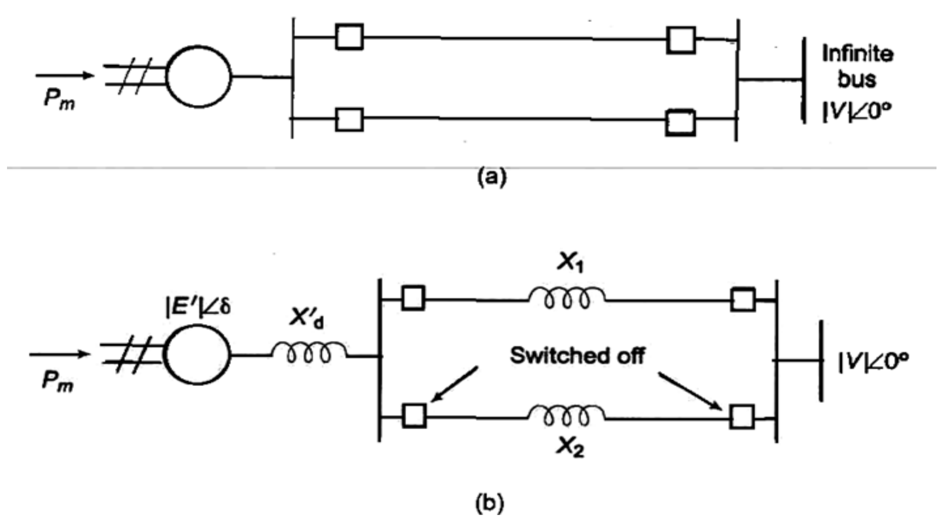


Fig. 4.11 Single machine tied to infinite bus through two parallel lines

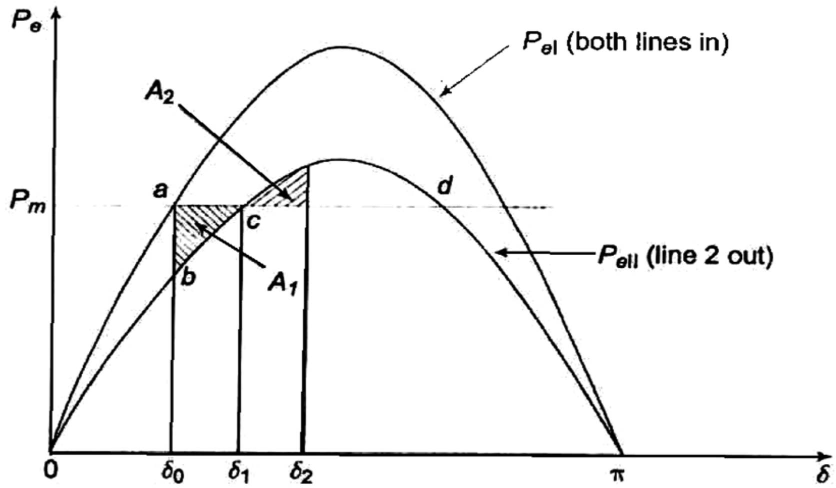


Fig. 4.12 Equal area criterion applied to the opening of one of the two lines in parallel

Both these curves are plotted in Fig. 4.12, wherein $P_{maxII} < P_{maxI}$ as $(X'_d + X_1) > (X'_d + X_1 \| X_2)$ .The system is operating initially with a steady power transfer P_e=P_m at a torque angle δ_0 on curve I. Immediately on switching off line 2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy for $\delta_1 > \delta_0$. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that A_1 = A_2, the system will be stable and will finally operate at *c* corresponding to a new, rotor angle $\delta_1 > \delta_1$. This is so because a single line offers larger reactance and larger rotor angle is needed to transfer the same steady power.

It is also easy to see that if the steady load is increased (line P_m is shifted upward in Fig. 4.12, a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one, For the limiting case of stability, δ_1 has maximum value given by

$$\delta_1 = \delta_{\max} = \pi - \delta_c$$

This is the same condition as in the previous example.

