Module - 04

Probability

We generally use the words probably, or probable or changes or likely or most probably, or perchance or astrologist or almost etc. These word convey the sense that the event is not certain to take place or in other words.

There are three type of approaches to theory probability, namely.

① Experimental or Empirical or observed frequency approach.

② Classical approach

③ Axiomatic approach

① Experimental or Empirical approach to probability

These are two type of experimental approach to probability.

① Deterministic

② Random or probabilistic

① Deterministic :— Deterministic experiment are those experiments which when re-peated under identical conditions produce the same result or outcome

Example:⇒ If we marked head (H) on both sides of a coin and it is tossed, then we always get the same outcome ~~Observing that~~

② Random or probabilistic :— If an experiment, when repeated under identical conditions do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes then it is known as a random or probabilistic experiment

Example:⇒ The tossing of a coin one is not sure of a head (H) or tail (T) will be obtained. So it is random experiment.

Trial :⇒ when we perform an experiment it is called a trial of the experiment.

For Example:— If a coin is tossed 10 time, then each toss is called a trial. If a die is thrown 5 times then each throw is called a trial.

Elementary event :- An outcome of a test is an elementary event.

(1) In tossing die a number which is multiple of 3 getting

(2) an even number (3) a multiple of 3

(4) an even number and a multiple of 3

(5) an even number and a multiple of 5

(6) A number less than 7 in one throw

(7) A number less than 7 in a single throw

Ans :- (6) or (7) a number between 1 and 6

(8) In a single throw of die we can get any one of the six numbers 1, 2, 3, 4, 5, 6 and no other. Therefore each of these elementary events is called with the random experiment of throwing a die is 6.

(9) Let A getting either the even number or multiple of 3 we say that A occurs if we obtain any one of 2, 3, 4, 6 as an outcome

Favourable number of elementary events $n(A) = 4$

Hence $P(A) = \frac{4}{6} = \frac{2}{3}$

(10) Let A denote the event getting a even multiple of 3 we say that the event A occurs if we obtain either 6 or 12 as an outcome

Favourable number of elementary event $n(A) = 2$

Hence $P(A) = \frac{2}{6} = \frac{1}{3}$

(11) An even number or a multiple of 3 is obtained if we obtain one of the numbers 2, 3, 4, 6, 12 as an outcome $n(A) = 5$; $n(S) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$

(12) Let A denote getting an even number and a multiple of 3 clearly event A happens if we get 6 or 12 as an outcome

Favourable number of elementary event $n(A) = 1$

Hence required probability $= \frac{n(A)}{n(S)} = \frac{1}{6}$

(13) Let $A = \{1, 2, 3, 4\}$; $n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

(14) $A = \{1, 2, 3, 4, 5\}$ $n(S) = 6$ $n(A) = 5$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$

(15) $A = \{1, 2, 3, 4\}$ $n(A) = 4$ $n(S) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

(16) $A = \{1, 2, 3, 4, 5\}$ $n(A) = 5$ $n(S) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$

(17) Two coins are tossed once or a coin is tossed twice. Find the probability of getting

(i) Two heads (ii) one head

(iii) one tail (iv) at least one head

(v) at most one head (vi) no head

Ans :- Two coins are tossed simultaneously we obtain any one of the following as an outcome

$S = \{(H,H), (H,T), (T,H), (T,T)\}$ $n(S) = 4$

Total number of elementary events $n(S) = 4$

(i) Elementary event only two heads $A = \{(H,H)\}$ $n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$

(ii) Here elementary event only one head $A = \{(H,T), (T,H)\}$

$n(A) = 2$ $n(S) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

(iii) Here elementary event only one tail $A = \{(H,T), (T,H)\}$

$n(A) = 2$ $n(S) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

(4) Here elementary event at least one head
$A = \{HH, HT, TH\}$ $n(A) = 3$, $n(S) = 4$
$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$ Ans

(5) Here elementary event at most one head
$A = \{TT, (TH), (HT)\}$, $n(A) = 3$ $n(S) = 4$
$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$

(6) Here elementary event of no head
$A = \{TT\}$, $n(A) = 1$ $n(S) = 4$
$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$

(3) Three unbiased coins are tossed together. Find the probability of getting
(1) all heads (2) two heads
(3) one heads (4) at least two heads
If we throw three coins then elementary event of random outcome
$S = \{(HHH), (HHT), (HTH), (THH), (TTT), (TTH), (THT), (HTT)\}$, $n(S) = 8$

(1) If A is event of elementary event of all heads outcome $A = \{HHH\}$, $n(A) = 1$
$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

(2) If A is elementary event outcome only two heads Then $A = \{(HHT), (HTH), (THH)\}$, $n(A) = 3$
$n(S) = 8$
$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

(3) If A is elementary event of only one two head at random outcome $A = \{(HTT), (THT), (TTH)\}$, $n(A) = 3$
$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

(4) If A is elementary event of only at least two heads
$A = \{(HH), (HHT), (HTH), (THH)\}$
$n(A) = 4$
$n(S) = 8$ $P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(A) Two dice are thrown simultaneously find the probability of getting
(1) the sum number as the sum 9 The sum is a prime number
(2) A total of the at least 10
(3) a double of same number
(4) A multiple of 2 on one dice and a multiple of 3 on the other
(5) Same number on both dice
(6) A multiple of 2 on the one.

Elementary events associated to the random experiment of throwing two dice are
(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) $n(S) = 36$

(1) $A = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$
Favourable number of elementary events $= 18$
$n(A) = 18$
$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

(2) let A be the sum of prime number then elementary event $A = \{(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)\}$
Favourable numbers of elementary events $= 15$
Hence required probability $P(E) = \frac{15}{36} = \frac{5}{12}$ Ans

(3) let A be the event of getting a total of at least 10 i.e. 10, 11, 12. Then the elementary event
$(5,6), (6,5), (5,5), (6,6), (5,5), (6,6)$ $n(A) = 6$
Favourable event of the outcome of elementary event $n(A) = 6$
$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(12) Let A be the event of even number. Then the favourable events $A = \{(2,2), (4,4), (6,6)\}$

$n(A) = 3$

Hence required probability $= \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(13) $A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$n(A) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$

(14) Find the probability that a leap year selected at random will contain 53 Sunday.

In a leap year are 366 days.

366 days = 52 weeks and 2 days

A leap year contain only 52 Sunday

Rest remaining 2 day can be be

(1) Sunday and Monday (2) Monday and Tuesday

(3) Tuesday and Wednesday (4) Wednesday and Thursday

(5) Thursday and Friday (6) Friday and Saturday

(7) Saturday and Sunday

There are seven elementary events associated with the random experiment

Total no of elementary events = 7

(1) Saturday and Sunday (2) Sunday and Monday

Favourable exact number of elementary events = 2

Hence, required probability $= \frac{2}{7}$

(15) Two dice are thrown together. Find the probability of getting a total of 12 from it.

Since one dice can be thrown in six way to obtain any one of the six number marked on its face. Therefore, if two dice are thrown the total number of elementary events $= 6 \times 6 = 36$

Let A be the event of getting a total of 12 from two dice. Then $A = \{(6,6)\}$

$n(A) = 1$

Hence required probability $= \frac{n(A)}{n(S)} = \frac{1}{36}$

(16) Find the probability that a leap year selected at random will contain 53 Sunday or 53 Monday.

In a leap year are 366 days.

366 days = 52 weeks and 2 days

A leap year contain only 52 Sunday

Rest remaining 2 day can be be

(1) Sunday and Monday (2) Monday and Tuesday

(3) Tuesday and Wednesday (4) Wednesday and Thursday

(5) Thursday and Friday (6) Friday and Saturday

(7) Saturday and Sunday

There are seven elementary events associated with the random experiment

Total no of elementary events = 7

(1) Saturday and Sunday (2) Sunday and Monday

Favourable exact number of elementary events = 2

Hence, required probability $= \frac{2}{7}$

(17) Two dice are thrown together. Find the probability of getting a total of 12 from it.

Since one dice can be thrown in six way to obtain any one of the six number marked on its face. Therefore, if two dice are thrown the total number of elementary events $= 6 \times 6 = 36$

Let A be the event of getting a total of 12 from two dice. Then $A = \{(6,6)\}$

$n(A) = 1$

Hence required probability $= \frac{n(A)}{n(S)} = \frac{1}{36}$

(18) Find the probability that a leap year selected at random will contain 53 Sunday or 53 Monday.

In a leap year are 366 days.

366 days = 52 weeks and 2 days

A leap year contain only 52 Sunday

Rest remaining 2 day can be be

(1) Sunday and Monday (2) Monday and Tuesday

(3) Tuesday and Wednesday (4) Wednesday and Thursday

(5) Thursday and Friday (6) Friday and Saturday

(7) Saturday and Sunday

There are seven elementary events associated with the random experiment

Total no of elementary events = 7

(1) Saturday and Sunday (2) Sunday and Monday

Favourable exact number of elementary events = 2

Hence, required probability $= \frac{2}{7}$

(19) Two dice are thrown together. Find the probability of getting a total of 12 from it.

Since one dice can be thrown in six way to obtain any one of the six number marked on its face. Therefore, if two dice are thrown the total number of elementary events $= 6 \times 6 = 36$

Let A be the event of getting a total of 12 from two dice. Then $A = \{(6,6)\}$

$n(A) = 1$

Hence required probability $= \frac{n(A)}{n(S)} = \frac{1}{36}$

(20) Find the probability that a leap year selected at random will contain 53 Sunday or 53 Monday.

In a leap year are 366 days.

366 days = 52 weeks and 2 days

A leap year contain only 52 Sunday

Rest remaining 2 day can be be

(1) Sunday and Monday (2) Monday and Tuesday

(3) Tuesday and Wednesday (4) Wednesday and Thursday

(5) Thursday and Friday (6) Friday and Saturday

(7) Saturday and Sunday

There are seven elementary events associated with the random experiment

Total no of elementary events = 7

(1) Saturday and Sunday (2) Sunday and Monday

Favourable exact number of elementary events = 2

Hence, required probability $= \frac{2}{7}$

Hence required probability = $\frac{1}{3}$

(2) Head and the number 6 is obtain in only one way i.e. when the outcome is (H, 6)
Favourable number of elementary events = 1
Hence required probability = $\frac{1}{3}$

(3) Head and an even number can be obtained in any one of the following way
(H, 2) (H, 4) (H, 6)
Favourable number of elementary events = 3
Hence required probability = $\frac{3}{3}$.

(11) A letter is chosen at random from the word ASSASSINATION. Find the probability that letter is (1) a vowel (2) a consonant.
There are 13 letters in the word ASSASSINATION out of which there are 6 vowels viz A, A, I, A, I, O and 7 consonants
Total no of way selected number from Assassination = 13 letters $n = 13$
No of ways selecting vowel out of 6 vowels = 6
No of way selecting consonant out of 7 consonant = 7

$P(V) = \frac{6}{13}$, $P(C) = \frac{7}{13}$

(12) In a lottery a person chosen six different natural numbers at random from 1 to 20 and if these six numbers match with the six numbers already fixed by the lottery committee the wins the prize. What is the probability of winning the prize in the game?
Out of 20 numbers six numbers can be chosen in $= {}^{20}C_6$ ways.
Total no of elementary events ${}^{20}C_6 = 38760$

It is given that a persons wins the prize if six selected numbers match with the six numbers already fixed by the committee
∴ Favourable number of ways = 1
Hence required probability = $\frac{1}{38760}$

(13) On her vacation veena visits four cities A, B, C and D in a random order. What is probability that she visits
(1) A before B (2) A before B and B before C
(3) A first and B last (4) A either first or second
(5) A just before B.
Veena can visit four cities A, B, C and D in any one of the following orders.
ABCD, ABDC, ACBD, ADBC, ADCB, ADBC
BACD
BACD, BCDA, BDAC, BDCA, BCAD, BADC
CABD, CADA, CDAB, CABD, CADB, CDBA, CDAB
DABC, DBCA, DCAB, DCBA, DBAC, DACB
Total no of arrangements order veena can visit four cities A, B, C and D is $4! = 24$.