We shall assume the fault to be a three-phase one. Before the occurrence of a fault, the power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X'_d + X_1 X_2} \sin\delta = P_{\max I} \sin\delta$$

This is plotted in fig. 16

Upon occurrence of a three-phase fault at the generator end of line 2 (see Fig. 15a), the generator gets isolated from the power system for purposes of power flow as shown by Fig. 15b. Thus during the period the fault lasts,

$P_{eII}=0$

The rotor therefore accelerates and angle δ increases. Synchronism will be lost unless the fault is cleared in time.

The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line.

The power flow is now restored via the healthy line (through higher line reactance X_2 in place of $X_l \parallel X_2$), with power angle curve

$$P_{eII} = \frac{|E'||V|}{X'_d + X_1 X_2} \sin\delta = P_{\max II} \sin\delta$$

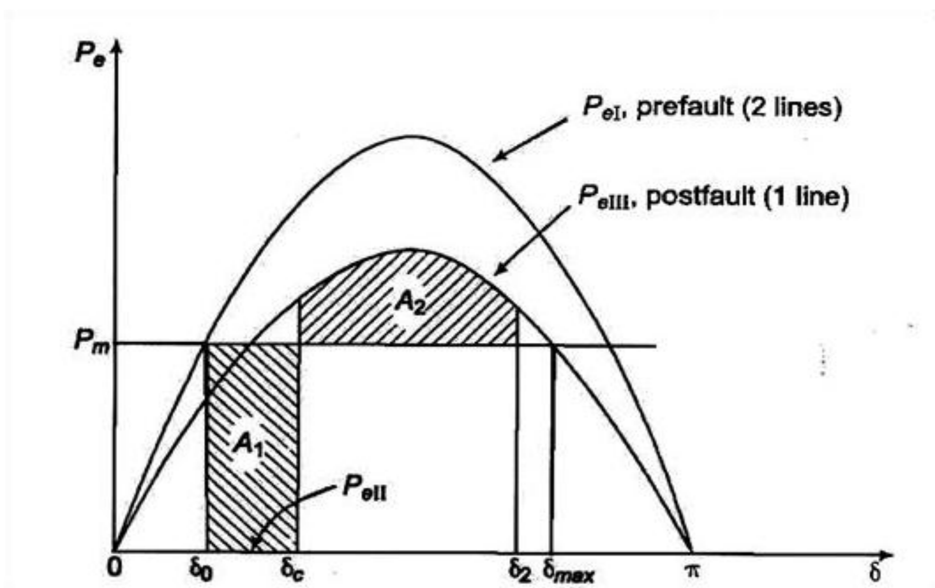


Fig. 4.13 Equal area criteria applied to the system, I system is normal, II fault applied, III faulted line isolated.

Obviously, $P_{maxII} < P_{maxI}$. The rotor now starts to decelerate as shown in Fig. 4.13. The system will be stable if a decelerating area A_2 can be found equal to accelerating area A_1 before δ reaches the maximum allowable value $\delta_{\max}$ .As area A_1 depends upon clearing time t_c (corresponding to clearing angle δ_c), clearing time must be less than a certain value (critical clearing time) for the system to be stable. It is to be observed that the equal area criterion helps to determine critical clearing angle and not critical clearing time. Critical clearing time can be obtained by numerical solution of the swing equation

It also easily follows that larger initial loading (P_m.) increases A_1 for a given clearing angle (and time) and therefore quicker fault clearing would be needed to maintain stable operation. The power angle curve during fault is therefore given by

$$P_{eII} = \frac{|E'||V|}{X_{II}} \sin\delta = P_{\max II} \sin\delta$$

P_{eI} , P_{eIII} and P_{eII} as obtained above are all plotted in Fig.4.14. Accelerating area A_1 corresponding to a given clearing angle δ is less in this case then in case *a* giving a better chance for stable operation. Stable system operation is shown in Fig. 4.14, wherein it is possible to find an area A_2 equal to A_1 for $\delta_2 < \delta_{\max}$. As the clearing angle δ_c is increased, area A_1 increases and to find $A_2 = A_1$, δ_2 increases till it has a value $\delta_{\max}$, the maximum allowable for stability This case of critical clearing angle is shown in Fig. 4.15

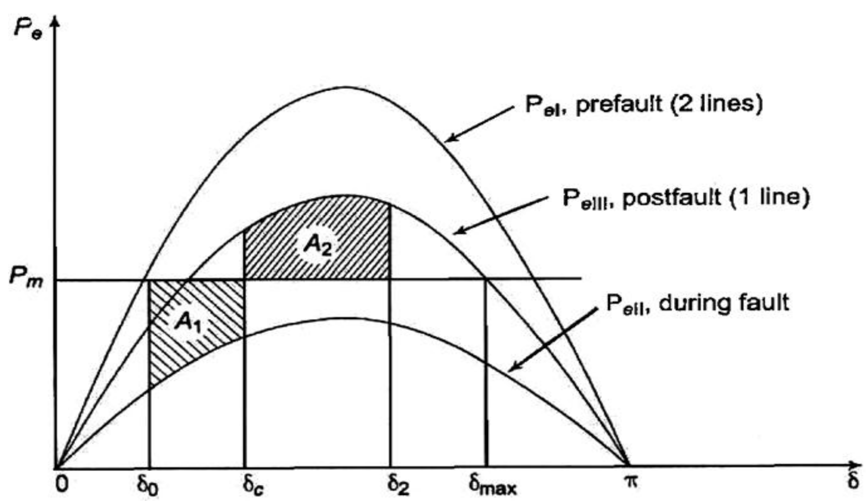


Fig. 4.14 Fault on middle of one line of the system with $\delta_c < \delta_{cr}$

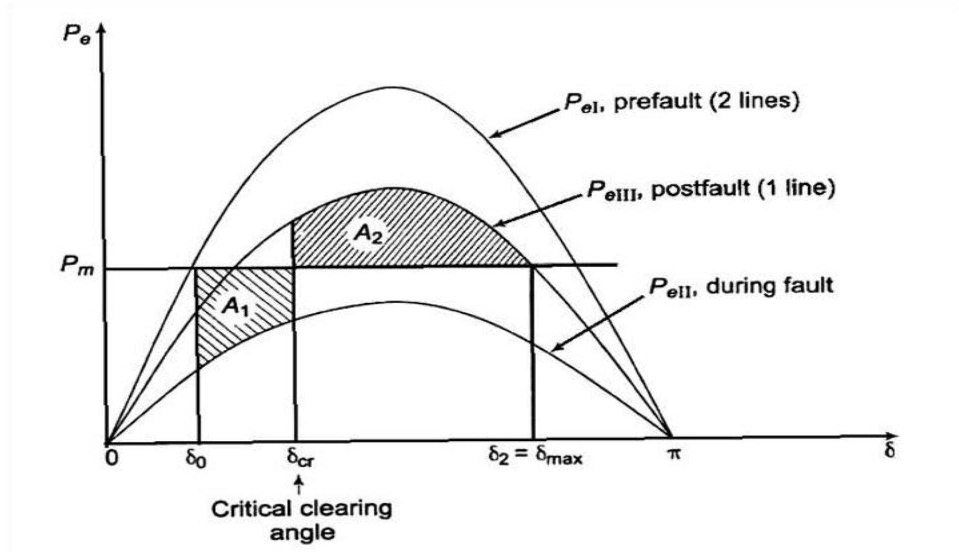


Fig.4.15 Fault on middle of one line of the system of, case of critical clearing angle

Applying equal area criterion to the case of critical clearing angle of Fig. 4.15 we can write

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{\max II} \sin\delta) d\delta = \int_{\delta_{cr}}^{\delta_{\max}} (P_{\max III} \sin\delta - P_m) d\delta$$

where

$$\delta_{\max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{\max III}}\right) \tag{4.63}$$