(1) Out of arrangements order veena can visit city A before city B in the following arrangements
ABCD, ABDC, ACBD, ADBC, ADCB, ADBC, CBAD
CBAD, CADB, DABC, DCAB, DACB
So there are 12 way in which veena can visit city A before city B.
Required probability = $\frac{12}{24} = \frac{1}{2}$

(2) Veena can visit A before B and B before C in any one of the following way
ABCD, ABDC, ADBC, ADBC
Required probability = $\frac{4}{24} = \frac{1}{6}$

③ Veena visit city A and city B (at) in any one of the following way.
ACBD, ADCB.
Required probability $= \frac{2}{4!} = \frac{1}{12}$

④ Veena can visit city A either first or second
ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, CABD, CADB, DABC, DACB
Required probability $= \frac{12}{4!} = \frac{1}{2}$

⑤ D just before B Taking AB together A, B, C and D can be arranged in 6 ways = 6
Required probability $\frac{6}{4!} = \frac{1}{4}$

⑪ A die has two faces with number 1 three faces each with number 2 and one face with number 3 If die is rolled once determine.
① P(2) ② P(1 or 3) P(not 3)
Total number of elementary event = 6
① Out of six faced 3 are marked with number 2
$P(2) = \frac{3}{6} = \frac{1}{2}$
② Two faces are marked with number 1 and one face with number 3. Therefore a face marked with 1 or 3 can be chosen in 3 ways.
$\therefore P(1 \text{ or } 3) = \frac{3}{6} = \frac{1}{2}$
③ There is one face marked with number 3
$P(\text{not } 3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$ Ans

⑫ 4-digit numbers greater than or equal to 5000 are randomly formed from the digits 0, 1, 3, 5 and 7 what is the probability of forming number divisible by 5 when
① The digits may be repeated
② The repetition of digits is not allowed

Total number of 4 digit numbers formed from the digits 0, 1, 3, 5 and 7 and greater than or equal to 5000 is
$2 \times 5 \times 5 \times 5 = 250$

A number is divisible by 5 if units digit is 0 or 5.
Therefore, number of 4 digit numbers formed from the digits 0, 1, 3, 5 and 7 divisible by 5 and greater than or equal to 5000 is $2 \times 5 \times 5 \times 2 = 100$

probability of forming a number divisible by $= \frac{100}{250} = \frac{2}{5}$

② If repetition of digits is not allowed, then the total number of 4 digit numbers formed from the digit 0, 1, 3, 5 and 7 is $2 \times 4 \times 3 \times 2 = 48$
Number of 4 digit numbers divisible by 5 having 0 at one's place $= 2 \times 3 \times 2 \times 1 = 12$
Number of 4 digit number divisible by 5 having 5 at one's place $= 1 \times 3 \times 2 \times 1 = 6$
No of 4 digit numbers with distinct digit and divisible by $5 = 12 + 6 = 18$

⑬ One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is
① an ace ② red ③ either red or king
④ red and a king
Out of 52 cards one card can be drawn in $52C_1 = 52$.
Total number of elementary events $= 52C_1 = 52$
① There are four aces in a pack of 52 cards. Out of which one ace can be drawn in $4C_1 = 4$ way.
Favourable number of elementary events $= 4C_1 = 4$
So required probability $= \frac{4}{52} = \frac{1}{13}$
② The pack of cards contain 26 red cards. In which one red card drawn then $26C_1 = 26$ ways.
Favourable number of elementary event $= 26C_1 = 26$
so required probability $= \frac{26}{52} = \frac{1}{2}$

(11) There are 26 red cards including 2 red kings and there are 2 more kings. Therefore total number of cards = 26+2 = 28

One can be drawn in $^{28}C_1 = 28$ way.

Favourable number of elementary event = 28

So required probability = $\frac{28}{364} = \frac{1}{13}$

(12) According to question a pack of cards contains red and king there are only 2 card.

The favourable event = $^2C_1 = 2$

So required probability = $\frac{2}{52} = \frac{1}{26}$

(13) An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random find the probability that

(i) both the balls are red
(ii) One ball is white
(iii) the balls are of the same colour
(iv) One is white and other red.

There are 20 balls in the bag out of which 2 balls can be drawn in $^{20}C_2$ way

$\frac{20 \times 19}{2} = 190$

Total number of elementary event = 190

(i) Here 9 balls are red then in which 2 balls drawn can be drawn in $^9C_2 = \frac{9 \times 8}{1 \times 2} = 36$

So required probability = $\frac{36}{190} = \frac{18}{95}$

(ii) Here no of white 7 ball is drawn one ball and rest no of balls 9+4 = 13 balls

Drawn one ball no of way drawn $^7C_1 = 7$

13 balls in one ball drawn then no of way drawn ball $= {}^{13}C_1 = 13$

Favourable event = $^7C_1 \times {}^{13}C_1 = 7 \times 13$

Hence required probability = $\frac{^7C_1 \times {}^{13}C_1}{^{20}C_2} = \frac{7 \times 13 \times 2}{20 \times 19} = \frac{91}{190}$

(14) According to question all balls are same colour i.e. including some colour two balls are red either 2 white & black, some colour two balls

Two balls drawn are of the same colour means either both are red or both are white or both black. Out of 9 red balls two red balls can be drawn in $^9C_2$ way. Similarly, two white can be drawn from 7 white balls in $^7C_2$ ways two black balls from 4 black ball in $^4C_2$ ways. Therefore, the number of way of drawing of the same colour

$^9C_2 + {}^7C_2 + {}^4C_2$

$\frac{9 \times 8}{2} + \frac{7 \times 6}{2} + \frac{4 \times 3}{2} = 36 + 21 + 6 = 63$

So required probability = $\frac{63}{190}$

(15) Favourable number of elementary event = $^7C_4$

So required probability = $\frac{63}{190}$

(10) A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box at random. Find the probability that (i) all will be blue (ii) at least one will be green.

Here total no of marbles = 10 + 20 + 30 = 60

5 marbles drawn at random in 60 ways thus $n = {}^{60}C_5$

(i) 5 marble drawn out of 20 blue marble drawn in any one $= {}^{20}C_5$ ways

Here required probability $= \frac{{}^{20}C_5}{{}^{60}C_5} = \frac{11}{5313}$

(ii) At least one will be green =

1 − probability that no ball is green.

= 1 − probability that 5 balls drawn are red or blue.