Integrating, we get

$$\left(P_m \delta + P_{\max II} \cos\delta\right)\Big|_{\delta_0}^{\delta_{cr}} + (P_{\max III} \cos\delta - P_m \delta)\Big|_{\delta_{cr}}^{\delta_{\max}} = 0$$

or

$$P_m\left(\delta_{cr} - \delta_0\right) + P_{\max II} (\cos\delta_{cr} + \cos\delta_0)$$
$$+ P_m\left(\delta_{\max} - \delta_{cr}\right) + P_{\max III} (\cos\delta_{\max} + \cos\delta_{cr}) = 0$$

or

$$\cos\delta_{cr} = \frac{P_m\left(\delta_{\max} - \delta_0\right) - P_{\max II} \cos\delta_0 + P_{\max III} \cos\delta_{\max}}{P_{\max III} - P_{\max II}} \tag{4.64}$$

Critical clearing angle can be calculated from Eq. (4.64) above. The angles in the equation are in radians. The equation modifies as below if the angle are in degrees.

$$\cos\delta_{cr} = \frac{\frac{\pi}{180} P_m\left(\delta_{\max} - \delta_0\right) - P_{\max II} \cos\delta_0 + P_{\max III} \cos\delta_{\max}}{P_{\max III} - P_{\max II}}$$

If the circuit breakers of line 2 are reclosed successfully (i.e., the fault was a transient one and therefore vanished on clearing the faulty line), the power transfer once again becomes

$$P_{eIV} = P_{eI} = P_{\max I} \sin\delta$$

Since reclosure restores power transfer, the chances of stable operation improve. A case of stable operation is indicated by Fig. 4.16. For critical clearing angle

$$\delta_1 = \delta_{\max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{\max I}}\right)$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{\max II} \sin\delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{rc}} (P_{\max III} \sin\delta - P_m)\, d\delta + \int_{\delta_{rc}}^{\delta_{\max}} (P_{\max I} \sin\delta - P_m)\, d\delta$$

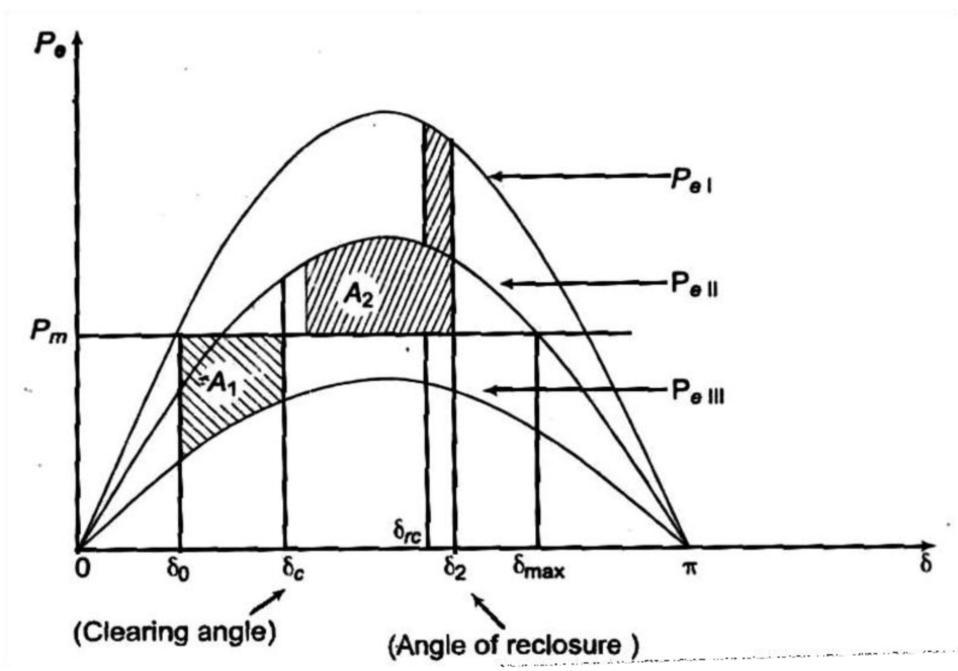


Fig. 4.16 fault in middle of a line of the system

Point To Point Method of Improvement of Transient Stability

In most practical systems, after machine lumping has been done, there are still more than two machines to be considered from the point of view of system stability. Therefore, there is no choice but to solve the swing equation of each machine by a numerical technique on the digital computer. Even in the case of a single machine tied to infinite bus bar, the critical clearing time cannot be obtained from equal area criterion and we have to make this calculation numerically through swing equation. There are several sophisticated methods now available for the solution of the swing equation including the powerful Runge-Kutta method. Here we shall treat the point-by-point method of solution which is a conventional, approximate method like all numerical methods but a well tried and proven one. We shall illustrate the point-by-point method for one machine tied to infinite bus bar. The procedure is, however, general and can be applied to every machine of a multi-machine system. Consider the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_{max}\sin\delta) = \frac{P_a}{M}$$

$$M = \frac{GH}{\pi}$$

Or in p.u $M = \dfrac{H}{\pi f}$

The solution $\delta(t)$ is obtained at discrete intervals of time with interval spread of At uniform throughout. Accelerating power and change in speed which are continuous functions of time are discretized as below:

1. The accelerating power P_a computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered as shown in Fig. 4.17.

2.The angular rotor velocity $\omega = d\delta/dt$ (over and above synchronous velocity ω_s) is assumed constant throughout any interval, at the value computed for the middle of the interval as shown in fig . 4.17