$= 1 - \frac{{}^{30}C_5}{{}^{60}C_5} = \frac{\text{favourable ways}}{\text{total ways}}$

(11) In a lottery there tickets are sold and ten equal prizes are announced what is the probability of not getting a prize if you buy

(i) 1 ticket (ii) two tickets (iii) ten tickets.

(i) out of 10,000 tickets, one ticket can be chosen in ${}^{10000}C_1$ 10000 ways.

There are 9990 tickets not containing a prize out of these 9990 tickets one can be chosen in ${}^{9990}C_1 = 9990$ ways.

probability of not getting prize is $\frac{9990}{10000} = \frac{999}{1000}$

(ii) 10 tickets can be drawn out of 10000 tickets in ${}^{10000}C_{10}$ ways. There are 9990 tickets without any prize out of these 10 tickets can be chosen in ${}^{9990}C_{10}$ ways. So, 10 drawn tickets will not contain any prize if they are chosen from 9990 tickets.

Hence required probability $= \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$

(12) Out of 10,000 tickets, ten tickets can be chosen in ${}^{10000}C_2$ ways. As there are 9990 tickets without any prize. Hence ten chosen tickets will not contain any prize if they are chosen from the remaining 9990 tickets. This can be done in ${}^{9990}C_{10}$ ways.

So required probability $= \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$

(13) 10 tickets can be drawn out of 10000 tickets in ${}^{10000}C_{10}$ ways. There are 9990 tickets without any prize. Out of these 10 tickets 10 can be chosen in ${}^{9990}C_{10}$ ways. So 10 chosen tickets will not contain any prize if they are chosen from 9990 tickets.

Hence required probability $= \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$

(14) The number lock of a suitcase has 4 wheels each labelled with ten digits i.e. from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is probability of a person getting the right sequence to open the suitcase.

There are ${}^{10}P_4 = 5040$ sequence of 4 distinct digits out of which there is only one sequence in which the lock opens.

∴ Required probability $= \frac{1}{5040}$

(2) An unbiased die is thrown what is the probability of getting
(i) an even number (ii) a multiple of 2
(iii) an even number or a multiple of 3
(iv) an even number and a multiple of 3.
(v) a number 2 or 4
(vi) an odd number
(vii) a number less than 5
(viii) a number greater than
(ix) a number between 2 and 6

In a single throw of a die we can get any one of the six numbers 1, 2, 3, 4, 5, 6 marked on its six faces. Therefore, the total number of elementary events associated with the random experiment of throwing a die is 6.

(1) One card is drawn from a pack of 52 cards and the 52 cards being equally likely to be drawn. Find the probability that the card drawn is
(i) an ace (ii) red (iii) either red or king
(iv) red and a king (v) a face card
(vi) a red face card (vii) a jack
(viii) a ten of clubs

Out of 52 cards, one card can be chosen in 52 ways.
So total number of elementary events = 52.

(i) There are four aces in a cards in a pack of 52 card. So one ace can be chosen in 4 ways.
Favourable number of elementary events = 4.

Hence, required probability = $\frac{4}{52} = \frac{1}{13}$

(ii) There are 26 red cards in red suit. So a pack of card one card drawn at random can be chosen in 26 way.
Favourable probability = $\frac{26}{52} = \frac{1}{2}$

(iii) There are 26 red cards including two red kings in a pack of 52 cards. Also there are 4 kings two red and two black. Therefore, card drawn will be a red card or a king if it is any one of 28 cards.

Favourable number of elementary events = 26+2 = 28

Hence, required probability = $\frac{28}{52} = \frac{7}{13}$

(iv) A card drawn will be red if card or king. If it is a red king. There are 2 red kings in a pack of 52 playing cards.

∴ Favourable event = 2
Hence required probability = $\frac{2}{52} = \frac{1}{26}$

(v) Well shuffle deck of cards contains 12 number of face card $n(E) = 12$.

Hence required probability = $\frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

(vi) 12 face card in 6 black face cards and 6 red face card. $n(E) = 6$, $n(S) = 52$
$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$

(vii) A pack of card cards contain 13 spade cards. only one card required is of spades.
Favourable number of elementary events = 1
Hence required probability = $\frac{13}{52}$

(viii) There are two suits of black cards spades and clubs. Each suit contains one card bearing number 10.

Favourable number of elementary event = 2
Hence required probability = $\frac{2}{52} = \frac{1}{26}$

The king, queen and jack of clubs are removed from a deck of 52 playing cards and the well shuffled. One card is selected from the cards. Find the probability of getting (i) a king (ii) a red

When removing king, queen and Jack of club from a deck of 52 playing cards there are 48 cards left in the deck. Out of these 48 cards one card can be chosen in 48 possible ways

Total no of elementary events = 48

(1) There are 13 heart cards are in well shuffled deck of card. Here total no of 48 cards out of which one heart card can be chosen in 13 ways.

Favourable no of elementary events = 13

Hence P(Getting a heart) = $\frac{13}{48}$

(2) When two aces are also removed then no of cards 52-2=50 Now total no of 50 cards one card chosen in 5 ways.

Favourable Numbers of elementary events = 5

Getting a king = $\frac{5}{50}$

(3) When removing king, queen and Jack, only 12 club cards are left in the deck out of these 12 club cards one club card is chosen in 12 ways.

P(Getting the 12 to $\frac{12}{48}$) = $\frac{12}{48}$

(4) There is only one 10 of hearts

Favourable number of elementary events = 1

Hence (Getting the 10 to hearts) = $\frac{1}{52}$

(5) A bag contains 3 red and 5 blue marbles A marble is drawn at random what is the probability of drawing a blue marble

There are 5 marbles in the bag, out of these 5 marbles one can be chose in 5 ways

Total number of elementary events = 5

The bag contains 5 blue marbles Therefore one blue marbles can be drawn in 5 ways

Favourable elementary events = 5

Hence P (Getting a blue marble) = $\frac{5}{5}$

(11) A bag contains 5 red balls and some blue balls if the probability of drawing a blue ball is double that of a red ball. Find the number of blue balls in the bag.

let there be x blue balls in the bag

Total no of balls in bag = 5+x

P_1 = probability of drawing a blue ball = $\frac{x}{5+x}$

P_2 = probability of drawing a red ball = $\frac{5}{5+x}$

According to question

$P_1 = 2P_2$

$\frac{x}{5+x} = \frac{2\times5}{5+x}$ $x = 10$

Hence, there are 10 blue balls in the bag.

(12) A bag contains 12 balls out of which x are white

(a) If one ball is drawn at random, what is the probability that it will be a white ball?

(b) If 6 more white balls are put in the bag, the probability of drawing a white ball will be thrice that it is now.

There are 12 balls in the bag, out of these 12 balls one can be chose in 12 ways.

Total number of elementary events = 12

Here x white balls in the bag, one ball can be selected x ways.

Favourable number of balls are selected = no of elementary events

P_1 = P (Getting a white ball) = $\frac{x}{12}$

(11) If 6 more white balls are put in the bag, then Total number of balls in the bag = 12+6=18
Number of white balls in the bag = x+6

P_1 = P(getting a white ball) = $\frac{x+6}{18}$

It is given that

$P_1 = 2P_2$

$\frac{x+6}{18} = \frac{2x}{12}$

$2x+12 = 6x$

$12 = 6x - 2x$

$12 = 4x$

$3 = \frac{12}{x}$

$x = 3$

It is known that a box of 600 electric bulbs contains 12 defective bulbs. One bulb is taken out at random from the box. What is the probability that it is a non-defective bulb?

Out of 600 electric bulbs, one bulb can be chosen in 600 ways.

Total no of elementary events = 600
no. of defective bulbs = 12
no of non-defective bulbs = 600 − 12 = 588

Favourable number of elements = 588
Hence P(Getting a non-defective bulb) = $\frac{588}{600} = q$

17 cards numbered 1, 2, 3, ..., 17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is

(i) odd (ii) even
(iii) divisible by 3 (iv) divisible by 3 and 2.

Out of 17 balls in a box, one ball drawn at random in 17 ways.

Total no of elementary events = 17

(i) There odd numbers = 9 and 17 namely {1, 3, 5, 7, 9, 11, 13, 15, 17}
Out of these 9 cards one card can be 9 ways.
Favourable number of events = $\frac{9}{17}$

Binomial distribution

The probability of the happening of an once, twice, thrice, ..., r times, ..., exactly in n trials

Let the probability of the happening of an event A in one trial be p and its probability of not happening be 1 − p = q

There are n trials and the happening of the event A 'is r times and its not happening' is n−r times.

This may be shown as follows

A, A, A, ..., A = $\bar{A}\,\bar{A}\,\bar{A}\cdots\bar{A}$

r times = n−r times ——— (1)

A indicates its happening, $\bar{A}$ its failure and P(A) = p and P($\bar{A}$) = q

We see that (1) has the probability

p.p.p.....p q.q.q.....q = $p^r q^{n-r}$

r times n−r times ——— (2)

clearly (1) is merely one order of arranging rA's

A and B = $A \cap B$

A or B (at least one of A and B) = $A \cup B$

A but not B = $A \cap \bar{B}$

Neither A nor B = $\bar{A} \cap \bar{B}$

At least one of A, B or C = $A \cup B \cup C$

Exactly one of A and B = $(A \cap \bar{B}) \cup (\bar{A} \cap B)$

All three of A, B and C = $A \cap B \cap C$

Exactly two of A, B and C = $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

(5) Twelve balls are distributed among three boxes. Find the probability that the first box will contain three balls.

Since each ball can be put into any one of the three boxes so the total number of ways in which 12 balls can be put into three boxes is 3^{12}.

Out of 12 balls, 3 balls can be chosen in $^{12}C_3$ ways and then these balls in the first box. Now remaining 9 balls are to be put in the remaining two boxes. This can be done in 2^9 ways.

The total no of ways in 3 balls can be put in the 1st box and the remaining 9 in other two boxes is $^{12}C_3 \times 2^9$

Hence, required probability $= \frac{^{12}C_3 \times 2^9}{3^{12}}$

Independent events

Let A and B are two events then probability of $P(A \cap B) = P(A) \cdot P(B)$

$P(A \text{ or } B) = P(A) + P(B)$

conditional probability

Let A and B are events of a sample space S. If $P(A) \neq 0$ then conditional probability of the event A given B denoted by $P(A|B)$ is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Theorem: If the events A and B defined on a sample space S of random experiment are independent, then

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B)$$

Proof: Let A and B are given to be independent events

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

	Verbal description of event	Equivalent set theoretic notation
①	Not A	$\bar{A}$
②	A or B (at least one of A or B)	$A \cup B$
③	A and B	$A \cap B$
④	A but not B	$A \cap \bar{B}$
⑤	Neither A nor B	$\bar{A} \cap \bar{B}$
⑥	At least one of A, B or C	$A \cup B \cup C$
⑦	Exactly one of A and B	$(A \cap \bar{B}) \cup (\bar{A} \cap B)$
⑧	All three of A, B and C	$A \cap B \cap C$
⑨	Exactly two of A, B and C	$(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

Let A, B and C are three arbitrary events. Find the expression for the events noted below in the context of A, B and C.

① only A are occurs = $A \cap \bar{B} \cap \bar{C}$

② both A and B, but not occur = $A \cap B \cap \bar{C}$

③ All the three events occurs = $A \cap B \cap C$

④ At least one occurs = $A \cup B \cup C$

⑤ At least two occur
$(A \cap B) \cup (B \cap C) \cup (A \cap C) \cup (A \cap B \cap C)$

⑥ one and no more occurs
$(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

⑦ Two and no more occurs
$(A \cap B \cap \bar{C}) \cup (\bar{A} \cap B \cap C) \cup (A \cap \bar{B} \cap C)$

⑧ None occurs $\bar{A} \cap \bar{B} \cap \bar{C} = \overline{A \cup B \cup C}$

⑨ Not more than two occurs
$(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C) \cup (A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) \cup (\bar{A} \cap \bar{B} \cap \bar{C})$

① From a group of 2 boys and 3 girls, two children are selected. Find the sample space associated to this random experiment.

A group contains two boy and three girls (B_1 and B_2) represents two boy and G_1, G_2 & G_3 are represents three girls. Two children are selected in both groups.