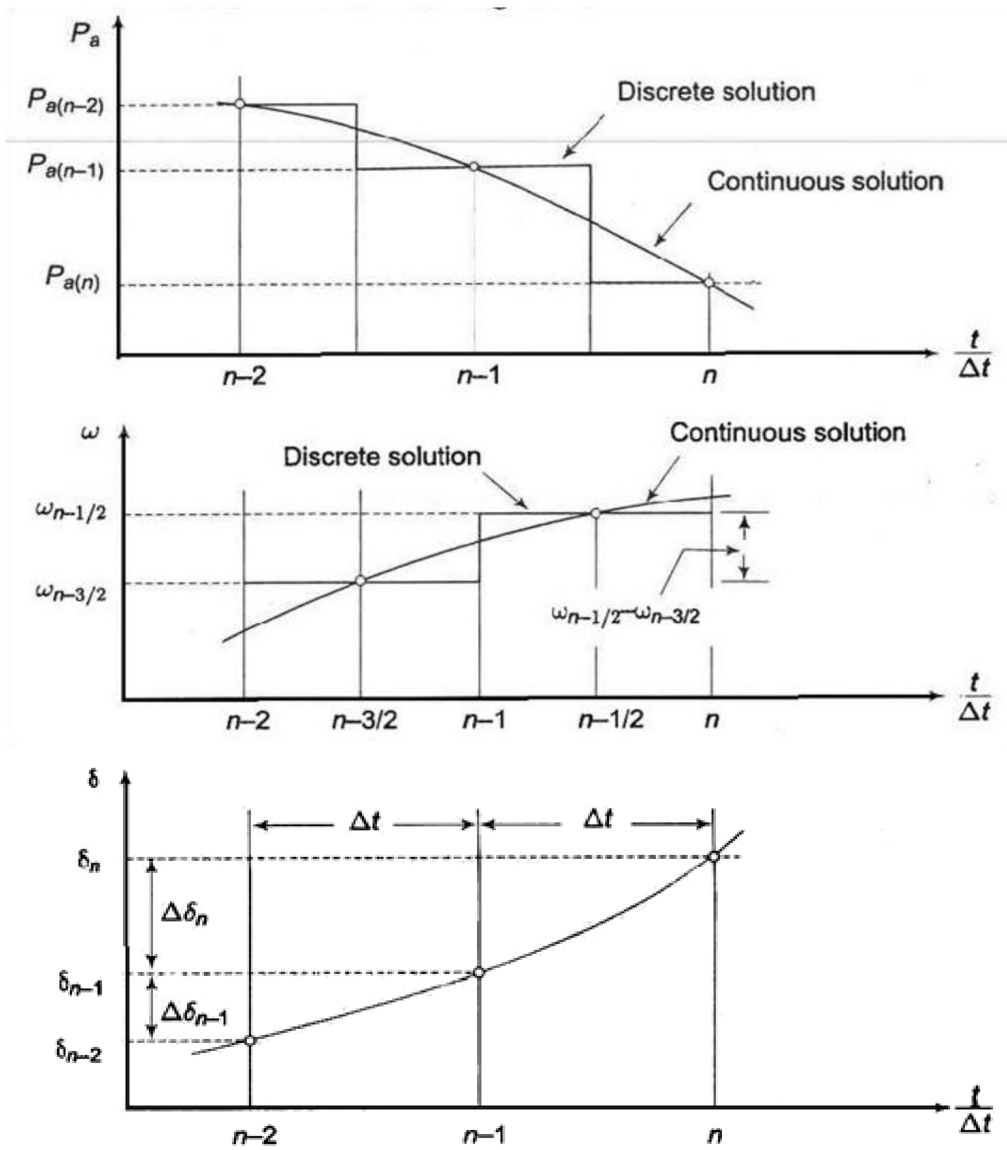


Fig. 4.17 Point-by-point solution of swing equation

In Fig.4.17, the numbering on t/Δt axis pertains to the end of intervals At the end of the *(n-1)th* interval, the acceleration power is

$$P_{a(n-1)} = P_m - P_{\max} \sin\delta_{n-1} \tag{4.65}$$

Where δ_{n-1} has been previously calculated. The change in velocity *(ω=dδ/dt),* caused by the $P_{(n-1)}$, assumed constant over Δt from (*n-3/2*) to (*n-1/2*) is

$$\omega_{n-1/2} - \omega_{n-3/2} = (\Delta t / M) P_{a(n-1)} \tag{4.66}$$

The change in δ during the (n-1)th interval is

$$\Delta\delta_n = \delta_{n-1} - \delta_{n-2} = \Delta t\omega_{n-3/2} \tag{4.67}$$

And during the nth interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t\omega_{n-1/2} \tag{4.68}$$

Subtracting Eq. (4.67) from Eq. (4.68) and using Eq. (4.65), we get

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \tag{4.69}$$

Using this, we can write

$$\delta_n = \delta_{n-1} + \Delta\delta_n \tag{4.70}$$

The process of computation is now repeated to obtain $P_{a(n)}$,$\Delta\delta_{n+1}$.and $\delta_{n+1.}$ The time solution in discrete form is thus carried out over the desired length of time, normally 0.5 s. Continuous form of solution is obtained by drawing a smooth curve through discrete values as shown in Fig. 4.17. Greater accuracy of solution can be achieved by reducing the time duration of intervals.

The occurrence or removal of a fault or initiation of any switching event causes a discontinuity in accelerating power P_a. lf such a discontinuity occurs at the beginning of an interval, then the average of the values of P_a before and after the discontinuity must be used. Thus, in computing the increment of angle occurring during the first interval after a fault is applied at t = 0, Eq. (4.69) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} + \frac{P_{a0+}}{2}$$

Where (P_{a0+}) accelerating power after fault. Immediately before the fault the system is in steady state, so that $P_{a0}^{-} = 0$ and δ_0 is a known value. If the fault is cleared at the beginning of the *n*th interval, in calculation for this interval one should use for $P_{a(n-1)}$ the value ½[$P_{a(n-1)}^{-}$ + $P_{a(n-1)}^{+}$], where $P_{a(n-1)}^{-}$ is the accelerating power immediately before clearing and $P_{a(n-1)}^{+}$ is that immediately after clearing the fault. If the discontinuity occurs at the middle of an interval, no special procedure is needed. The increment of angle during such an interval is calculated, as usual, from the value of P_a at the beginning of the interval.