Total no of children = 2+3 = 5 in two children selected. $5P_2 = \frac{5!}{3!} = 20$

selection of children is two way

$S = \{(B_1, G_1), (B_1, G_2), (B_1, G_3), (B_2, G_1), (B_2, G_2), (B_2, G_3), (G_1, G_2), (G_1, G_3), (G_2, G_3)\}$

Theorem (ii) A and B are two events associated with a random experiment and S = sample space. If C is an event such that $P(C) \neq 0$

$P(A \cup B | C) = P(A|C) + P(B|C) - P(A \cap B | C)$

If A and B are mutually exclusive events. Then
$P(A \cup B | C) = P(A|C) + P(B|C)$

We know that by conditional probability
$P(A \cap B) = P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(A) \cdot P(B|A)$

$P(A \cup B | C) = \frac{P[(A \cup B) \cap C]}{P(C)}$

$= \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)}$

$P(A \cup B | C) = \frac{P(A \cap C) + P(B \cap C) - P[(A \cap C) \cap (B \cap C)]}{P(C)}$

$\frac{P(A \cup B)}{P(C)} = \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P(A \cap B \cap C)}{P(C)}$

$P(A|C) + P(B|C) - P(A \cap B | C)$

② If A and B are two events such that $P(A) = .5$, $P(B) = .6$ and $P(A \cup B) = .8$. Find $P(A|B)$ and $P(B|A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.8 = .5 + .6 - P(A \cap B)$

$.8 - 1.1 = -P(A \cap B)$

$-.3 = -P(A \cap B)$ $\quad P(A \cap B) = .3$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{5}} = \frac{5}{6}$

② If A and B are two events such that $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{2}$ and $P(B/A) = \frac{5}{6}$ find $P(A \cap B)$ and $P(A \cup B)$

$P(A \cap B) = P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

③ On her vacation Veena visits four cities A, B, C and D in a random order what is the probability that she visits

① A before B ② A before B and B before C ③ A first one is last ④ A either first or second

Veena can visit four cities A, B, C and D in any one of the following orders

ABCD, ACBD, ADBC, BACD, BCAD, BDAC, CABD, CBAD, CDAB, DABC, DBCA, DCAB

Total no of ways in which Veena can visit four cities A, B, C and D is $4! = 24$

① Out of 24 ways Veena visits city A before city B in the following

ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB

So there are 12 ways in which Veena can visit city A before city B

Required probability $= \frac{12}{24} = \frac{1}{2}$

④ Veena can visit A before B and B before C in any one of the following ways

ABCD, ADBC, ADBC, DABC

Required probability $= \frac{4}{24} = \frac{1}{6}$

⑤ Veena can visit city A first and city B last in any one of the following way

ACDB, ADCB

Required probability $= \frac{2}{24} = \frac{1}{12}$

⑥ Veena can visit city A as a city and then

BACD, BADC, BCAD, BCDA, BDBC, BDCA

CABD, CADB, CBAD, CBDA, CDAB, CDBA

DABC, DACB, DBAC, DBCA, DCAB, DCBA

Required probability $= \frac{12}{24} = \frac{1}{2}$

A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card

Let A = getting a king

B = getting a heart card

C = getting a red card

No of kings = 4

No of heart card = 13

No of red card = 26

$P(A) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$P(B) = \frac{^{13}C_1}{^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

$P(C) = \frac{^{26}C_1}{^{52}C_1} = \frac{26}{52} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{52}$, $P(B \cap C) = \frac{13}{52}$

$P(A \cap C) = \frac{2}{52}$, $P(A \cap B \cap C) = \frac{1}{52}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{1}{13} + \frac{1}{4} + \frac{1}{2} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$ Ans

Find the probability of drawing a one-rupee coin from a purse with two compartments one of which contains 3 five-rupee coins and 2 one-rupee coins and other contains 2 fifty paise coins and 3 one-rupee coins.

Let A one rupee coin be drawn in two mutually exclusive ways.

① Selecting compartment I and then drawing a rupee coin from it

② Selecting compartment II and drawing a rupee coin from it

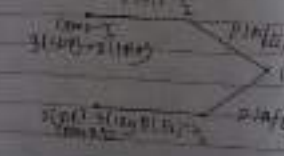
Let E_1 = the 1st compartment of the purse is chosen

E_2 = the 2nd compartment of the purse is chosen

$P(E_1) = P(E_2) = \frac{1}{2}$

$P(A|E_1)$ = probability drawing a rupee coin given that the 1st compartment of the purse is chosen

$= \frac{3}{5}$



$P(A|E_2)$ = probability of drawing a rupee coin given that the second compartment of the purse is chosen

$P(A|E_2) = \frac{1}{2}$

By the law of total probability

$P(A) = P(E_1) P(A|E_1) + P(E_2) P(A|E_2)$

$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{10} + \frac{1}{4} = \frac{12+5}{40} = \frac{17}{40}$

A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.

Let S be the sample space. Then $n(S) = 52$

Let E_1 = event of getting a spade

And E_2 = event of getting a king

Then $E_1 \cap E_2$ = event of getting a king of spade

Then $n(E_1) = 13$ $P(E_1) = \frac{13}{52} = \frac{1}{4}$

$n(E_2) = 4$ $P(E_2) = \frac{4}{52} = \frac{1}{13}$

and $n(E_1 \cap E_2) = 1$ $P(E_1 \cap E_2) = \frac{1}{52}$

$\therefore P$(getting a spade or a king)

$P(E_1 \cup E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{13+4-1}{52} = \frac{16}{52}$

$= \frac{4}{13}$ Ans

Two cards are drawn at random from a well shuffled pack of 52 cards. What is the probability that either both are red or both are kings.

Two cards drawn from a pack of 52 cards

Let E_1 = event that both are red cards

and E_2 = event that both are kings

$n(E_1) = {}^{26}C_2$ $P(E_1) = \frac{13 \times 25}{52 \times 51}$

$n(E_2) = {}^{4}C_2$ $P(E_2) = \frac{1}{221}$

$P(E_1) = \frac{25}{102}$

$A \cap B = B \cap A$

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)}$

$P(A|E_i) = \frac{P(A \cap E_i)}{P(E_i)}$

$P(E_i) P(A|E_i) = P(A \cap E_i)$

$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + \cdots + P(A \cap E_n)$

$P(A) = \frac{P(E_1 \cap A)}{1} + \frac{P(E_2 \cap A)}{1} + \cdots + \frac{P(E_n \cap A)}{1}$ ——— (1)

$= \sum_{i=1}^{n} \frac{P(E_i) P(A|E_i)}{P(A)}$

$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)}$

$= \sum_{i=1}^{n} P(E_i) P(A|E_i)$

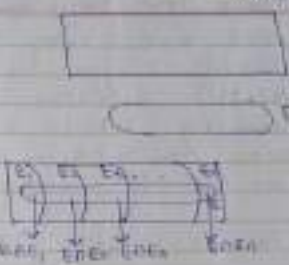
$E_i \cap E = E \cap E_i$

$P(E_i|E) = P(E \cap E_i)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ Bayes' theorem

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(A \cap B) = P(A|B) P(B)$



$S = E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n$

$E = E \cap S$

$E = E \cap (E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n)$

$E = (E \cap E_1) \cup (E \cap E_2) \cup (E \cap E_3) \cup \cdots \cup (E \cap E_n)$

$P(E) = P\{(E \cap E_1) \cup (E \cap E_2) \cup (E \cap E_3) \cup \cdots \cup (E \cap E_n)\}$

$P(E) = P(E \cap E_1) + P(E \cap E_2) + P(E \cap E_3) + \cdots + P(E \cap E_n)$

$P(E) = P(E_1) P(E|E_1) + P(E_2) P(E|E_2) + P(E_3) P(E|E_3) + \cdots + P(E_n) P(E|E_n)$

$P(E) = \sum_{i=1}^{n} P(E_i) P(E|E_i)$ ——— (1)

$P(E_i|E) = \frac{P(E_i \cap E)}{P(E)}$ $\quad P(E_i \cap E) = P(E \cap E_i)$

$P(E_i|E) = \frac{P(E_i \cap E)}{\sum_{i=1}^{n} P(E_i) P(E|E_i)} = \frac{P(E \cap E_i)}{\sum_{i=1}^{n} P(E_i) P(E|E_i)}$

$P(E_i|E) = \frac{P(E_i) P(E|E_i)}{\sum_{i=1}^{n} P(E_i) P(E|E_i)}$

(1) Assume that the probability of an individual coal miner being killed in a mine accident during a year is $\frac{1}{2000}$ use appropriate statistical distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

Here $p = \frac{1}{2000}$, $n = 200$

$m = np = 200 \times \frac{1}{2000} = \frac{1}{10}$

$P(\text{at least one}) = P(1) + P(2) + P(3) + \ldots + P(200)$

$= 1 - P(0)$

$= 1 - \frac{e^{-m} m^0}{0!} = 1 - e^{-0.1}$

$= 1 - 0.9048 = 0.0952$

(2) Using poisson distribution find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive tests.

probability of the ace of spades $= p = \frac{1}{52}$, $n = 104$

$m = np = \frac{104}{52} = 2$

$P(x) = \frac{e^{-m} m^x}{x!}$, $P(2) = \frac{e^{-2} 2^2}{2!} = \frac{e^{-2} 2^2}{2!}$

$P(\text{at least once}) = P(1) + P(2) + P(3) + \ldots + P(104)$

$= 1 - P(0)$

$= 1 - \frac{1}{e^2} = 1 - 0.135 = 0.865$

(3) In a certain factory packing cycle type there is a small chance of 1 in 500 tyres to be defective. The tyres are sold in lots of 10. Using poisson distribution, calculate the approximate number of lots of containing no defective one defective and two defective tyres respectively in a consignment of 10,000 lots.

$p = \frac{1}{500}$, $n = 10$

$m = np = 10 \times \frac{1}{500} = \frac{10}{500} = 0.02$

$P(x) = \frac{e^{-m} m^x}{x!}$

S no	probability of defective	no of lots containing defective
1.	$P(0) = \frac{e^{-0.02}(0.02)^0}{0!} = e^{-0.02}$ $= 0.9512$	10000×0.9512 $= 9512$ lots
2.	$P(1) = \frac{e^{-0.02}(0.02)^1}{1!}$ $= 0.9512 \times 0.02$ $= 0.019024$	$10,000 \times 0.019024 =$ 190 lots
3.	$P(2) = \frac{e^{-0.02}(0.02)^2}{2!}$ $= 0.9512 \times 0.0004 = 0.0003804$	same poisson 3.804

(4) Suppose 2% of bolts made by a machine are defective the defects occurring at random during production of bolts are packed in box. Find (i) Binomial distribution / exact probability (ii) poisson probability

$p = 3\% = \frac{3}{100} = 0.03$

(a) $p + q = 1$

$q = 1 - p = 1 - 0.03 = 0.97$

Hence the probability for 5 defective bulb in a lot of 50

$^{50}C_5 (.03)^5 (.97)^{45}$ Ans

(b) poisson approximation

$m = np =$

$= 50 \times \frac{3}{100} = \frac{3}{2} = 1.5$

Required poisson approximation

$\frac{m^r e^{-m}}{\lfloor r} = \frac{(1.5)^5 e^{-1.5}}{\lfloor 5}$

$= 0.0471 = Ans$

The number of arrivals of customers during any day follows poisson distribution with a mean 4. Find (i) the probability that the total number of customers on two days selected at random is less than 3.

Here $m = 5$.

$P(x) = \frac{e^{-m} m^x}{\lfloor x}$

According to question the number of customers on two days $= 1.0.5$

First day	Second day	Total
0	0	0
0	1	1
1	0	1

Required probability $= P(0)P(0) + P(0)P(1) + P(1)P(0)$

$= \frac{e^{-5}(5)^0}{\lfloor 0} \frac{e^{-5}(5)^0}{\lfloor 0} + \frac{e^{-5}(5)^0}{\lfloor 0} \frac{e^{-5}(5)^1}{\lfloor 1} + \frac{e^{-5}(5)^1}{\lfloor 1} \frac{e^{-5}(5)^0}{\lfloor 0}$

$\frac{e^{-5}(5)^1}{\lfloor 1} \times \frac{e^{-5}(5)^1}{\lfloor 1}$

$= 1 + 5e^{-5}$

$e^{-5} e^{-5} + e^{-5} e^{-5} + e^{-5} e^{-5}$

$= e^{-10} [1 + 5 + 5] = 11 e^{-10} = 11 \times 4.53999 \times 10^{-5}$

$= 4.994 \times 10^{-5}$

If the probability that an individual suffers a bad reaction from a certain injection is 0.001 determine the probability that out of 2000 individuals (a) exactly 3 (b) more than 2 individuals (c) more than one individual will suffer a bad reaction

Solution $p = 0.001, n = 2000$

$m = np = 2000 \times 0.001 = 2$

$p(x) = \frac{e^{-m} m^x}{\lfloor x} = \frac{e^{-2} 2^x}{\lfloor x} = \frac{1}{e^2} \times \frac{2^x}{\lfloor x}$

$p(\text{exactly 3}) = p(3) = \frac{1}{e^2} \times \frac{2^3}{(3)(2)(1)} = \frac{4}{3}$

Mean deviation

Show that in a poisson distribution with unit mean and the mean deviation about the mean is $\left(\frac{2}{e}\right)$ times the standard deviation.

Solution :- $p(x) = \frac{e^{-m} m^x}{\lfloor x}$ mean $m = 1$

i.e. $m = 1$ and S.D $= \sqrt{m} = \sqrt{1} = 1$

Here $p(x) = \frac{e^{-m}}{\lfloor x} m^x = \frac{e^{-1}}{\lfloor x} 1^x = \frac{1}{e} \frac{1}{\lfloor x}$

x	p(x)	|x-1|	p(x) |x-1|
0	$\frac{1}{2}$	1	$\frac{1}{2}$
1	$\frac{1}{4}$	0	0
2	$\frac{1}{8}$	1	$\frac{1}{8}$
3	$\frac{1}{16}$	2	$\frac{2}{16}$
4	$\frac{1}{32}$	3	$\frac{3}{32}$
...	...	...	...
x	$\frac{1}{2^{x+1}}$	x-1	$\frac{(x-1)}{2^{x+1}}$

Mean deviation $= \sum p(x)|x-1|$

$= \frac{1}{2} + 0 + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \cdots + \frac{x-1}{2^{x+1}}$

$= \frac{1}{2}\left[1 + 0 + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \cdots + \frac{x-1}{2^x}\right]$

$= \frac{1}{2}\left[1 + \left(\frac{1}{4} - \frac{1}{4}\right) + \left(\frac{2}{8} - \frac{1}{8}\right) + \left(\frac{3}{16} - \frac{2}{16}\right) + \left(\frac{4}{32} - \frac{3}{32}\right) + \cdots\right]$

$= \frac{1}{2}\left[1 + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots + \frac{1}{2^x}\right]$

$= \frac{1}{2}\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^x}\right]$

$= \frac{1}{2}\left[\frac{\frac{1}{2}\left(1 - \frac{1}{2^x}\right)}{1 - \frac{1}{2}}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{2^x}\right] = \frac{1}{2}$

Recurrence formula for poisson distribution

$$P(x) = \frac{e^{-m} m^x}{\lfloor x}$$ ——— (1)

$$P(x+1) = \frac{e^{-m} m^{x+1}}{\lfloor x+1}$$ ——— (2)

On dividing (2) by (1) we get

$$\frac{P(x+1)}{P(x)} = \frac{e^{-m} m^{x+1}}{\lfloor x+1} \times \frac{\lfloor x}{e^{-m} m^x} = \frac{m}{x+1}$$

$$\frac{P(x+1)}{P(x)} = \frac{m}{x+1}$$

$$P(x+1) = \frac{m}{x+1} P(x)$$

If the variance of the poisson distribution is 2 find the probabilities for $x = 0, 1, 2, 3$ from the recurrence relation of the poisson distribution. Also find $P(x>3)$

Variance $= m = 2$

Mean $= 2$

$P(x+1) = \frac{m}{x+1} P(x) = \frac{2}{(x+1)} P(x)$

$P(x) = e^{-m}$

$= 0.1353$

If $x = 0$, $P(1) = \frac{2}{0+1} P(0) = 2 \times 0.1353$

$= 0.2706$

$x = 1$, $P(2) = \frac{2}{1+1} P(1) =$

$= \frac{2}{2} \times 0.2706 = 0.2706$

$x = 2$, $P(3) = \frac{2}{2+1} = \frac{2}{3} P(2) = \frac{2}{3} \times 0.2706 = 0.1804$

$x = 3$, $P(4) = \frac{2}{3+1} P(3) = \frac{1}{2} \times 0.1804 = 0.0902$

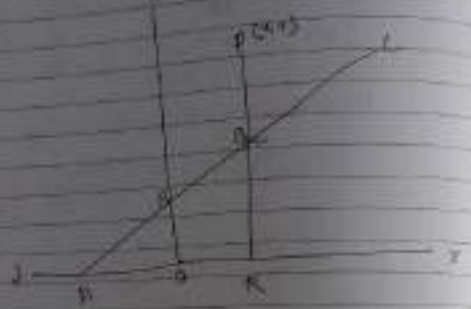
$P(x>3) = 0.1353 + 0.2706 + 0.2706 + 0.1804$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - [0.1353 + 0.2706 + 0.2706 + 0.1804]$

$= 1 - 0.8569 = 0.1431$

Separation (P= head, coin toss)



Probability distribution

A description giving the values of a random variable along with the corresponding probabilities is called the probability distribution of the random variable.

A random variable X takes the values $x_1, x_2, \ldots x_n$ with respective probabilities $P_1, P_2, \ldots P_n$. Then the probability distribution of X is given by

X	x_1	x_2	x_3	...	...	x_n
P(X)	P_1	P_2	P_3	...	...	P_n

(1) The probability distribution of X is defined by each $P(x_i)$, $0 \le P_i \le 1$ & $\sum_{i=1}^{n} P_i = 1$

(2) Variance is denoted by σ^2
$\sigma^2 = \sum x_i^2 P_i - \mu^2$

(3) $\mu = \sum_{i=1}^{n} P_i x_i$

(4) Standard deviation $\sigma = \sqrt{\text{variance}}$

Q.=> Find the mean, variance and standard deviation of the number of tails in two tosses of a coin.

Solution:- In two tosses of a coin, the sample space is given by
$S = \{HH, HT, TH, TT\}$
$n(S) = 4$
every simple outcome has a probability of

Let X = num